INDIAN AGRICULTURAL
RESEARCH INSTITUTE, NEW DELHI

I.A.R I.6.
GIP NLK—H-3 I.A.R.I.—10-5-55—15,000

28 FEB 1949

Journal
of
The Franklin Institute

DEVOTED TO

SCIENCE AND THE MECHANIC ARTS

EDITOR, HENRY BUTLER ALLEN, MET.E., D.Sc.
EDITORIAL ASSISTANT, NANCY S. GLENN, M.A.

ASSOCIATE EDITORS·

WILDER D. BANCROFT, PH.D.
C. B. BAZZONI, PH D
ARTHUR L. DAY, SC.D.
R. EKSERGIAN, PH.D
PAUL D. FOOTE, PH.D.
W. J. HUMPHREYS, PH D.
C. E. K. MEES, D.SC.
WILLIAM B. MELDRUM, PH.D
HENRY C. SHERMAN, SC.D.
W. F. G. SWANN, D.SC.
HUGH S. TAYLOR, D.SC
A. F. ZAHM, PH D.
JOHN ZELENY, PH D.

COMMITTEE ON PUBLICATIONS

G. H. CLAMER, CHAIRMAN
RICHARD HOWSON
C. L. JORDAN
LIONEL F. LEVY
MALCOLM LLOYD, JR.
HIRAM S. LUKENS
RICHARD H. OPPERMANN
WINTHROP R. WRIGHT

VOL. 246.—Nos. 1471–1476
(123d Year)

JULY–DECEMBER, 1948

PHILADELPHIA
THE FRANKLIN INSTITUTE
Benjamin Franklin Parkway at 20th Street
1948

◆

LANCASTER PRESS, INC., LANCASTER, PA.

13 SEP 1948

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 246 JULY, 1948 No. 1

LANGUAGES FOR SCIENCE.

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER.[2]

As everyone knows, the scientific world had at one time a universal language -Latin. A scientific publication from any country could be read without translation by any scientist, no matter what his nationality. Direct communication, verbally or by letter, was possible for all scientists. For five centuries this internationality existed, to be gradually replaced by a narrow nationalism. Since 1850 practically no scientific work has been published in Latin. In fact, the twentieth century has seen a growing provincialism in the face of a glorious development in communication and transportation. It has become the custom of even the smallest countries to expect their scientists to publish in the national tongue, though such a step reduces the usefulness of the publication.

As a result, the scientist of today is isolated from his fellows in other lands. Even though he has mastered English, French, and German, he may be unable to understand a scientist from South America, Italy, Poland, or Russia. An important research, published in Dutch or Swedish or Japanese, may be entirely overlooked, with resulting loss to the whole scientific world. Language difficulties intrude also in every attempt at international standardization and in all international scientific congresses.

The revolt against Latin as the language of science was part of the humanistic revolt against class privilege (1).[3] As such it was commend-

---

[1] Massachusetts Institute of Technology, Cambridge, Mass.

[2] Brown University, Providence, R. I.

[3] The boldface numbers in parentheses refer to the references appended to this paper

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute

TABLE I.—*Languages Used for Scientific Publications.*

Author		Important Publication	Date	Language
Italy:				
Leonardo of Pisa	1175–1250	Liber abaci	1202	Latin
Galileo Galilei	1564–1642	Saggiatore	1623	Italian
Francesco Grimaldi	1618–1663	Physico-mathesis de lumine, coloribus, et iride	1665	Latin
G. Saccheri	1667–1733	Euclides ab omni naevo vindicatus	1733	Latin
Luigi Galvani	1737–1798	De viribus electricitatis in motu musculari commentarius	1791	Latin
L. Cremona	1830–1903	Various papers on geometry	—	Italian
France:				
Franciscus Vieta	1540–1603	In artem analyticam isogoge	1591	Latin
		De numerosa potestatum resolutione	1600	Latin
		De aequationum recognitione et emendatione	1615	Latin
Albert Girard	1595–1632	Invention nouvelle en l'algébra	1629	French
Girard Desargues	1593–1662	Brouillon project d'une atteinte aux événemens des rencontres d'un cône avec un plan	1639	French
Réné Descartes	1596–1650	Discours de la méthode	1637	French
P. de la Hire	1640–1718	Sectiones conicae	1685	Latin
G.F.A. l'Hôpital	1661–1704	Analyse des infiniment petits	1696	French
A. de Moivre	1667–1754	Miscellanea analytica	1730	Latin
Pierre Bouguer	1698–1758	Traité d'optique sur la gradation de la lumière	1760	French
A. C. Clairaut	1713–1765	Théorie de la figure de la terre	1743	French
J. d'Alembert	1717–1783	Traité de dynamique	1743	French
C. A. Coulomb	1736–1806	Various papers	—	French
J. L. Lagrange	1736–1813	Mécanique analytique	1788	French
Gaspard Monge	1746–1818	Géometrie descriptive	1798	French
P. S. Laplace	1749–1827	Mécanique céleste	1799–1825	French
A. L. Cauchy	1789–1857	Cours d'analyse	1821	French
J. B. J. Fourier	1758–1830	La théorie analytique de la chaleur	1822	French
Great Britain:				
William Gilbert	1540–1603	De magnete	1600	Latin
John Napier	1550–1617	Mirifici logarithmorum canonis descriptio	1614	Latin
Thomas Harriot	1560–1621	Artis analyticae praxis	1631	Latin
Francis Bacon	1561–1626	Novum organum	1620	Latin

TABLE I.—*Continued.*

Author		Important Publication	Date	Language
John Wallis	1616–1703	Arithmetica infinitorum	1655	Latin
Isaac Barrow	1630–1677	Various papers	—	Latin
Robert Hooke	1635–1703	Micrographia	1665	Latin
Roy. Soc. of London	1622–	Phil. Trans.	1665–	English
Isaac Newton	1642–1727	Principia	1687	Latin
Brook Taylor	1685–1731	Methodus incrementorum directa et inversa	1715	Latin
Colin Maclaurin	1698–1746	Treatise on fluxions	1742	English
Thomas Simpson	1710–1761	New treatise on fluxions	1737	English
Robert Smith	—	Compleat system of optiks	1738	English
W. R. Hamilton	1805–1865	Elements of quaternions	1866	English
Arthur Cayley	1821–1895	Various papers	—	English
Germany:				
Johann Kepler	1571–1630	Mysterium cosmographicum	1596	Latin
G. W. Leibniz	1646–1716	De arta combinatoria	1666	Latin
Berlin Academy	1700–		1700–	Latin
Leonhard Euler	1707–1783	700 Scientific papers	—	Latin
J. H. Lambert	1728–1777	Photometria	1760	Latin
C. F. Gauss	1777–1855	Papers to about 1830	—	Latin
		Later papers	—	German
A. F. Möbius	1790–1868	Papers	—	German
C. G. J. Jacobi	1804–1851	Papers	—	Latin
Other Countries:				
Nicolaus Copernicus (Poland)	1473–1543	De revolutionibus orbium coelestium	1543	Latin
Tycho Brahe (Denmark)	1546–1601	De nova stella	1573	Latin
Christian Huygens (Netherlands)	1629–1695	Papers	—	Latin Dutch French
Johann Bernoulli (Switzerland)	1667–1748	Lectiones mathematicae de methodo integralium	1742	Latin
Daniel Bernoulli	1700–1782	Hydrodynamica	1738	Latin
Wolfgang Bolyai (Hungary)	1775–1856	Papers	—	Latin
P. L. Tchebychef (Russia)	1821–1894	Papers	—	French

able. But the use of the national languages for scientific purposes merely replaces a previous class distinction by a much worse national distinction. The present state of affairs shows very clearly that the scientific world has been moving in the wrong direction linguistically for the past century. Something should be done. In the following pages, an attempt is made to outline the problem in an unbiased manner and to study possible solutions.

LATIN.

In the middle ages, Latin was the language of culture throughout Europe. Thus when educated people began to write about scientific subjects, they wrote in Latin (2). Leonardo (Italy, 1175–1250) in writing his treatise on algebra in 1202 used Latin, as did Roger Bacon (England, 1214–1292) in advocating the experimental method.

Latin was employed in nearly all scientific publications up to 1630, and in some cases it was used even after 1830. Since the study of Latin was an important part of the schooling of every educated person, there was no trouble with noninternationality in scientific publications. That the use of a single language for science was an advantage can hardly be doubted by anyone who examines a part of the voluminous scientific correspondence that was carried on throughout Europe (3). It was in Latin. While the merchants of Amsterdam, London, Venice, and Lisbon might tear their hair over language difficulties, the scientists were serenely free from such impediments. But by the twentieth century this freedom had disappeared. Now, even in peace times, it seldom occurs to a scientist to compare notes with anyone outside his own country.

Table I lists some of the great physicists and mathematicians. In most cases, an important book of each is noted and its language is stated. In this way, the table gives an idea of when the transition occurred from Latin to the national language. There is, of course, no sudden break. The language used in a scientific work depended to some extent on the class of readers to whom it was addressed and the country in which it was published. For example, William Gilbert, court physician to Queen Elizabeth, wrote his celebrated treatise on magnetism (1600) in Latin; but Robert Norman, writing a practical description of the compass ("The newe attractive," 1581), used English.

Also, the transition did not occur simultaneously in the different countries of Europe. For example, Jacobi (Germany, 1804–1851) wrote most of his important mathematical papers, throughout his life, in Latin. But the great French mathematicians were employing French exclusively, two centuries before Jacobi. Desargues (1593–1662), Descartes (1596–1650), de Moivre (1667–1754), Lagrange (1736–1813), Laplace (1749–1827), Cauchy (1789–1857), all used French. Another example is the famous treatise on photometry by Lambert (Germany) published in Latin in 1760, and the equally important work on the same subject by Bouguer (France) which was published in the same year but in the author's native language.

One concludes from Table I, then, that:

1. Scientists in all European countries employed Latin before about 1630,
2. Practically no Latin was used in science after 1850,
3. The transition from Latin to the national speech occurred at different times in different countries. It took place first in France and last in Germany. Table II indicates the approximate dates.

TABLE II.—*Approximate Date of Transition from Latin to a National Language for the Publication of Scientific Works.*

Country	Date
France	1630
England	1700
Germany	1830

If an international language is desirable in science, why was Latin abandoned after it had been used for several centuries? The change was caused by the wave of humanism that swept Europe (1). The special privileges of the aristocracy were to be abolished and the common man was to be given a "new deal." This idea had been felt strongly by that radical, Galileo (1564–1642), who wrote his scientific works in the language of the people. The change in Italy, however, did not in general occur until much later, largely because of the influence of the Church.

The feeling grew that the results of scientific research should not be the property of the learned but should be written in a language that could be understood by the common man. A potent instrument in making the transition was the national academies. The French Academy was founded in 1666. It published in French. The Royal Society of London was established in 1662 and its *Philosophical Transactions* have been published in English from 1665 to the present. The Berlin Academy (1700–), however, published for many years in Latin.

SYNTHETIC LANGUAGES.

Latin is dead as a scientific language. It possesses all the difficulties of a modern ethnic language, plus a grammatical complexity that modern languages have long since abolished. Furthermore, classical Latin lacks thousands of technical words that are required in science and engineering. What the world needs is a universal auxiliary language that can be learned by anyone, anywhere, in a minimum of time.

The famous mathematician, Gottfried Wilhelm Leibniz (1646–1716), spent much thought on a universal scientific language. He was interested primarily in an artificial language that could be used as a precision tool in philosophical and logical research (2). He felt that Latin, which had grown up in a haphazard manner, was too blunt a tool to satisfy the needs of the future. Descartes had a similar idea in 1629, but neither man tried actually to devise such a language. Bishop Wilkins, one of the founders of the Royal Society, developed an *a priori* artificial language in 1668. Dalgarno, Sudre, Ochando, and others worked out similar projects, all of which were found to be impracticable because of the difficulty in memorizing the synthetic words, which had no resemblance to existing words in the ethnic languages (4).

The first artificial language to be learned and used by all kinds of people was Volapük, invented by a priest, Johann Martin Schleyer (4), in 1879. Its vocabulary was based largely on English, but with the words so shortened as to be unrecognizable in most cases. Despite its peculiar words and its complicated grammar, Volapük seemed to fill a human need and it achieved an immense success. Volapük clubs were formed throughout Europe and international congresses were held. Today, however, Volapük has been superseded by other languages.

Table III.—*Some Important Universal-Language Projects.*

Author	Date	Name of Language	Principal Basis for Vocabulary
Réné Descartes	1629	—	—
G. W. Leibniz	1702	—	—
George Dalgarno	1661	—	Synthetic
John Wilkins	1668	—	Synthetic
François Sudre	1817	Solresol	Synthetic
Sotos Ochando	1852	Langue universelle	Synthetic
J. M. Schleyer	1879	Volapük	English
L. L. Zamenhof	1887	Esperanto	Latin
Giuseppe Peano	1903	Latino	Latin
L. de Beaufront	1907	Ido	Latin
W. Ostwald			
O. Jespersen			
G. Peano			
L. Couturat			
L. Leau			
Edgar de Wahl	1922	Occidental	Latin
Otto Jespersen	1928	Novial	Latin
Int. Aux. Lang. Assn.	—	Iala	Latin

The principal artificial languages at the present time (4) are *Esperanto* (5), *Latino sine Flexione* (6), *Ido* (7), *Occidental* (8), and *Novial* (9) (Table III). Hundreds of other artificial languages have been developed, however, and a bibliography (10) of over 6000 items is available. Rather than discuss the differences in these languages, we shall pursue the more fruitful course of indicating what they have in common. As Otto Jespersen pointed out (9), artificial languages are converging to a common mean, so that any new development that makes use of previous research will differ from the others by less than the differences that exist between dialects of a single ethnic language. In other words, the five formulations named above (and many others) are merely *variations of a single international language.*

Some characteristics of this language are as follows:

1. The language is *phonetic*, so that difficulties in spelling and pronunciation are eliminated. In place of the large number of vowel sounds existing in French and in English, the international language employs only five:

a	always as in	*father,*
e		*met,*
i		*machine,*
o		*rose,*
u		*rule.*

The consonants are generally pronounced as in English.

2. Sounds that are difficult for any nationality are omitted. These omissions include the *th* of English, the *ü* and *ö* of German, the gutteral *ch*, and some consonant clusters that occur in Polish and Czech.

3. The vocabulary is based principally on Latin. This choice of words lightens the memorizing task for anyone who knows English, French, Italian, Spanish, or Portuguese. In Latino (6), the vocabulary is almost entirely Latin, while in Esperanto, Ido, and Novial it contains words also from German, Russian, and other sources.

4. International words are accepted. This includes many scientific words, as well as such words as *taxi, aeroplane, academy, chocolate, coffee, hotel, jazz, cabaret.*

5. Grammar is greatly simplified. Irregular verbs are abolished, grammatical gender is eliminated, case endings are generally omitted, and everything is covered by invariable rules. Peano said (6), "La grammatica minima è la grammatica nulla," though most versions of the international language have not yet reached this ideal.

All the modern versions agree on these five points and are therefore very similar. And any of them is immensely easier to learn than a natural language. Any one of the five could be adopted today by an international organization, to the great advantage of mankind.

Many people seem to feel, however, that ten or twenty years further study of the problem will suddenly reveal something different and vastly superior. It is practically certain that such is not the case. In an international language, every word is a compromise, every rule is a compromise—a compromise between logical simplicity and "naturalness" or among the needs of several language groups. The ethnic languages are not simple and they are not logical. Thus, at every step, the maker of an international language must choose between what appears obvious and natural and what is simple and regular. He must also balance the requirements of speakers of different languages so that the new language is not too difficult for any group. At every step a choice must be made and this choice is arbitrary. In Occidental (8), most choices have been made in favor of what is "natural" for speakers of the Romance languages; while in Esperanto (5) and Ido, the choice has usually favored simple, invariable rules rather than naturalness. Different people favor different compromises, which is the reason for the five variants and many others. But the important point is that no further research can possibly change this fundamental fact—*the international language is necessarily an arbitrary compromise.* Let us start using whichever of the dialects appears most attractive (11), secure in the knowledge that it is a dialect of *the* international language that will some day become the official auxiliary language of the world.

SCIENTIFIC AND POPULAR NEEDS.

The international auxiliary language was developed primarily for popular use. Evidently, the language difficulties of tourists, business men, and delegates to international gatherings would be eliminated if everyone knew two languages—his native tongue and the international auxiliary language.

The language problem confronting the human race is a very real one. As Guerard (4) says, "The fact which seems to blind us with excess of evidence is that the language obstacle is most emphatically not a theory but a condition." It is inconceivable that the human animal has insufficient brains to solve the problem. It must be done, and the only feasible method seems to be the use of an auxiliary language. This language can be taught in the elementary schools so that within a generation every literate person would know it. The study of the international language is of considerable cultural value and is an excellent preparation for the study of an ethnic language. The use of the national languages would not be affected by the study of the auxiliary language.

In contradistinction to this idea of a popular international language, we have the Descartes-Leibniz conception of a constructed language for precise scientific use. There seems to be no reason, however, why these two ideas cannot be combined. Among scientists who have worked toward the establishment of the universal scientific language are Wilhelm Ostwald, Nobel prize winner in chemistry; W. Rosenberger, Russian engineer; Giuseppe Peano, outstanding Italian mathematician; Couturat and Leau, French logicians and mathematicians; Eugene Wüster, Austrian engineer; F. G. Cottrell, American chemist and inventor.

Esperanto or Ido may not have quite the precision that Leibniz had in mind, but it is greatly superior to any of the ethnic languages in that respect. Any English-speaking person who has tried scientific writing in the international language is impressed by how vague, inexact, and sloppy is English in comparison with the constructed language. French has the reputation for logic and clarity, but it cannot compare in that respect with the artificial language.

Thus to obtain a universal scientific language, we need only take one of the variants devised for popular use. Such a procedure has the advantage that it will never get the scientist into the trouble that was experienced with Latin. Unlike Latin, the international language is easy to learn and is a language of the people.

ADVANTAGES.

The foremost advantages to be gained by the use of a universal scientific language are

1. Internationality,
2. Clarity.

The advantages of scientific publication in a single language instead of a dozen languages is obvious. The possibilities of direct communication—by telephone, cable, letter—among scientists and technicians throughout the world are of far-reaching consequence. The standardization of nomenclature, symbols, weights and measures has always been hampered by the lack of a universal language. Usually, international technical conferences are conducted in three languages (12), with consequent loss of time and frequent misunderstandings. All these difficulties would be eliminated by the use of an international auxiliary language (13).

The foregoing advantages are self evident. Less obvious is the advantage of *clarity*. In this respect, the principal difference between the constructed language and the natural languages is the smaller number of meanings attached to a given word in the constructed language. In fact, the attempt is made to have only one meaning for each word instead of the dozen or more meanings sometimes associated with an English word. Semantics (14) teaches some of the dangers to exact thought hidden in the four meanings of "is" and in multiple meanings of most other words. The constructed language does not eliminate all these possibilities of misunderstanding but it does make for more exact scientific thought. In the elimination of ambiguity, one may mention also the proposed international names of scientific concepts (15), used either with the ethnic languages or with the international language.

As a single example of the innumerable ones that might be written, consider the English preposition *of* and its equivalents in Esperanto (16).

1. "Two *of* the three generators were inoperative."
   Esperanto—*el* (part of a whole).
2. "The cost *of* electric energy is being reduced."
   Esperanto—*de*.

3. "Add 1.2 liter *of* water."
   Esperanto—*da* (measure of amount).
4. "He was glad *of* it."
   Esperanto—*pro* (because of).
5. "The article contains nothing *of* interest."
   No preposition used in Esperanto.
6. "The machinery was ordered by the City of Los Angeles."
   No preposition used in Esperanto.

SUMMARY.

Because of the vital importance of science and technology in the modern world, an international language for scientific publication and scientific intercourse is needed as never before. For five centuries, the need was satisfied by Latin; but for the past hundred years, the world has had no international language and the linguistic condition has become steadily more muddled.

During the last eighty years, a large amount of research has been devoted to devising an international auxiliary language, and a number of satisfactory projects have been developed. It is encouraging to note that all the modern variants have much in common and are really all dielects of *the* international language. In the absence of authoritative international agreement on the exact details of the universal auxiliary language, one may study any one of them with the belief that whatever variant is finally standardized will not differ much from the forms now available.

A simple example of Esperanto text is now given, with its English translation.

Esperanto:

LA LEĜO DE OHM.

Konsideru metalfadenon de unuforma sekcio, kaj kunligu ĝin al baterio. La baterio konservas konstantan elektran tension trans la ekstremajojn de la fadeno kaj produktas konstantan elektran fluon tra la fadeno. Se ni ŝanĝus la elektran tension, la fluon ŝanĝus ankaŭ.

Ni konduktas eksperimenton: ni uzas plurajn valorojn de elektra tensio kaj ni mezuras la fluon pro ĉiu tensio. Se la temperaturo de fadeno estas konstanta, kaj ni duobliĝas la tensio, ni trovas, ke duobliĝas ankaŭ la fluo. Matematike, oni skribas

$$I = V/R,$$

kie R estas konstanto, kiu dependas je la materialo de la fadeno kaj je ĝiaj dimensioj. La konstanto R estas nomata, la *rezisto.* La matematika ekvacio estas la *lego de Ohm.*

English:

OHM'S LAW.

Consider a metal wire of uniform cross-section, and connect it to a battery. The battery maintains a constant voltage across the ends of the wire and produces a constant electric current through the wire. If we change the voltage, the current changes also.

We conduct an experiment: we use several values of voltage and we measure the current for each voltage. If the temperature of the wire is constant and we double the voltage, we find that the current doubles also. Mathematically, one writes

$$I = V/R,$$

where R is a constant that depends on the material of the wire and on its dimensions. The constant R is called the *resistance*. The mathematical equation is *Ohm's law*.

•

The use of a universal language for all scientific publication may seem visionary, but it is really as feasible as it is inevitable. Obviously the change cannot be made in a day. But any scientist or engineer can help to bring it about by:

(*a*) Learning the language. Many excellent textbooks and dictionaries are available, and classes are held in the larger cities. Phonograph records and international-language broadcasts may also be used.

(*b*) Teaching the language. Most people find it to be a fascinating subject. Even divorced from any practical application, it is excellent mental training, gives an insight into the structure of English and other languages, is splendid preparation for the study of natural languages, and is fun.

(*c*) Publishing extracts in the language. An international scientific journal, giving abstracts (in the international language) of scientific papers appearing in all languages would be a step toward the eventual publication of all scientific papers in the universal language. As a still humbler beginning, one may consider the possibility of publishing IL summaries of his own papers as they appear in existing journals. The Bulletin Française des Electriciens, for instance, published abstracts in Esperanto; and it is possible that other journals will do likewise.

REFERENCES.

(1) A. Wolf, "A History of Science, Technology, and Philosophy in the 16th and 17th Centuries," Macmillan, New York, 1939; "A History . . . in the 18th Century," New York, 1939.

(2) E. T. Bell, "Men of Mathematics," Simon & Schuster, New York, 1937; "The Development of Mathematics," McGraw-Hill Book Co., New York, 1940.

(3) See, for instance, "Leibnizens mathematische Schriften," Berlin, 1849, 7 vols.

(4) L. Couturat and L. Leau, "Histoire de la langue universelle," Hachette et Cie, Paris, 1903; "Les nouvelles langues internationales," Paris, 1907; A. L. Guérard, "A Short History of the International Language Movement," Boni and Liveright, New York, 1923; H. Jacob, "A Planned Auxiliary Language," Dennis Dobson, Ltd., London, 1947; "On the Choice of a Common Language," London, 1946; Henry Sweet, Articles "Esperanto," "Universal Languages," and "Volapük," in 11th Ed. of *Encyclopaedia Britannica;* E. Drezen, "*Historio de la mondlingvo*," Leipzig, 1931; E. D. Durrant, "The Language Problem," London, 1943; L. H. Dyer, "The Problem of an International Auxiliary Language and Its Solution in Ido," Pitman, London, 1924; Frederick Bodmer, "The Loom of Language," Allen and Unwin, London, 1943.

(5) L. L. Zamenhof, "Langue internationale Esperanto," Paris, 1897; "Krestomatio de la lingvo Esperanto," Paris, 1939; C. A. Connor and D. T. Connor, "Esperanto," S. F. Vanni, New York, 1944; Max Talmey, "Practical and Theoretical Esperanto," New York, 1906; George Cox, "Esperanto, Grammar and Commentary," Esperanto Pub.

Co., London, 1944; M. C. Butler, "Step by Step in Esperanto," London, 1933; H. S. Eaton, "General Language Course," Dallas, 1934; Paul Fruictier, "Kompleta gramatiko kaj vortfarado de Esperanto," Jaslo, Poland, 1930; Henrik Seppik, "La tuta Esperanto," Budapest, 1938; K. Kalocsay and G. Waringhien, "Plena gramatiko de Esperanto," Budapest, 1938; "Edinburgh Esperanto Pocket Dictionary," Tho. Nelson & Sons, London, 1939; "Plena vortaro de Esperanto," Paris, 1934.

(6) "Key and Primer of Interlingua," Kegan Paul, London, 1931; G. Canesi and L. Weber, "Lectiones de lingua auxiliare internationale Interlingua," Lwów, 1938; Nicola Mastropaolo, "Schola et vita," Milan, 1936–1939, Academia pro Interlingua.

(7) L. de Beaufront, "Complete Manual of the Auxiliary Language Ido," Pitman, London, 1919; "Kompleta gramatiko detaloza di la linguo internaciona Ido," Luxembourg, 1925; L. H. Dyer, "Ido-English Dictionary," Pitman, London, 1924; H. Jacob, "Esperanto-Ido vortaro," London, 1934.

(8) E. Pigal, "Okzidental, die Weltsprache," Stuttgart, 1931; Ilmari Federn, "Occidental—Grundlage für einem internationalen terminologischen Code," Copenhagen, 1936; Federn, Kemp, and Haislund, "Occidental—English Vocabulary," Chapelle, 1937.

(9) Otto Jespersen, "An International Language," Allen and Unwin, London, 1928; "Novial lexike," London, 1930; Otto Jespersen and E. de Wahl, "Discussion pri international lingue," Stockholm, 1935.

(10) P. E. Stojan, "Bibliografio de internacia lingvo," Geneva, 1929.

(11) "Variantes de la lingua internationale," Iala, New York, 1947; "Iala General Report, 1945," International Auxiliary Language Assn., Lexington Ave., New York, 1945; A. Martinet and J. P. Vinay, "Questionnaire," Iala, New York, 1946.

(12) H. Shenton, "Cosmopolitan Conversation," Columbia Univ. Press, New York, 1934.

(13) Jesperson, Couturat, Lorenz, Ostwald, and Pfaundler, "International Language in Science," London, 1908; International Electrotechnical Commission, "International Electrotechnical Vocabulary," London, 1935; Eugene Wüster, "Konturoj de la lingvonormigo en la tekniko," Budapest, 1936.

(14) C. K. Ogden and I. A. Richards, "The Meaning of Meaning," Harcourt, Brace & Co., New York, 1936; Alfred Korzybski, "Science and Sanity," Lancaster Press, Lancaster, 1933; I. J. Lee, "Language Habits in Human Affairs," Harper & Bros., New York, 1941; Rudolf Carnap, "Introduction to Semantics," Harvard Univ. Press, 1942.

(15) Parry Moon and D. E. Spencer, "Internationality in the Names of Scientific Concepts: A Method of Naming Concepts," *Am. J. Phys.*, Vol. 14, p. 285 (1946). "A Proposed International Photometric System," *Ibid.*, p. 431; "Comparison of Photometric Systems," *Ibid.*, Vol. 15, p. 84 (1947); "A Study of Photometric Nomenclature," *J. Optical Soc. Am.*, Vol. 36, p. 666, (1946); "Modern Terminology for Physics," *Am. J. Phys.*, Vol. 16, p. 100 (1948).

(16) J. E. Johnsen, "Basic English," The Reference Shelf, Vol. 17, No. 1, H. W. Wilson Co., New York, 1944.

MEASUREMENT OF THE DIELECTRIC PROPERTIES OF SOILS AND WATER AT 3.2 CM. WAVE LENGTH.*

BY

A. W. STRAITON [1] AND C. W. TOLBERT.[1]

I. INTRODUCTION.

1. A knowledge of the dielectric properties of soils and water is essential in the study of the propagation of radio waves through these materials or through the atmosphere adjacent to them. The dielectric properties measured at relatively long wave lengths require reevaluating for the recently introduced wave lengths of a few centimeters.

2. This paper describes the measurement of the dielectric constant and the conductivity of certain soils and water at a wave length of 3.2 cm. Particular attention is given to the measurement of the dielectric properties of the soil on the Navy Electronics Laboratory's Radio Site near Gila Bend, Arizona.

3. The measurements described in the present paper are believed to be the first made in the microwave region by actually measuring the phase shift and attenuation of a radio wave traveling through a sample of the material. Other measurements in this region have been made by a combination of reflection and transmission measurements (**2**, **3**, **4**).[2]

II. MEASUREMENT PROCEDURE.

1. Equipment Arrangement. The phase difference equipment designed by the Electrical Engineering Research Laboratory for phase front and angle-of-arrival measurements was used (**1**). The phase difference equipment indicates changes in relative phase of X-band signals applied to its two channels. This is accomplished by heterodyning the microwave signals down to 12 mc. and applying them to a cathode ray tube. The 12-mc. signal from one of the channels is applied to the horizontal and vertical deflection plates in such a manner as to produce a circle and the 12-mc. signal from the other channel applied is to the control grid of the cathode ray tube. This produces an arc on the screen of the cathode ray tube which is displaced around the screen of the tube in proportion to the phase difference between the microwave signals applied to the two channels. The materials whose dielectric constant and conductivity were to be measured were introduced between the signal source and one of the channels. The resulting changes in

---

* Prepared under U. S. Navy Contract N5ori-136, T. O. 1.

[1] Electrical Engineering Research Laboratory, The University of Texas, Austin, Tex.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

relative phase and attenuation, produced by varying the thickness of the materials, were plotted. An example of these curves is shown in Fig. 1.

2. *Calculations.* The dielectric constant and conductivity were determined from the curves of phase shift and attenuation *versus* material thickness as follows:

The attenuation and phase shift per meter due to the presence of the sample were determined from the slope of the best straight lines drawn through the points of the graphs as shown in Fig. 1. Since the transmission loss was negligible before the sample was inserted, the attenuation, α, in nepers[3] per meter was determined directly from the graph. However, the phase shift in radians per meter due to the air path was added to the phase shift due to the insertion of the sample to get the total phase shift per meter, β.

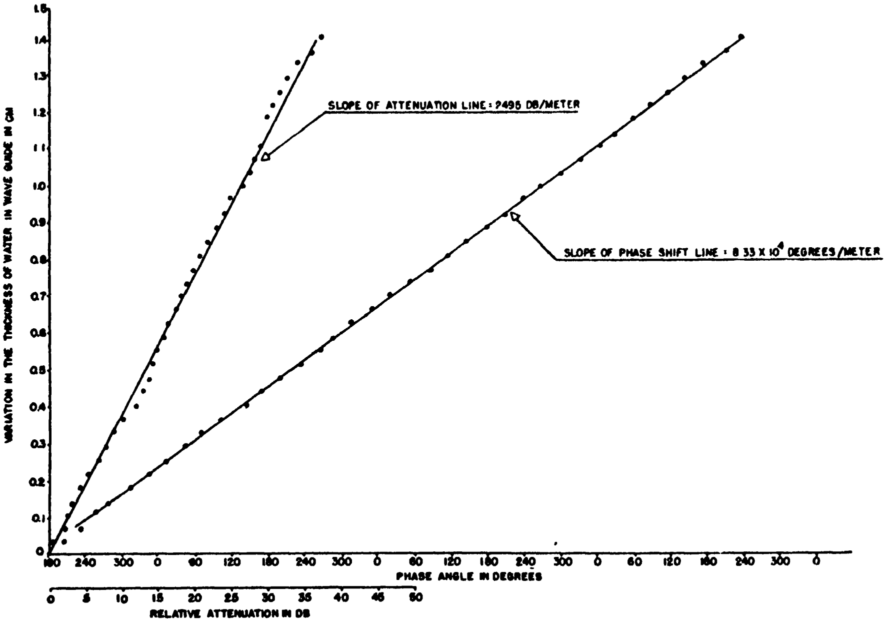


FIG. 1. Phase shift and attenuation produced by various thicknesses of distilled water.

The expressions for attenuation, α, and phase shift, β, are given in numerous textbooks as functions of the dielectric constant, k, and conductivity, σ, since k and σ are usually considered to be known data. These relationships for free space may be found in Stratton's "Electromagnetic Theory," p. 276 and for a rectangular wave guide may be found in Ramo and Whinnery's "Fields and Waves in Modern Radio,"

---

[3] Attenuation in nepers is the natural logarithm of the ratio of the field intensities and is equal to 0.115 times the attenuation in decibels.

p. 345. By simple algebraic transformations, k and σ may be expressed in terms of α and β as shown below:

Free space equations:

$$k = (\beta^2 - \alpha^2)(\lambda/2\pi)^2$$
$$\sigma = \alpha\beta\lambda/\pi n.$$

Wave guide equations:

$$k = (\lambda\beta/2\pi)^2 + (\lambda/2a)^2$$
$$\sigma = \frac{2\alpha}{n}\sqrt{k - (\lambda/2a)^2}$$

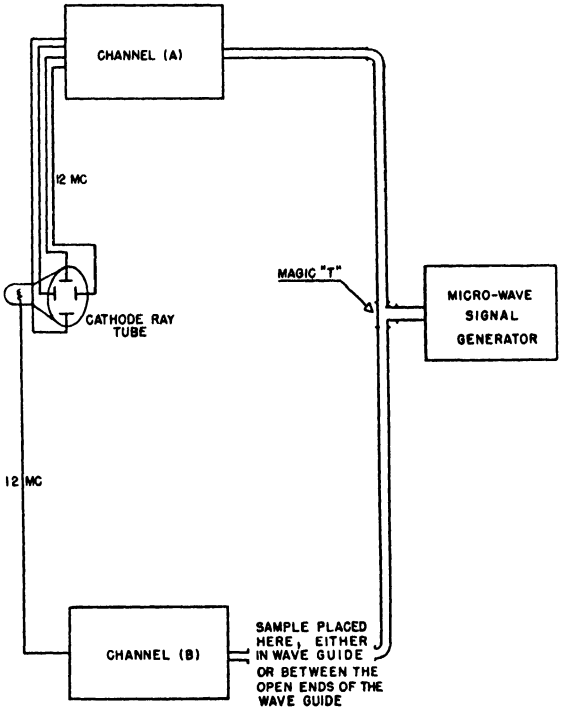

CHANNEL (A)
12 MC
CATHODE RAY TUBE
MAGIC "T"
MICRO-WAVE SIGNAL GENERATOR
12 MC
CHANNEL (B)
SAMPLE PLACED HERE, EITHER IN WAVE GUIDE OR BETWEEN THE OPEN ENDS OF THE WAVE GUIDE

FIG. 2. Apparatus for measuring dielectric characteristics.

Where λ = the wave length in air (0.0322 m.)
n = the intrinsic impedance of air (377 ohms)
a = the rectangular guide width (0.0228 m.).

3. Location of Sample. Three arrangements for measuring the samples were used as follows:

a. By supplying a signal to the two channels through wave guide from a common signal source and placing the material in the wave guide supplying one of the channels.

b. By supplying a signal to the two channels through wave guide from a common source and placing the material between open ends of the wave guide feeding one of the channels.

c. By supplying a signal to the two channels from a signal source 400 ft. from their antennas and placing the material between 3-ft. square glass plates adjacent to and in front of the antenna of one channel.

The apparatus arrangement for methods (a) and (b) was as shown in Fig. 2. The arrangement for (c) was the same except for the 400-ft. propagation path between the generator and the two channels.

4. *Reduction of Reflections.* In each of the three arrangements used for measuring the dielectric constant and conductivity of the materials, a fairly large sample was introduced before incremental changes in the thickness of the sample were made. This eliminated or reduced the effect on the measured values of phase shift and attenuation of the reflected signal from the discontinuity at the two faces of the samples.

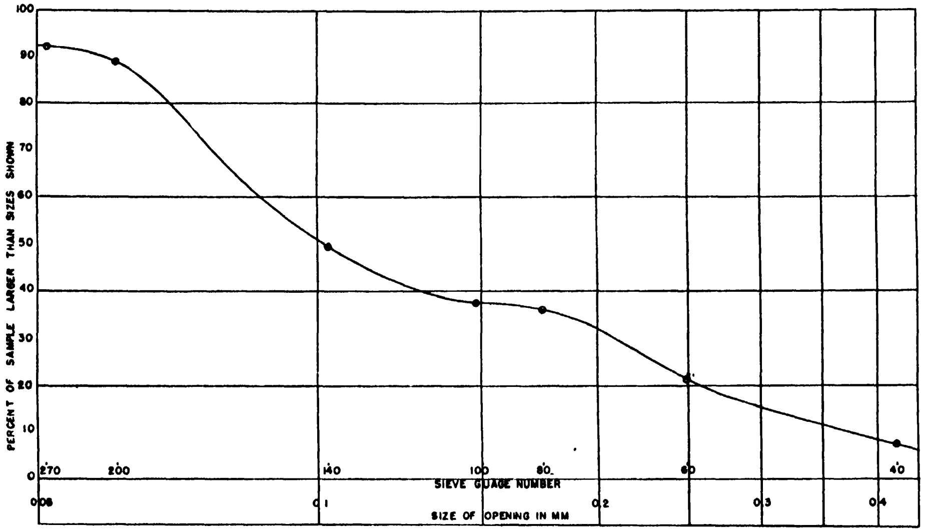


FIG. 3. Sieve analysis of Arizona soil sample.

5. *Soil Sample Methods.* The soil samples were measured by using the three methods above and also by filling a slotted section with a sample and taking off the signal for one of the channels with a traveling wave detector, thereby measuring the wave length in the soil-filled wave guide. The wave length was measured by observing the displacement of the traveling wave detector required to produce a 360° rotation of the arc on the screen of the cathode ray tube.

6. *Water Sample Methods.* The characteristics of the water samples were measured by placing the sample in a glass tray with one open end

of the wave guide directly beneath the tray and the other open end a short distance above the water. The measurements were repeated with the upper wave guide pushed into the water against the tray. Water was added to the tray in small increments and was permitted to rise in the wave guide.

7. *Agreement of Results.* The measured values of dielectric constant and conductivity agree to two significant figures for the various methods of measurement. The attenuation due to the losses in the walls of the wave guide was negligible as compared to the losses in the dielectric.

III. MEASUREMENTS OF ARIZONA SOIL.

1. *Nature of Sample.* The soil sample used was obtained on the Navy Electronics Laboratory's Desert Site near Gila Bend, Arizona. The soil was ordinarily very dry and finely powdered. Scattered pebbles and the surface vegetation were removed before taking the sample. A sieve analysis of the sand was made and the distribution of particle sizes is shown in Fig. 3.

2. *Measurements on Sample as Found.* Measurements were made on a large mass of the sand in Arizona by passing the microwaves through a 3-ft. square box containing the sample. The separation of the walls was varied to change the thickness of the sample.

Measurements were also made in Austin, Texas, by putting a sample of the same soil in a wave guide. The results of these measurements are as follows:

	Dielectric Constant	Conductivity, Mhos per meter
Wave guide measurements	3.2	0.10
Larger mass measurements .	3.2	0.10

3. *Effect of Moisture.* The properties of the Arizona soil when wet were considered for two combinations of soil and water. The first was a combination of 16 parts by weight of soil and 1 part by weight of water. This represented the condition of a medium wet soil. The second condition considered was that of 4 parts of sand to 1 part of water representing the saturated condition. The results of these measurements are as follows:

	Dielectric Constant	Conductivity, Mhos per meter
16 parts sand, 1 part water .	8.1	1.1
4 parts sand, 1 part water	. 19	2.5

4. *Effect of Particle Size.* The soil samples as separated by the sieve analysis were tested individually with the following results:

Particle Size	Dielectric Constant	Conductivity, Mhos per meter
Between 2.00 and 0.42 mm...	3.2	0.086
Between 0.42 and 0.25 mm....	3.0	0.091
Between 0.105 and 0.074 mm.. ...	3.0	0.103

5. Effect of Packing of Sample. The increasing loss with decreasing particle size seemed to be due to increased packing of the sample due to the smaller size. As a check of this, the middle sample in the list above was remeasured with extremes of packing with the following results:

	Dielectric Constant	Conductivity, Mhos per meter
Very loosely packed	2.7	0.07
Very tightly packed	3.0	0.12

IV. MEASUREMENT OF SOIL SAMPLES FROM AUSTIN, TEXAS.

1. Black Sandy Loam. A sample of sandy loam from the ground surface near the plant of the Electrical Engineering Research Laboratory was baked dry and its constants measured. One part of water was then added to ten parts of soil by weight and the measurements repeated. The results of these tests are as follows:

	Dielectric Constant	Conductivity, Mhos per meter
Black sandy loam, very dry	2.8	0.0074
Black sandy loam, 1 part of water to 10 parts of soil	4.6	0.22

2. Colorado River Sand and Gravel Mix. The dielectric characteristics of sand and gravel mix were measured with the following results:

	Dielectric Constant	Conductivity, Mhos per meter
Sand and gravel mix	3.0	0.0072

V. DISCUSSION OF SOIL SAMPLE MEASUREMENTS.

1. Comparison of Results. Ford and Oliver (2) reported a value of dielectric constant of 2 and conductivity of 0.033 mhos per meter for very dry sandy loam at 9 centimeters. This is in the same range as measured for Arizona and Austin soil samples. The Arizona soil samples showed a conductivity about 3 times as high as Ford and Oliver's value while the Austin soil conductivity was approximately one-fourth their value.

2. Iron Content of Arizona Soil. The Arizona soil contained a considerable amount of iron as shown by iron slivers sticking to a magnetized screw driver. This was evident in the larger sizes only and did not appear in the particles which passed through a 140 mesh screen (0.105-mm. openings). The relative permeability was measured at 550 kilocycles to be approximately 1.01 and was, therefore, taken as 1 in the calculations.

VI. WATER SAMPLES.

1. Measurements. Measurements of the dielectric constant and conductivity were made on distilled water and on a sample of water from the Gulf of Mexico by transmission through a shallow tray and by filling a wave guide. Identical results for the two methods were obtained and are shown below.

Sample	Dielectric Constant	Conductivity, Mhos per meter
Fresh water (23° C.)	67	12
Sea water (28° C.) (2.2 per cent salt)	65	16

2. Discussion. The measurements for fresh water agree fairly closely with the value of 72 for dielectric constant and the value of 13.1 mhos per meter for conductivity as obtained from Saxton's work (3) for 3.2 cm. and 25° C.

The salt water sample was obtained from the surface layer on the Gulf side of Galveston Island and contained a lower than average salt content because of the method of sampling. The measured value of conductivity of 16 mhos per meter agrees very well with the value of 17 suggested by von Hippel (4) for sea water, but the dielectric constant of reference 4 is 35 as compared with the authors' measurement of 65. The relatively small effect of the salt concentration on the values is shown by the conductivities of 12 and 16 and dielectric constants of 67 and 65 for fresh and sea water, respectively.

VII. SUMMARY.

The dielectric properties of soils and water at 3.2 cm. wave length as measured by the Electrical Engineering Research Laboratory are listed in the following table:

Sample	Dielectric Constant	Conductivity, Mhos per meter
Arizona Soil		
1. As found	3.2	0.10
2. 16 parts soil and 1 part water	8.1	1.1
3. 4 parts soil and 1 part water	19	2.5
4. Particle size: 0.42 to 2.00 mm.	3.2	0.086
5. Particle size: 0.25 to 0.42 mm.	3.0	0.091
6. Particle size: 0.074 to 0.105 mm.	3.0	0.104
7. Very loosely packed Particle size: 0.25 to 0.42 mm.	2.7	0.07
8. Very tightly packed Particle size: 0.25 to 0.42 mm.	3.0	0.12
Austin, Texas Soil		
9. Very dry	2.8	0.0074
10. 10 parts of soil to 1 part of water	4.6	0.22
11. Colorado River sand and gravel mix	3.0	0.0072
Water		
12. Fresh (23° C.)	67	12
13. Sea (28° C.) (2.2 per cent salt)	65	16

VIII. REFERENCES.

(1) F. E. BROOKS, JR., AND C. W. TOLBERT, "Equipment for Measuring Angle-of-Arrival by the Phase Difference Method," *Elec. Eng. Research Lab.*, Report No. 2, May, 1946.

(2) L. H. FORD AND R. OLIVER, "An Experimental Investigation of the Reflection and Absorption of Radiation of 9 cm. Wave Length," *Proc. Phys. Soc.*, Vol. 58, Part 3, May, 1946.

(3) J. A. SAXTON, "The Dielectric Properties of Water at Wave Lengths From 2 mm. to 10 cm. and Over the Temperature Range 0° C. to 40° C.," *Radio Research Board*, Paper No. R.R.B./C. 115.

(4) W. T. FISHBACK AND P. J. RUBENSTEIN, "Further Measurements of 3- and 10-cm. Reflection Coefficients of Sea Water at Small Grazing Angles," *Radiation Lab.*, Report No. 568, Mass. Inst. Tech.

STABILIZATION OF CARRIER-FREQUENCY SERVOMECHANISMS. I. GAIN-PHASE MARGIN DIAGRAMS OF CONTROLLER CHARACTERISTICS.*

BY

ANDREW SOBCZYK, Ph.D.[1]

ABSTRACT.

In this first part a method of analysis of linear carrier-frequency systems is indicated, and applied in particular to electrical servomechanisms using a two-phase alternating-current control motor. The performance of such a servo, as regards stability and fidelity of the output as a copy of the input, depends on the action of a controller-demodulator unit including the motor. A calculation of the steady-state effect of a linear transfer characteristic on the suppressed-carrier modulated error signal makes possible application of the Nyquist stability criterion.

The method does not require symmetry of the characteristics about the carrier frequency; if such symmetry is present the system is similar to an ordinary or d-c. servo. Thus a band-rejection network tuned to the carrier frequency has an approximate proportional plus derivative action on the envelope of the error signal, resulting in an "anti-hunt" or stabilizing effect similar to that due to a translated band-rejection characteristic (center at zero-frequency) in a corresponding d-c. servo.

A procedure is given for construction of transformed Nyquist diagrams, or gain-phase margin diagrams. The effect of the correctly tuned band-rejection network, as well as the effects of bandwidth, of the network being tuned off the carrier frequency, of increasing gain or velocity constant k_v, and of the use of phase-shifting networks, are clearly shown by a series of such diagrams.

1. INTRODUCTION.

A servomechanism is a power-amplifying device, employing a feedback arrangement, by means of which the error, or difference of the input and output quantities, controls a motor which drives the output in such a manner as to maintain the error as nearly zero as possible.

In an ideal servomechanism, the output would be an exact power-amplified copy of the input, without time delay, whatever the form of the independent input as a function of time. Thus for sinusoidal inputs an ideal servomechanism would have a flat frequency amplitude characteristic, and zero phase shift at all frequencies.

The problem of servomechanism design is to obtain high power gain with small time lag of the output in following the input quantity, and with satisfactory freedom from a tendency toward instability or "hunting," that is freedom from excitability of steady or only slightly damped oscillations of the output. In a servomechanism which is properly designed and adjusted, the output quantity is a "slave" of the independent input quantity. These requirements will be met if the frequency-phase characteristic is a sufficiently good approximation to the ideal characteristic of identical transmission at all frequencies.

---

* This paper and Parts II and III to follow in the August and September issues consist essentially of the material contained in the author's Radiation Laboratory report, No. 811, "Parallel 'T' Stabilizing Networks for AC Servos."

[1] Formerly with Radiation Laboratory, Massachusetts Institute of Technology; now at Watson Laboratories, Cambridge Field Station, Cambridge, Mass.

The theory and analysis of ordinary servomechanisms, based on the steady state response to sinusoidal inputs, has been presented by A. C. Hall in a recent paper in this JOURNAL (1).[2]

A servomechanism may be of a mechanical, hydraulic, pneumatic, or electrical nature, or a combination of elements of two or more kinds. A d-c. servomechanism, requiring a d-c. control amplifier, is indicated in Fig. 1. The input θ_i and output θ_o are potentiometer shaft rotations.

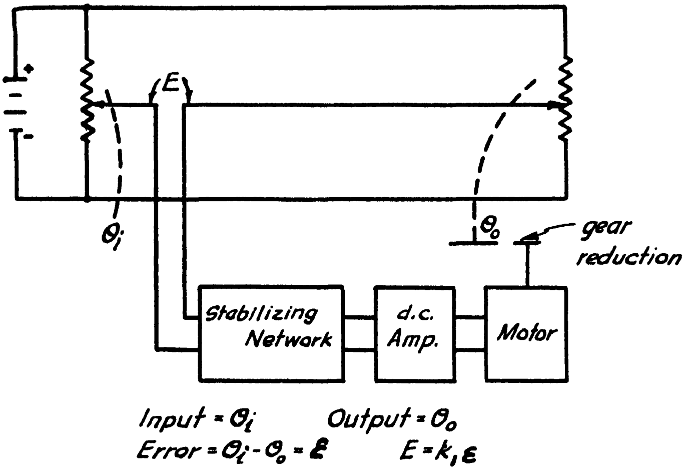


FIG. 1. A d-c. servomechanism.

The error voltage is the difference of potential between the contact arms of the linear input and output potentiometers. This voltage drives the d-c. motor to correct the error, through a stabilizing network and the d-c. amplifier. With a stabilizing or anti-hunt network well suited to the time lag characteristics of the d-c. motor and amplifier, and suitable gear reduction, the output θ_o will be a faithful copy of the input θ_i, and will be capable of driving a heavy load.

It is well known that the design of a stable d-c. amplifier (to drive a d-c. motor) requires extreme care (it is in itself a problem of the same nature as the design of a servomechanism, with added difficulties), and the construction of the amplifier is expensive; while the design of a high-gain, ordinary resistance-coupled feedback audio amplifier (satisfactory for driving an a-c. motor at say the standard frequency of 60 cps.) is easy, and the construction inexpensive. For most applications it is therefore highly advantageous to use a-c. servomechanisms.

---

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Figure 2 is a schematic drawing of an a-c. servomechanism of the general type which the writer had in mind when developing the analysis contained in this paper. The potentiometers may be replaced by selsyns in the standard generator-control transformer connection (Fig. 3).

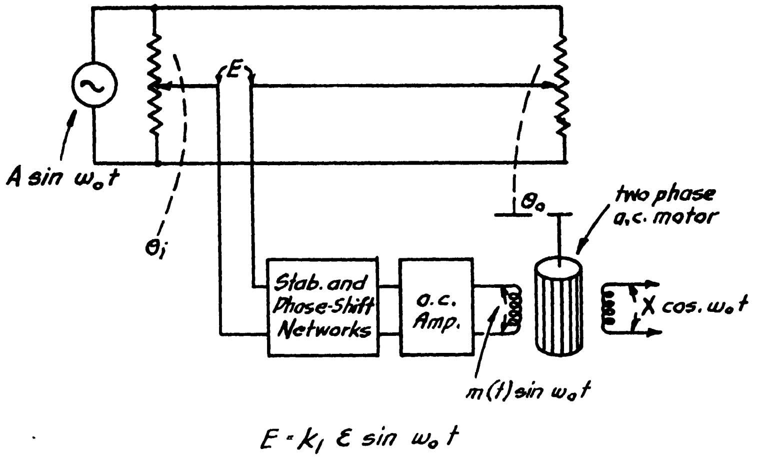


FIG. 2. An a-c. servomechanism.

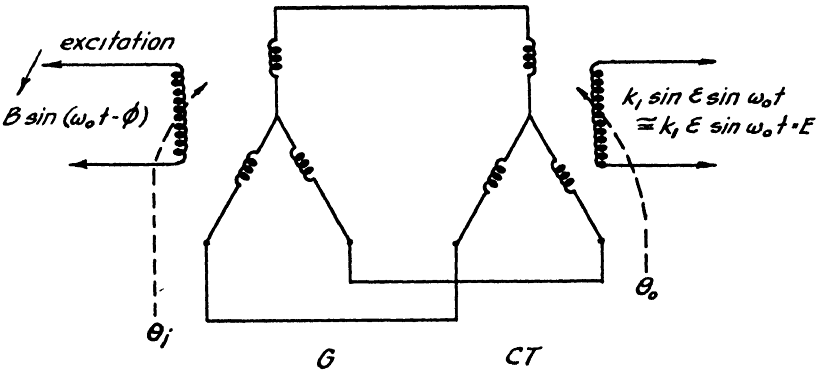

excitation
$B \sin (\omega_0 t - \phi)$
θ_i
$k_1 \sin \varepsilon \sin \omega_0 t$
$\cong k_1 \varepsilon \sin \omega_0 t = E$
θ_o
G
CT

FIG. 3. Connection of selsyn generator and control transformer (to replace potentiometers in Fig. 2).

In both Figs. 1 and 2, the driven load on the output shaft θ_o is not shown. This load of course may have a large effect on the nature of the stabilizing characteristic which is desirable.

2. SERVO AND SERVO CONTROLLER; STABILITY.

Figure 4 is a familiar general diagram for a linear servo, which includes as instances the linear approximations to the d-c. and a-c. servos of Figs. 1 and 2. The input and output are subtracted in the symbol for a differential gear, and the difference or error $\epsilon = \theta_i - \theta_o$ is fed into the portion of the servo marked $kH(p)$. This portion is called the *servo controller*. The output $\theta_o(t)$ is determined by the error $\epsilon(t)$ acting through the servo controller.

In the terminology and notation of Laplace transforms (2), the servo controller characteristic is represented by the function $kH(p)$ of the complex variable p, called the *system transform* or transfer characteristic of the controller, and the transform $\bar{\theta}_o(p) = \int_0^\infty \exp(-pt)\theta_o(t)dt$ of the output and the transform $\bar{\epsilon}(p) = \int_0^\infty \exp(-pt)\epsilon(t)dt$ of the error are related by the equation

$$\bar{\theta}_o = kH(p)\bar{\epsilon}. \tag{1}$$

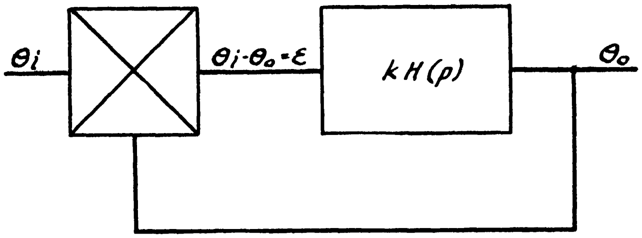


FIG. 4. General servo diagram.

To find the relation between output and input transforms, we have

$$\bar{\epsilon} = \bar{\theta}_i - \bar{\theta}_o, \quad 1 = \frac{\bar{\theta}_i}{\bar{\epsilon}} - \frac{\bar{\theta}_o}{\bar{\epsilon}}, \quad 1 + \frac{\bar{\theta}_o}{\bar{\epsilon}} = 1 + kH(p) = \frac{\bar{\theta}_i}{\bar{\epsilon}}, \tag{2}$$

and

$$\frac{\bar{\theta}_o}{\bar{\theta}_i} = \frac{\bar{\theta}_o/\bar{\epsilon}}{\bar{\theta}_i/\bar{\epsilon}} = \frac{kH(p)}{1 + kH(p)}. \tag{3}$$

In the case of a zero displacement error servo [(1), p. 297], that is, a servo such that the error of the response $\theta_o(t)$ to a unit-step displacement input $\theta_i(t) = u(t)$ approaches zero as a limit as $t \to \infty$, while the error for an instantaneous velocity input $\theta_i(t) = \int_0^t u(t)dt$ [equals t for $t \geqq 0$, equals 0 otherwise] does not approach zero, $H(p)$ may be normalized so that the gain constant k is the velocity error constant k_v (equals reciprocal of the limiting time lag of θ_o behind θ_i for an instantaneous velocity input). For if $\theta_i = u(t)$, the unit step at $t = 0$, then $\bar{\theta}_i = 1/p$, and from (2) above

$$\frac{\bar{\epsilon}}{\bar{\theta}_i} = \frac{1}{1 + kH(p)}, \qquad \bar{\epsilon} = \frac{1}{p[1 + kH(p)]}.$$

By the final-value theorem for Laplace transforms [(2), p. 265],

$$\lim_{t\to\infty}\epsilon(t) = \lim_{p\to 0} p\bar{\epsilon} = 0 = \lim_{p\to 0}\frac{1}{1 + kH(p)},$$

$kH(p) \to \infty$ as $p \to 0$. Thus $H(p)$ must be of the form

$$\frac{A(p)}{p}, \qquad A(0) \neq 0 \quad \text{and} \quad \neq \infty.$$

For an instantaneous velocity input, $\bar{\theta}_i = 1/p^2$, and

$$\lim_{t\to\infty}\epsilon(t) = \lim_{p\to 0} p\bar{\epsilon} = \lim \frac{1}{p + pkH(p)} = \frac{1}{kA(0)}.$$

Thus if $\lim_{p\to 0} pH(p) = A(0) = 1$, then k will be the velocity error constant k_v.

If $G(p)$ is the transfer characteristic of a system, then if $\cos \omega t$ is applied to the input, the steady-state output is $|G(j\omega)|\cos(\omega t + \phi)$, and if $\sin \omega t$ is applied, the steady-state output is $|G(j\omega)|\sin(\omega t + \phi)$, where $\phi = \angle G(j\omega)$. For by the convolution and inversion theorems [(2), pp. 228, 105] if $\exp(j\omega t)$ is applied to $G(p)$, the output is

$$\begin{aligned}\int_0^t e^{j\omega(t-\tau)}g(\tau)d\tau &= e^{j\omega t}\int_0^t e^{-j\omega\tau}g(\tau)d\tau \\ &= e^{j\omega t}\left[\int_0^\infty - \int_t^\infty\right] = e^{j\omega t}\left[G(j\omega) - \int_t^\infty\right],\end{aligned}$$

where

$$g(\tau) = (2\pi j)^{-1}\int_{\tau - j\infty}^{\tau + j\infty} e^{p\tau}G(p)dp.$$

The second integral in the bracket approaches zero as $t \to \infty$; therefore the steady-state output is $G(j\omega)\exp(j\omega t)$. The stated results follow since the real and imaginary components of this output are due respectively to the real and imaginary components of $\exp(j\omega t)$. Thus in dealing with sinusoidal inputs, the equations in transforms apply also to the time functions if p is replaced by $j\omega$. Accordingly the bars over θ_o, θ_i, ϵ indicating transforms will be omitted. A ratio, as for example θ_o/ϵ, for each ω is a complex number of which the magnitude and angle are, respectively, the amplitude and phase of the output when the error ϵ is a unit sinusoid of frequency ω.

The plot of $\theta_o/\epsilon = kH(j\omega)$ in the complex plane, as ω ranges from 0 to ∞, is the well-known Nyquist diagram for the servo. If the origin is shifted to the point -1, the same plot is also the graph of θ_i/ϵ, since $\epsilon = \theta_i - \theta_o$, or $1 = \theta_i/\epsilon - \theta_o/\epsilon$ (see Fig. 5).

The degree of stability of the servo is expressed by the peak height of the frequency characteristic, that is the value of $M = \max|\theta_o/\theta_i|$.

The greater the value of the ratio M, the less stable is the servo. Also, for any particular application, of course the frequency ω for which the maximum value M is attained, is important. Stability corresponding to values of M between 1.0 and 2.0 usually is satisfactory. A general design rule is stated on p. 287 of (1).

In the Nyquist plane, the system of curves of equal values of the ratio $M = |\theta_o/\theta_i|$ are circles of diameter $(M + 1)^{-1} + (M - 1)^{-1}$, with centers at $-1 - \frac{1}{2}[(M - 1)^{-1} - (M + 1)^{-1}]$. These circles are plotted in Fig. 6 on p. 291 of (1). Given the Nyquist graph for a stable servo, the peak height M of the frequency-amplitude characteristic is the value of M for that circle of the system which is just tangent to the graph. The transition between stability and instability of a servo occurs as the gain is varied so that the Nyquist graph passes through the point -1 (where the ratio θ_o/θ_i becomes infinite).

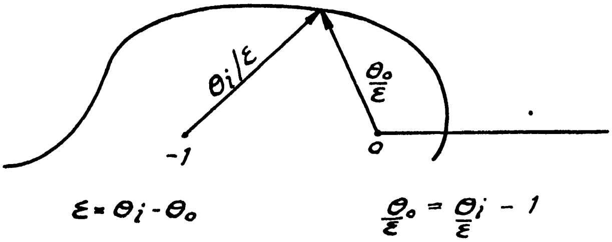


FIG. 5. A Nyquist diagram.

For easy determination of appropriate values of the gain factor k, and also for other reasons, it is advantageous for our purposes to transform the Nyquist plane and diagram as follows. The *phase margin* is the difference of phase between θ_o/ϵ and the phase π of the critical point -1. The rectangular coordinates in the transformed plane are the gain ratio $|\theta_o/k\epsilon|$ expressed in decibels, and the phase margin $\pm [\angle (\theta_o/\epsilon) - \pi]$. For any controller characteristic $kH(p)$, the transformed Nyquist graph is obtained by plotting $20 \log |H(j\omega)|$ *versus* the phase margin $\pm [\angle H(j\omega) - \pi]$, with frequency ω as parameter. Let the chart with this graph be referred to as A.

Figure 6 gives the curves which correspond, under the above transformation of the Nyquist plane (the rectangular coordinates are $|\theta_o/\epsilon|_{db}$ and $\phi = \angle (\theta_o/\epsilon) - 180°$), to the system of circles of equal values of M. To determine the gain possible without exceeding a specified peak M, and the frequency at which the peak is attained, Fig. 6 is superposed on A and translated up or down until the curve corresponding to the desired peak $|\theta_o/\theta_i| = M$ just touches the graph in A. The position of 0 *db* in Fig. 6 on A then is read, to determine the value of the gain factor k in decibels. If 0 *db* in Fig. 6 corresponds to $-x$ *db*, then $20 \log k = x$.

This follows since $20 \log |H(j\omega)| = 20 \log |\theta_o/k\epsilon| = 20(-\log k + \log |\theta_o/\epsilon|) = -|k|_{db} + |\theta_o/\epsilon|_{db}$. (For the sliding process it would be convenient to have the curves of Fig. 6 on a sheet of transparent material.)

The ratio of error to input, $E = |\epsilon/\theta_i|$, for any gain setting k and any frequency of the input, may be read by comparing the graph of the servo controller in A with the dashed-line curves in Fig. 6. (The

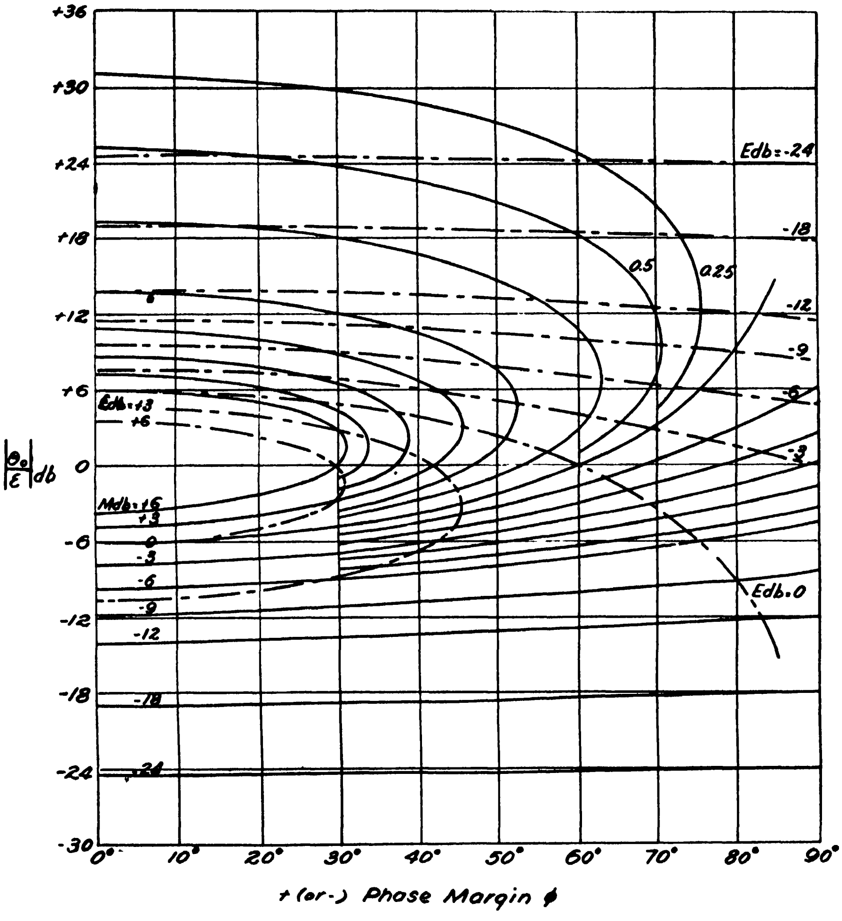


FIG. 6. Contours of constant ratios of output to input.

——————— $|\theta_o/\theta_i| = M$

– – – – ——— $|\epsilon/\theta_i| = E$

E-curves are reflections in the 0 db axis of the corresponding M-curves.) In Fig. 6(a) the M-curves are replotted, together with curves of constant phase shift ψ of output with respect to input, $\psi = \angle\ (\theta_o/\theta_i)$. The phase shift characteristic of the servo thus may be determined by superposing A on Fig. 6(a) and reading the intersections of the transformed Nyquist graph with the dashed-line curves.

If the Nyquist plane is the complex z-plane, then the above transformed plane is essentially the w-plane for the transformation $w = \log_e z = \ln z$. For if $z = r \exp(j\theta)$, $\ln z = \ln r + j\theta$. Since $\ln z$ is analytic, if rectangular coordinate scales in the w-plane were chosen such that

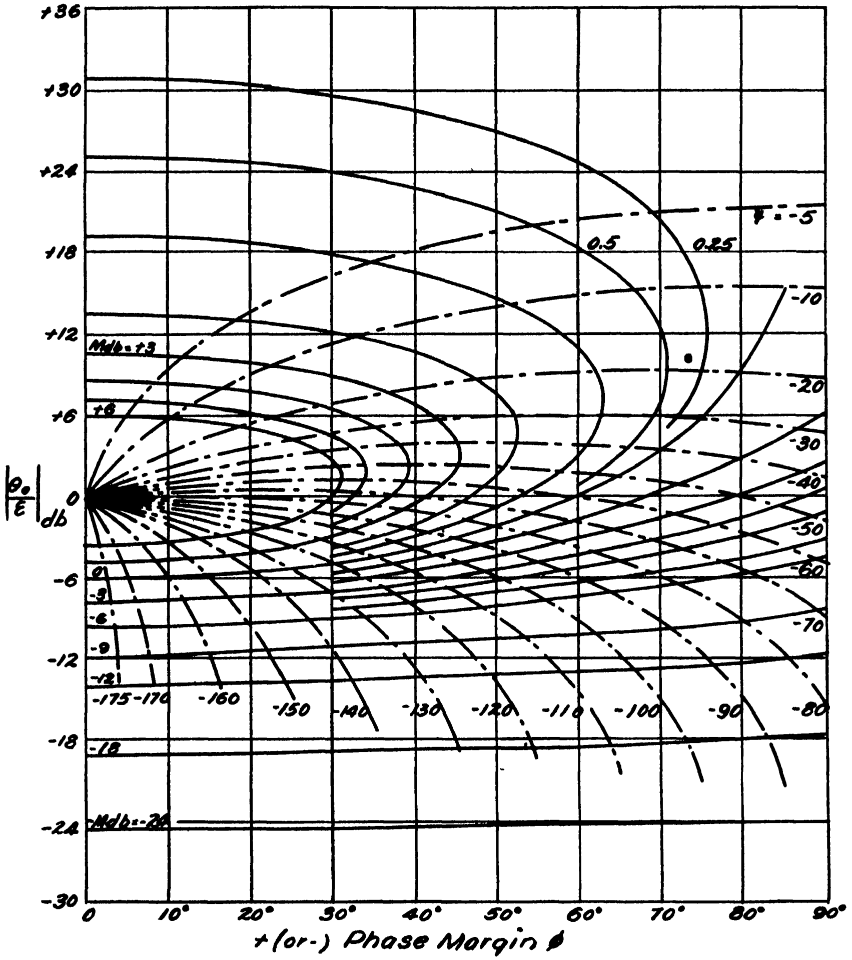


FIG. 6(a). Contours of constant ratio of output to input and of constant phase difference of output and input.

———— $|\theta_o/\theta_i| = M$

— — — $\angle(\theta_o/\theta_i) = \psi$

(Each constant phase curve is labeled with the (−) value of ψ in degrees, corresponding to positive ϕ. For negative ϕ, ψ is positive with the same numerical value.)

one radian and one neper (8.686 *db*) were equal units of distance, then angles would be preserved by the transformation. In particular, an orthogonal intersection of curves in the Nyquist z-plane would be transformed into an orthogonal intersection.

For controller characteristics of the rational form:

$$H(j\omega) = \frac{(T_1 j\omega + 1)(T_2 j\omega + 1)\cdots}{(Tj\omega)^n(S_1 j\omega + 1)(S_2 j\omega + 1)\cdots}$$

a fairly accurate sketch of the gain-phase margin diagram or transformed Nyquist diagram may be very rapidly obtained by first sketching the Bode chart, that is, the graph of $|H(j\omega)|_{db}$ as a function of $\log \omega$, or $|H(j\omega)|_{db}$ plotted against frequency on a logarithmic scale. The construction of such a chart is quite easy because of the fact that for high frequencies, each linear factor in $H(j\omega)$ is approximated very closely by a straight line, of slope $+6$ *db* per frequency octave for numerator factors, -6 *db* per octave for denominator factors. The procedure may be best indicated by an example.

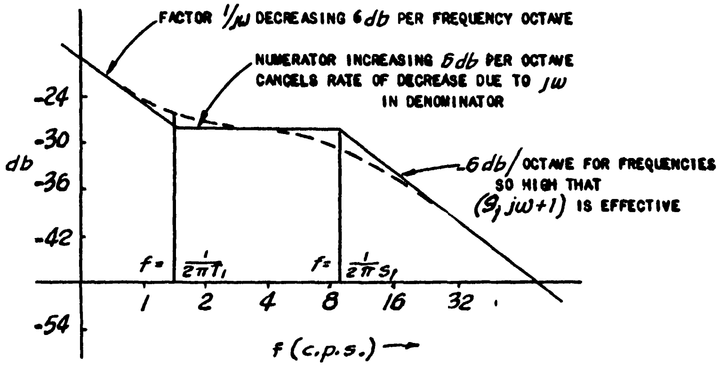


FIG. 7. Bode frequency-amplitude chart for controller characteristic.

Suppose the controller characteristic is:

$$H(j\omega) = \frac{T_1 j\omega + 1}{j\omega(S_1 j\omega + 1)},$$

corresponding to a zero-displacement error servo. Assume $T_1 > S_1$. At frequency $f = 1/(2\pi T_1)$, the numerator has a value $+ 3$ *db* $= 20 \log\sqrt{2}$; at lower frequencies, the numerator contributes less than 3 *db* to $|H(j\omega)|_{db}$. Each time ω is doubled, the $j\omega$ in the denominator subtracts 6 *db* from $|H(j\omega)|_{db}$, since 6 *db* $= 20 \log 2$. Therefore, for frequencies up to $f = 1/(2\pi T_1)$, $|H(j\omega)|_{db}$ is approximately a straight line of slope -6 *db* per octave, as indicated in Fig. 7. For frequencies above $f = 1/(2\pi T_1)$, the factor $(T_1 j\omega + 1)$ approximately is equal to $T_1 j\omega$, and contributes $+6$ *db* per octave, cancelling the -6 *db* per octave contribution of $j\omega$ in the denominator; therefore, the graph of $|H(j\omega)|_{db}$ is approximately flat to the frequency $f = 1/(2\pi S_1)$ where the value of the factor $(S_1 j\omega + 1)$ in the denominator has increased to 3 *db*. Above this frequency,

$(S_1 j\omega + 1)$ contributes approximately -6 *db* per octave, so that as $f \rightarrow \infty$ the graph of $|H(j\omega)|_{db}$ is asymptotic to a straight line of slope -6 *db* per octave. The actual graph of $|H(j\omega)|_{db}$ everywhere differs from the asymptotic line approximation by less than 3 *db*, with maximum positive deviation near $f = 1/(2\pi T_1)$, and maximum negative deviation near $f = 1/(2\pi S_1)$.

The phase of $H(j\omega)$ also may easily be sketched approximately, as indicated in Fig. 8. Up to $f = 1/(2\pi T_1)$, the phase of $H(j\omega)$ is principally the $-90°$ of the factor $j\omega$ in the denominator (dashed line). From $f = 1/(2\pi T_1)$ to $f = 1/(2\pi S_1)$, the $-90°$ is cancelled by approximately $+90°$ due to $(T_1 j\omega + 1)$; and beyond $f = 1/(2\pi S_1)$ the phases of $(T_1 j\omega + 1)$ and $j\omega$ approximately cancel, so that the phase of $H(j\omega)$ is nearly equal to the phase $-90°$ of the factor $(S_1 j\omega + 1)$. From this the phase margin characteristic may easily be sketched, as indicated in Fig. 8.

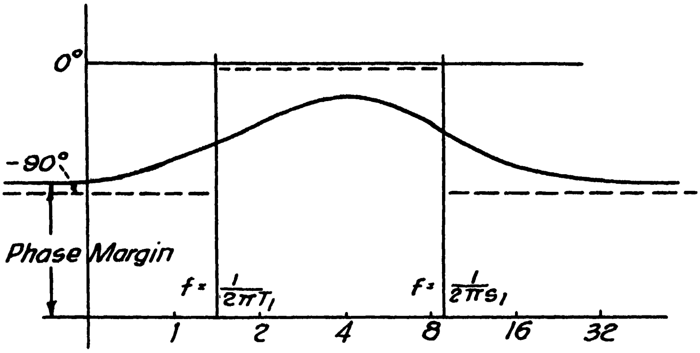


FIG. 8. Phase chart for controller characteristic.

Having the approximate frequency amplitude and frequency phase characteristics, corresponding values of $|H(j\omega)|_{db}$ and phase margin ϕ may be plotted in the *db*-phase margin plane, to obtain the transformed Nyquist diagram for the servo controller.

Since the phase margin for the above controller characteristic is always $\geqq 90°$, the servo will be stable no matter how high the gain setting k (equals velocity error constant k_v). (In an actual servo, other small time lag factors would be present in the denominator so that the gain could not be increased without limit with preservation of stability.)

If the numerator time lag T_1 were smaller instead of larger than the denominator time lag S_1, the phase margin would be less than 90° between $f = 1/(2\pi S_1)$ and $f = 1/(2\pi T_1)$, which would be a less favorable situation as regards stability of the servo. This case will be discussed in the third example of the next section.

3. EXAMPLES OF SERVO CONTROLLERS.

In order to obtain examples of gain-phase margin diagrams for servo controllers and their comparison with the M, E, and ψ curves to analyze the servo performance, we shall consider several cases of the d-c. servo of Fig. 1. For simplicity, in all examples let us assume that the amplifier is an ideal amplifier with gain μ.

1. As a first example, suppose the motor is ideal or an exact integrator; that is, that its speed is proportional to the voltage V which is applied to the control field or armature, and suppose that the stabilizing network is omitted. For the controller in Fig. 1, we have then

$$\dot{\theta}_o = k_1 V = k_1 \mu \epsilon, \qquad p\bar{\theta}_o = k\bar{\epsilon} \qquad \text{where} \qquad k = k_1 \mu,$$

and the controller characteristic

$$\frac{\theta_o}{\epsilon} = \frac{k}{j\omega}.$$

The gain-phase margin diagram for this controller is the straight line $|H(j\omega)|_{db} = -|\omega|_{db}$ of constant (negative) phase margin 90°, extending from $+\infty$ *db* at $\omega = 0$ to $-\infty$ *db* at $\omega = \infty$. As may be determined by comparison with the M-curves in Fig. 6, the ratio $|\theta_o/\theta_i|_{db}$ of output to input amplitudes decreases with increasing frequency, starting with 0 *db* at $\omega = 0$. At frequency $\omega = k$, the output is 3 *db* below the input. Thus the entire servo acts as a low-pass filter or band-pass filter with center at zero frequency; the higher the gain k, the greater the bandwidth. Comparing with Fig. 6(a), the phase shift and error are seen to be 0 (error in $db = -\infty$) at $\omega = 0$, increasing to, respectively, 45° and -3 *db* at $\omega = k$. In this case it is also convenient to solve directly for $(\theta_o/\theta_i) = kH(j\omega)/[1 + kH(j\omega)] = k/(j\omega + k)$, and for $(\epsilon/\theta_i) = 1/[1 + kH(j\omega)] = j\omega/(j\omega + k)$. From these we obtain

$$\left|\frac{\theta_o}{\theta_i}\right| = \frac{k}{\sqrt{\omega^2 + k^2}}, \qquad \angle \frac{\theta_o}{\theta_i} = \text{arc tan}\,\frac{\omega}{k}, \qquad \left|\frac{\epsilon}{\theta_i}\right| = \frac{\omega}{\sqrt{\omega^2 + k^2}}.$$

2. Suppose that the differential equation of the motor is

$$T\ddot{\theta}_o + \dot{\theta}_o = k_1 V, \qquad T = J/f$$

where J is the moment of inertia of the armature and load, f is the viscous damping coefficient. Suppose again that the stabilizing network is omitted. Substituting $V = \mu\epsilon$ and taking transforms, we have

$$(Tp + 1)p\bar{\theta}_o = k\bar{\epsilon}$$

so that the controller characteristic is:

$$\frac{\theta_o}{\epsilon} = \frac{k}{j\omega(Tj\omega + 1)}.$$

The straight-line approximation to the controller amplitude characteristic in the Bode chart has a slope of -6 *db* per octave for frequencies up to $\omega = 1/T$; above this frequency the slope is -12 *db* per octave. The phase margin is approximately 90° (270° due to $1/j\omega$ minus 180°) up to $\omega = 1/T$, and approximately 0° at frequencies above $\omega = 1/T$. The gain phase-margin diagram is curve (*a*) in Fig. 9, for the case of motor time constant $T = 0.2$ sec. For a peak $M = |\theta_o/\theta_i|$ not exceeding 2.0,

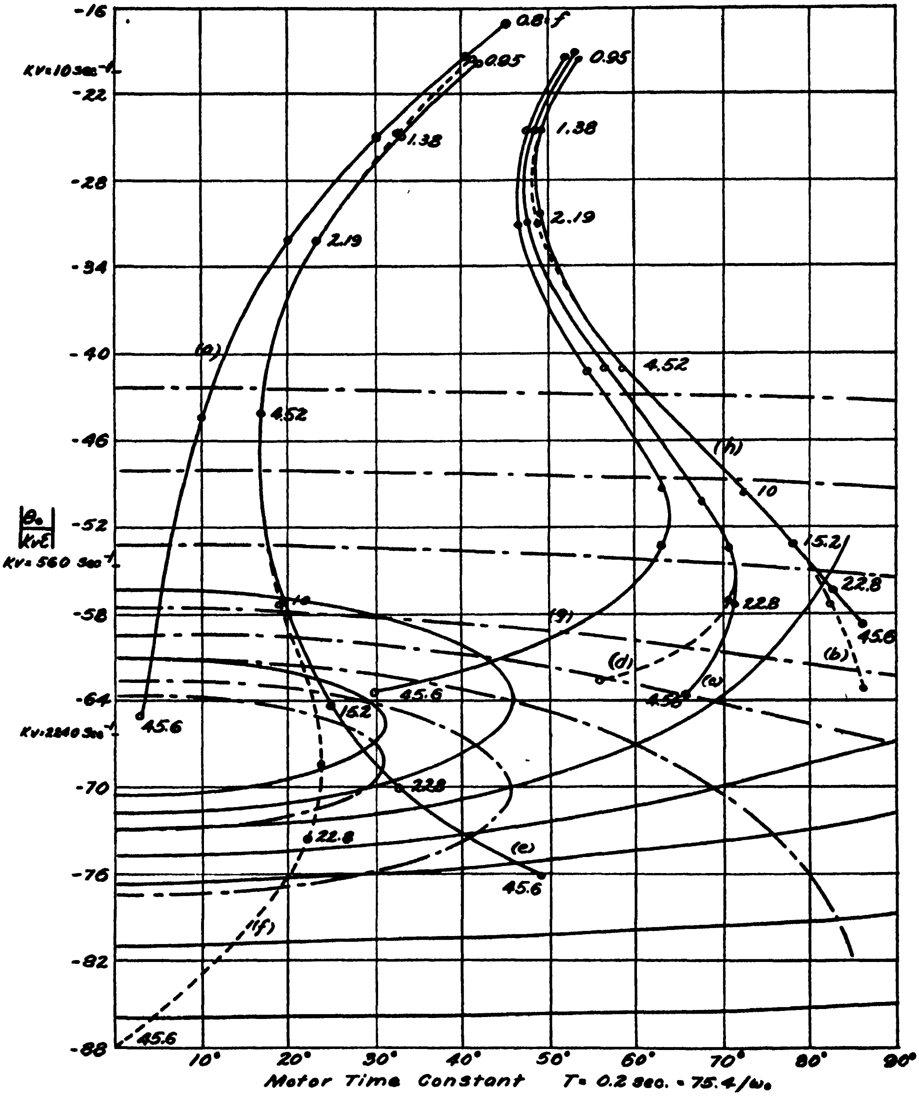


FIG. 9. (*a*) Proportional controller; (*b*) Ideal proportional-derivative characteristic; (*c*) True proportional-derivative characteristic with a time lag; (*d*) Approximate proportional-derivative characteristic obtainable with a parallel "T"; (*e*) 50-cycle parallel "T" with 60-cycle carrier frequency; (*f*) 70-cycle parallel "T" with 60-cycle carrier frequency; (*g*) Ideal proportional-derivative characteristic with 100° phase lag network (equal roots); (*h*) Ideal proportional-derivative characteristic with 90° phase lag. In (*b*) through (*h*), $T_d\omega_o = 15$, and in (*c*) through (*f*) the time lag $= 0.5/\omega_0$.

due to the presence of the time lag factor $(Tj\omega + 1)$ in the denominator, the gain or velocity constant $k = k_v$ cannot be increased beyond approximately 10 sec.$^{-1}$. For $k_v = 10$ sec.$^{-1}$, the peak value is attained at a frequency in the vicinity of 1 cps.

3. Suppose the controller is as in (2), except that the stabilizing network is an ideal proportional-derivative network, that is a network with characteristic such that for an input voltage proportional to ϵ, the output is $k_2(\epsilon + T_d\dot{\epsilon})$, where T_d is the "derivative time constant." The controller characteristic is:

$$\frac{\theta_o}{\epsilon} = \frac{k(T_dj\omega + 1)}{j\omega(Tj\omega + 1)}.$$

For $T_d > T$, this characteristic has been discussed at the end of the last section ($S_1 = T$, $T_1 = T_d$). For $T_d < T$, specifically for $T = 0.2$ sec., $T_d = 15/377$ sec., the gain-phase margin diagram is curve (*b*) in Fig. 9.

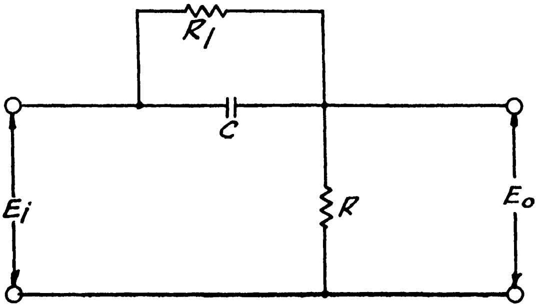


FIG. 10. A d-c. proportional-derivative stabilizing network.

Increasing the value of T_d improves stability and diminishes error, that is, improves fidelity, in the above ideal case. After the next example it will be clear, however, that in the case of an actual proportional-derivative characteristic, where a time lag factor is present in the denominator, although increasing T_d increases phase margin at the lower frequencies, it increases $|\theta_o/k_v\epsilon|_{db}$ at higher frequencies, with the effect that the value of $|\theta_o/k_v\epsilon|$ is large at a high frequency where the phase margin has decreased to a very small value, so that, for a prescribed degree of stability, the highest value of k_v which is possible with a high value of T_d is much lower than the k_v which is possible with a medium value of T_d. Figure 12 is a sketch illustrating this situation.

4. Suppose that the stabilizing network is an actual proportional-derivative network having a transfer characteristic of the form:

$$\frac{1}{n}\frac{(T_dj\omega + 1)}{(lj\omega + 1)}$$

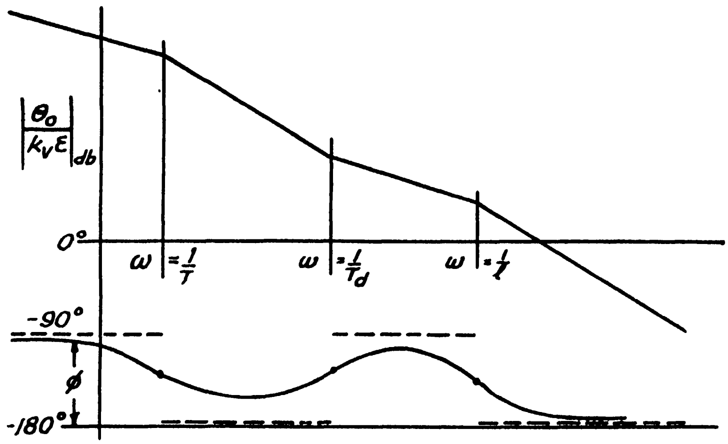


FIG. 11. Bode chart and phase characteristic for proportional-derivative controller (including time lag).

where $T_d = nl$. Suppose that the controller otherwise is as in (2). Figure 10 is a possible circuit for the stabilizing network. The transfer characteristic for Fig. 10 is:

$$\frac{E_o}{E_i} = \frac{R(R_1Cp + 1)}{R(R_1Cp + 1) + R_1} = \frac{R}{(R + R_1)} \frac{(R_1Cp + 1)}{[RR_1/(R + R_1)]Cp + 1}$$

which is of the preceding form with $T_d = R_1C$, $l = [RR_1/(R + R_1)]C$, $n = (R + R_1)/R$. The characteristic for the entire controller is

$$\frac{\theta_o}{\epsilon} = \frac{k(T_dj\omega + 1)}{j\omega(Tj\omega + 1)(lj\omega + 1)}.$$

The Bode chart and the graph of phase margin against frequency in octaves are of the nature indicated in Fig. 11. For the case of motor time constant $T = 0.2$ sec., $T_d = 15/377$ sec., $l = 0.5/377$ sec., the gain-phase margin diagram is curve (*c*) in Fig. 9.

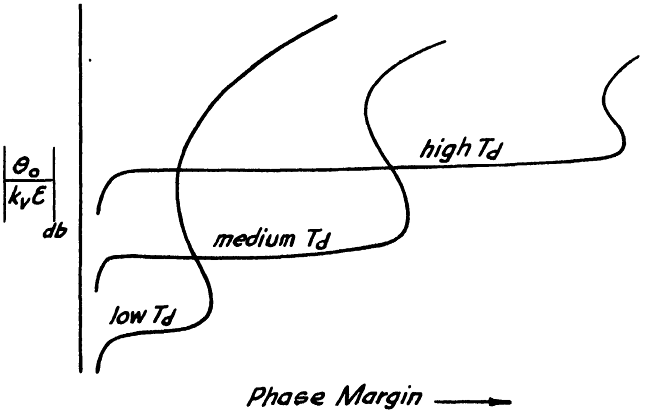


FIG. 12. Gain-phase margin diagrams for high, medium, and low values of T_d.

The general effect of varying T_d on the gain-phase margin diagram is depicted in the sketch of Fig. 12. For the preceding values of motor constant T and lag l, the value $T_d = 15/377$ sec. is an appropriate medium value.

4. OUTPUT OF A LINEAR CHARACTERISTIC FOR SUPPRESSED-CARRIER MODULATED INPUT.

In order to analyze the a-c. servomechanism of Fig. 2, it is necessary to calculate the effect of a linear transfer characteristic on a suppressed-carrier modulated voltage.[3] [Compare below with (2), p. 248.]

Let $G(j\omega)$, Fig. 13, be any linear transfer characteristic, defined for positive frequencies. This means that for a real input cos ωt, the output is $R[G(j\omega)\exp(j\omega t)]$, and for a real input sin ωt, the output is $I[G(j\omega) \times \exp(j\omega t)]$. Suppose the modulation of the suppressed-carrier modulated input is sinusoidal, with angular frequency m; let the input be of the form cos mt cos $\omega_0 t$.

FIG. 13. A transfer characteristic.

By a formula of elementary trigonometry, the input may be written in the form:

$$E_i = \cos mt \cos \omega_0 t = \tfrac{1}{2}[\cos(\omega_0 + m)t + \cos(\omega_0 - m)t].$$

The output is:

$$\begin{aligned} E_o &= \tfrac{1}{2}R\{G[j(\omega_0 + m)]e^{j(\omega_0+m)t} + G[J(\omega_0 - m)]e^{j(\omega_0-m)t}\} \\ &= \tfrac{1}{2}\{|G[j(\omega_0 + m)]|\cos[(\omega_0 + m)t + \phi] \\ &\qquad + |G[j(\omega_0 - m)]|\cos[(\omega_0 - m)t + \psi]\} \end{aligned}$$

where $\phi = \angle G[j(\omega_0 + m)]$, $\psi = \angle G[j(\omega_0 - m)]$. This may be expanded in the form $E_o = \frac{1}{2}|G[j(\omega_0 + m)]|\{\cos(mt + \phi_1)\cos(\omega_0 t + \phi_2) - \sin(mt + \phi_1)\sin(\omega_0 t + \phi_2)\} + \frac{1}{2}|G[j(\omega_0 - m)]|\{\cos(mt + \psi_1) \times \cos(\omega_0 t + \psi_2) + \sin(mt + \psi_1)\sin(\omega_0 t + \psi_2)\}$ where ϕ_1, ϕ_2, ψ_1, ψ_2 are arbitrary angles such that $\phi_1 + \phi_2 = \phi$, $\psi_2 - \psi_1 = \psi$. We take $\psi_1 = \phi_1$, $\psi_2 = \phi_2$ to obtain the form $E_o = \frac{1}{2}\{|G[j(\omega_0 - m)]| + |G[j(\omega_0 + m)]|\}$

---

[3] In the a-c. servo, the error-signal is suppressed-carrier modulated since it is of the form $k\epsilon(t)\cos\omega_0 t$, where the modulation envelope $k\epsilon(t)$ is proportional to the servo error $\epsilon(t) = \theta_i(t) - \theta_o(t)$. An ordinary amplitude modulated signal would be of the form $[1 + k\epsilon(t)]\cos\omega_0 t$, where k would be the modulation index. In the previous form, the carrier component $\cos\omega_0 t$ is not present.

$\times \cos (mt + \phi_1) \cos (\omega_0 t + \phi_2) + \frac{1}{2}\{ |G[j(\omega_0 - m)]| - |G[j(\omega_0 + m)]| \}$ $\times \sin (mt + \phi_1) \sin (\omega_0 t + \phi_2)$, $\phi_1 = \frac{1}{2}(\phi - \psi)$, $\phi_2 = \frac{1}{2}(\phi + \psi)$, which is a resolution into a main component and a quadrature component having the minimum possible amplitude.

The amplitude of the main component is the average of the amplitudes $|G^+|$ and $|G^-|$ of the upper and lower sidebands; the phase of the carrier is the average of the phases of the respective sidebands, while the modulation phase is one half the difference. The amplitude of the quadrature component is one half the difference of the amplitudes of the lower and upper sidebands; if $|G^-| > |G^+|$ the quadrature component lags the main component, and the carrier and modulation phases of the quadrature component lag those of the main component by 90°. The relationships between the sidebands, main component (m.c.), quadrature component (q.c.), and their phase angles, are indicated vectorially in Fig. 14.

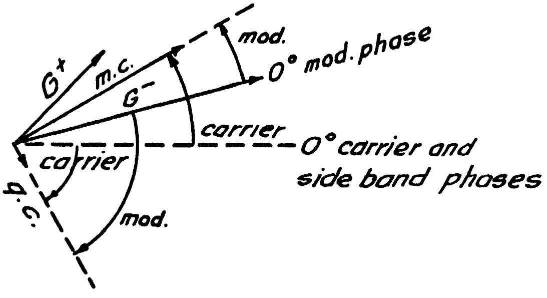


FIG. 14. Vector diagram for main and quadrature components.

Evidently if the amplitude characteristic of the linear system $G(p)$ is symmetrical about the carrier frequency, $|G[j(\omega_0 - m)]| = |G[j(\omega_0 + m)]|$, the quadrature component disappears, and the effect of $G(p)$ is similar to the effect of a d-c. system on the modulation, with a phase shift ϕ_2 of the carrier. In this form, the phase shift ϕ_2 may vary with the modulating frequency; if desired a different resolution may be made with a constant carrier phase shift angle in the main component.

5. EQUATION FOR TWO-PHASE MOTOR; DEMODULATOR CHARACTERISTICS.

A two-phase motor like the one used in the a-c. servo of Fig. 2 is depicted in Fig. 15. Suppose that $H_x = X \cos \omega_0 t$, X = constant, is the voltage applied to the fixed winding, and that $H_y = X\beta(t)$ is the voltage applied to the control winding. The differential equation relating the output θ_o of the motor and the input voltage $\beta(t)$ has been derived by N. B. Nichols; it is

$$T\ddot{\theta}_o + [1 + \cos 2\omega_0 t + 2\beta^2 (t)]\dot{\theta}_o = 2\omega_0\beta(t) \sin \omega_0 t + 2\dot{\beta}(t) \cos \omega_0 t$$

where T is a time constant associated with the motor and the load on the output. (In the derivation of this equation, the effect of circulating rotor currents on the stator field was neglected.)

For small amplitudes of the input $\beta(t)$, if the high-frequency term $\cos 2\omega_0 t$ is neglected, the above equation simplifies to the linear equation

$$T\ddot{\theta}_o + \dot{\theta}_o = 2\omega_0\beta(t)\sin\omega_0 t + 2\dot{\beta}(t)\cos\omega_0 t.$$

If $\beta(t) = \beta_1 \sin\omega_0 t$ [$\beta(t)$ then is 90° out of phase with the fixed winding excitation], the motor will run at constant speed $2\omega_0\beta_1$. If $\beta(t) = \beta_1 \times \sin(\omega_0 t + \theta)$, the right-hand side of the differential equation is $2\omega_0\beta_1[\sin\omega_0 t \sin(\omega_0 t + \theta) + \cos\omega_0 t \cos(\omega_0 t + \theta)] = 2\omega_0\beta_1 \cos\theta[\sin^2\omega_0 t + \cos^2\omega_0 t] = 2\omega_0\beta_1\cos\theta$, so that the motor will run at constant speed $2\omega_0\beta_1\cos\theta$. If $\theta = \pi/2$, then $\beta(t) = \beta_1\sin(\omega_0 t + \pi/2) = \beta_1 \cos\omega_0 t$, the right-hand side is zero, and the motor will not run.

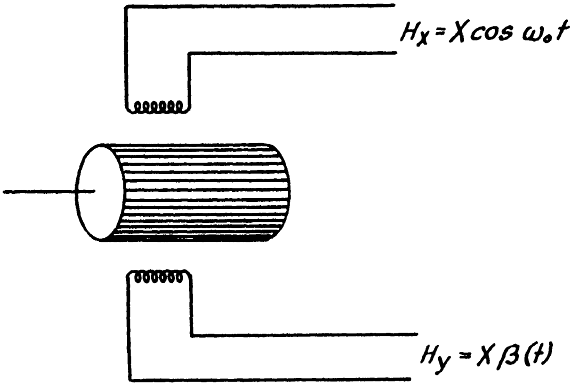


FIG. 15. Two-phase motor.

If the voltage applied to the fixed winding is $H_x = X\sin(\omega_0 t + \phi)$ instead of $X\cdot\cos\omega_0 t$, then by a change of origin of time t, the simplified differential equation of the motor becomes

$$T\ddot{\theta}_o + \dot{\theta}_o = -2\omega_0\beta(t)\cos(\omega_0 t + \phi_o) + 2\dot{\beta}(t)\sin(\omega_0 t + \phi_o).$$

For a control voltage input $\beta(t) = \cos(mt + \theta)\cos(\omega_0 t + \phi_o)$, the right-hand side of the differential equation is:

$$-2\omega_0\cos(mt + \theta)\cos^2(\omega_0 t + \phi_o) - m\sin(mt + \theta)\sin 2(\omega_0 t + \phi_o)$$
$$-2\omega_0\cos(mt + \theta)\sin^2(\omega_0 t + \phi_o) = -2\omega_0\cos(mt + \theta),$$

on neglecting the high frequency term. For a control voltage input $\beta(t) = \cos(mt + \theta)\sin(\omega_0 t + \phi_o)$, the right-hand side is:

$$-2\omega_0\cos(mt + \theta)\sin(\omega_0 t + \phi_o)\cos(\omega_0 t + \phi_o)$$
$$-2m\sin(mt + \theta)\sin^2(\omega_0 t + \phi_o)$$
$$+2\omega_0\cos(mt + \theta)\sin(\omega_0 t + \phi_o)\cos(\omega_0 t + \phi_o) = -m\sin(mt + \theta)$$

on substituting $\sin^2(\omega_0 t + \phi_o) = (1 - \cos 2\omega_0 t)/2$ and neglecting the high frequency term. (The assumed justification for neglecting the high frequency terms is that their average or macroscopic effect is zero.)

For a control voltage input $\beta(t) = \cos(mt + \theta)\cos(\omega_0 t + \phi_o + \phi) = \cos(mt + \theta)[\cos(\omega_0 t + \phi_o)\cos\phi - \sin(\omega_0 t + \phi_o)\sin\phi]$, by the preceding paragraph, the right-hand side is:

$$\cos\phi[-2\omega_0\cos(mt + \theta)] = \sin\phi[-m\sin(mt + \theta)].$$

Thus the a-c. motor is effectively equivalent to a demodulating device followed by a d-c. motor.

The general linear demodulator characteristic would be that for an input $\beta(t) = X\cos(mt + \theta)\cos(\omega_0 t + \phi_o + \phi)$, the output is $|P(\phi, m)|X \times \cos(mt + \theta + \rho)$, where $\rho = \angle P(\phi, m)$. In the above case of the right-hand side of the differential equation of the two-phase motor, the output is:

$$\begin{aligned}|P(\phi, m)|X\cos(mt + \theta + \rho)\\ = -2\omega_0 X\cos\phi\cos(mt + \theta) + mX\sin\phi\sin(mt + \theta)\\ = XR[-2\omega_0\cos\phi\, e^{j(mt+\theta)} + m\sin\phi\, e^{j(mt+\theta-\pi/2)}].\end{aligned}$$

Therefore in this case

$$P(\phi, m) = -2\omega_0\cos\phi + m\sin\phi\, e^{-j\pi/2} = -2\omega_0\cos\phi - jm\sin\phi.$$

Any linear carrier frequency servo may now be analyzed. The characteristic of the servo controller for the a-c. servo is the combination of a characteristic $G(p)$ as in the previous section with the demodulator and motor characteristics. The output of $G(p)$ is applied to the control winding of the motor (or more generally the output of $G(p)$ is applied to a demodulator characteristic $P(\phi, m)$ and the demodulated output is applied to the motor). As in the previous section, if the input to $G(p)$ is $\epsilon_1\cos mt\cos\omega_0 t$, that is $\epsilon = \epsilon_1\cos mt$, the output is

$$\begin{aligned}E_o = \epsilon_1[A(m)\cos(mt + \phi_1)\cos(\omega_0 t + \phi_o + \phi)\\ + B(m)\sin(mt + \phi_1)\sin(\omega_0 t + \phi_o + \phi)]\end{aligned}$$

where

$$\begin{gathered}A(m) = (|G^-| + |G^+|)/2 = (|G[j(\omega_0 - m)]| + |G[j(\omega_0 + m)]|)/2,\\ B(m) = (|G^-| - |G^+|)/2, \quad \phi^+ = \angle G^+, \quad \phi^- = \angle G^-,\\ \phi_1 = (\phi^+ - \phi^-)/2, \quad \phi = -\phi_o + (\phi^+ + \phi^-)/2, \quad \phi_o = \angle G(j\omega_0).\end{gathered}$$

For an input:

$$\begin{aligned}\beta(t) = A\cos(mt + \phi_1)\cos(\omega_0 t + \phi_o + \phi)\\ + B\cos(mt + \phi_1 - \pi/2)\cos(\omega_0 t + \phi_o + \phi - \pi/2)\end{aligned}$$

to the demodulator characteristic $P(\phi, m)$, the demodulator output (right-hand side of the motor differential equation) is:

$$\begin{aligned}|P(\phi, m)|A \cos (mt + \phi_1 + \rho_1) \\ + |P[(\phi - \pi/2), m]|B \cos (mt + \phi_1 + \rho_2 - \pi/2) \\ = R\{[P(\phi, m)Ae^{j\phi_1} + P[(\phi - \pi/2), m]Be^{j(\phi_1-\pi/2)}]e^{jmt}\} \\ = |Q(m)| \cos (mt + \tau), \\ Q(m) = P(\phi, m)Ae^{j\phi_1} + P[(\phi - \pi/2), m]Be^{j(\phi_1-\pi/2)},\end{aligned}$$

$\tau = \angle Q(m)$. (The quantities ϕ, ϕ_1, A, B depend on m through $G(p)$.) The linear characteristic $Q(m)$ is the over-all characteristic of the portion of the servo controller including $G(p)$ and the demodulator; for an error $\epsilon = \cos mt$, the differential equation of the motor is:

$$T\ddot{\theta}_o + \dot{\theta}_o = |Q(m)| \cos (mt + \tau), \quad \tau = \angle Q(m).$$

Thus the servo controller characteristic is $\theta_o/\epsilon = Q(m)/(-Tm^2 + jm)$. From this θ_o/ϵ may be easily calculated, and the Nyquist or gain-phase margin diagram constructed, following the procedure outlined in the next section.

In the case of the two-phase motor,

$$P(\phi, m) = -(2\omega_0 \cos \phi + jm \sin \phi),$$

and

$$\begin{aligned}Q(m) &= -(2\omega_0 \cos \phi + jm \sin \phi)Ae^{j\phi_1} + j(2\omega_0 \sin \phi - jm \cos \phi)Be^{j\phi_1} \\ &= [(-2\omega_0 A \cos \phi + mB \cos \phi) + j(2\omega_0 B \sin \phi - mA \sin \phi)]e^{j\phi_1}.\end{aligned}$$

6. CONSTRUCTION OF GAIN-PHASE MARGIN DIAGRAM FOR ANY $G(p)$ WITH TWO-PHASE SERVO MOTOR.

From the servo controller characteristic and calculation of $Q(m)$ in the preceding section, the angle of $\theta_o/k_v\epsilon$ evidently is:

$$\angle \frac{\theta_o}{\epsilon} = \phi_1 + \alpha - 90° - \arctan mT$$

where

$$\tan \alpha = -\frac{[2\omega_0 B(m) - mA(m)] \sin \phi}{[2\omega_0 A(m) - mB(m)] \cos \phi}.$$

Thus we have:

$$\text{phase margin} = 90° + \phi_1 + \alpha - \arctan mT.$$

By a preceding discussion, the reciprocal of the error for a step-velocity input is the velocity error constant

$$\begin{aligned}k_v = \lim_{m\to 0} jm(\theta_o/\epsilon + 1) &= \lim_{m\to 0}[Q(m)/(Tjm + 1) + jm] \\ &= Q(0) = -2\omega_0 A(0) = -2\omega_0|G(j\omega_0)|.^4\end{aligned}$$

---

[4] The minus sign means that actually the servo corresponding to the two-phase motor demodulator characteristic $P(\phi, m)$ is unstable; for stable operation the connections to the control winding of the motor would merely be reversed, so that the demodulator characteristic would be $-P(\phi, m)$. Or in the case of synchros, the normal position of the output shaft would rotate 180° with respect to the input shaft, reversing the sign of the error.

Therefore, we have also:

$$\left|\frac{\theta_o}{k_v\epsilon}\right| = \frac{\{[2\omega_0 A(m) - mB(m)]^2 \cos^2\phi + [2\omega_0 B(m) - mA(m)]^2 \sin^2\phi\}^{\frac{1}{2}}}{2\omega_0 |G(j\omega_0)| m(1 + m^2T^2)^{\frac{1}{2}}}$$

$$= \frac{[2\omega_0 A(m) - mB(m)] \cos\phi \cos(\text{arc tan } mT)}{2m\omega_0 |G(j\omega_0)| \cos\alpha}.$$

In proceeding with the calculation of the required numerical values for the construction of the gain-phase margin diagram, it is convenient to read arc tan x and cos (arc tan x) $= 1/(1 + x^2)^{\frac{1}{2}}$ simultaneously from a table of natural trigonometric functions. A tabular procedure is recommended for obtaining the values of phase margin and corresponding ratios $|\theta_o/k_v\epsilon|$. For successive values of modulating frequency m, find and enter in the table: ϕ^+, ϕ^-, ϕ, $\cos\phi$, $|G^-|$, $|G^+|$, $A(m)$, $B(m)$, $[2\omega_0 A(m) - mB(m)]$, $\tan\alpha$, $\cos\alpha$ from $G(p)$, mT, cos (arc tan mT) from the motor time constant T, then finally phase margin, ratio $|k_v\epsilon/\theta_o|$, and $|\theta_o/k_v\epsilon|$ in $db = -|k_v\epsilon/\theta_o|$ in db.

For a carrier frequency of 60 cycles, that is $\omega_0 = 2\pi 60 = 377$, Table I, used in the construction of the gain-phase margin diagrams in the next section, will be useful to the reader in constructing diagrams for any additional cases of $G(p)$ and $P(\phi, m)$ in which he may be interested.

TABLE I.

arc tan mT	mT	$(1 + m^2T^2)^{-1/2}$ $= \cos(\text{arc tan } mT)$	m for $T = 0.2$ sec.	$\frac{m}{2\pi}$ in cps.	$\frac{2\omega_0 - m}{2(\omega_0 - m)}$	$\frac{2\omega_0 + m}{2(\omega_0 + m)}$	$\frac{\frac{11}{6}\omega_0 - m}{2(\omega_0 - m)}$	$\frac{\frac{11}{6}\omega_0 + m}{2(\omega_0 + m)}$	$\frac{\frac{13}{6}\omega_0 - m}{2(\omega_0 - m)}$	$\frac{\frac{13}{6}\omega_0 + m}{2(\omega_0 + m)}$	$\frac{\omega_0 - m}{\omega_0}$	$\frac{\omega_0 + m}{\omega_0}$	arc tan $\frac{15}{\omega_0}m$	$\cos\left(\text{arc tan } \frac{15}{\omega_0}m\right)$
50°	1.192	0.643	5.96	0.95	1.008	0.992	0.924	0.910	1.093	1.074	0.985	1.015	13.3°	0.973
60°	1.732	0.500	8.66	1.38	1.012	0.989	0.925	0.909	1.097	1.070	0.978	1.022	19.0°	0.9455
70°	2.75	0.342	13.75	2.19	1.019	0.982	0.933	0.902	1.105	1.063	0.963	1.036	28.65°	0.8775
80°	5.67	0.1736	28.35	4.52	1.041	0.965	0.951	0.886	1.131	1.043	0.925	1.075	48.4°	0.664
85°	12.56	0.0794	62.8	10.0	1.100	0.929	1.00	0.857	1.200	1.000	0.835	1.167	68.2°	0.3714
87°	19.08	0.0523	95.4	15.2	1.169	0.899	1.06	0.832	1.281	0.966	0.741	1.252	75.2°	0.255
88°	28.64	0.0349	148.3	22.8	1.324	0.859	1.16	0.806	1.462	0.919	0.607	1.392	80.4°	0.167
89°	57.29	0.0175	286.5	45.6	2.583	0.784	2.23	0.737	2.930	0.831	0.240	1.76	85°	0.0872

7. PROPORTIONAL-DERIVATIVE FREQUENCY-PHASE CHARACTERISTICS.

The ideal proportional-derivative stabilizing characteristic for a carrier frequency servo is $G(j\omega) = 1 + jT_d(\omega - \omega_0)$. It may be shown that if a modulated carrier voltage $m(t) \cos\omega_0 t$ is impressed on this characteristic, the output is $[m(t) + T_d\dot{m}(t)] \cos\omega_0 t$. The ratio T_d of the coefficients of the differentiated and proportional components is called the "derivative time constant."

A parallel "T" network (Fig. 16) may be used in $G(p)$ to obtain an approximation to the ideal proportional-derivative characteristic. Part II of this paper will be concerned with the design of parallel "T" networks for this purpose (and with the allowable tolerances in the values of the R and C components).

The approximation to the ideal proportional-derivative characteristic which it is possible to obtain with the parallel "T" network is

$$G(j\omega) = \frac{l}{T_d} \frac{1 + jT_d \dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)}{1 + jl\dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)} \tag{7-1}$$

where the time lag l cannot be larger than $1/\omega_0$.

It has been found experimentally that different servo setups require different values of T_d for best stability; however T_d is not at all critical. Values of $T_d\omega_0$ in the range of 5 to 50 have been found useful.

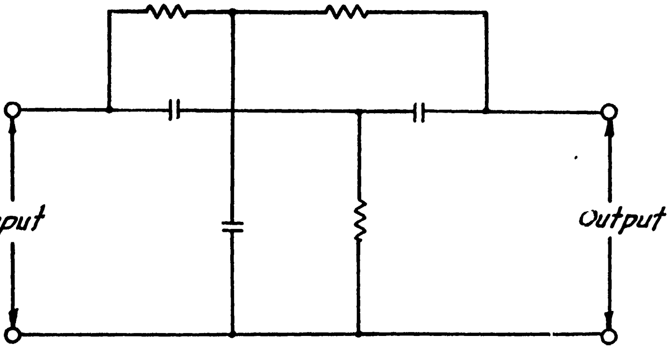


FIG. 16. Parallel "T" network.

As mentioned in section 3, curve (*a*) in Fig. 9 is the gain-phase margin diagram for a simple proportional controller, that is, a controller in which $G(p)$ is identically 1 or a constant, so that the controller characteristic $H(p)$ is the motor characteristic $1/(Tp + 1)p$. Comparing with Fig. 6, evidently even with low gains (k_v in the vicinity of 10 sec.$^{-1}$), there are peaks of large amplitude at low frequencies. In an actual servo system, where there are time lags not included in curve (*a*) (as for example a small time lag in the coupling circuits between stages of a vacuum tube amplifier), as the gain is increased, the servo output quickly will begin to oscillate violently and continuously, with no input; that is, the servo will be unstable.

Curve (*b*) in Fig. 9 is the corresponding plot for the servo controller consisting of the ideal proportional-derivative characteristic with $T_d\omega_0 = 15$, and the simplified motor of section 5, with $T = 0.2$ sec. This value of T is approximately the value for the Diehl FPE-49-2 motor. Comparing with Fig. 6, evidently the servo now is very stable at all values of k_v. That is, if k_v = 560 sec.$^{-1}$ ~ 55 *db,* the frequency response $|\theta_o/\theta_i|$ is nearly flat from 0 to 20 cps. or higher, and the output θ_o of the servo is a very faithful copy of the input θ_i, even though θ_i may be of such a nature as to have components of rather high frequency. In particular the output for a step displacement of the input θ_i will also be very nearly a step, and will have no overshoot. This is unlike the situation for the proportional servo (with the same velocity error constant), where for a step input the output has many overshoots and only gradually settles down to the displacement corresponding to the input displacement.

Curve (*c*) is the decibel-phase margin plot for the proportional-derivative characteristic $G(j\omega) = \dfrac{1+jT_d(\omega-\omega_0)}{1+jl(\omega-\omega_0)}$ with time lag $l = 0.5/\omega_0$, and $T_d\omega_0 = 15$. Curve (*d*) is the plot for the approximate proportional-derivative characteristic (7-1), which may be obtained with the parallel "*T*" network, for the same values of l and T_d.

In all curves of Fig. 9, the carrier phase is assumed to have the proper value for the control winding of the motor. To examine the effect of incorrect carrier phase, let us compare the right-hand side of the equation in section 5,

$$2\omega_0 \cos \phi \cos mt - m \sin \phi \sin mt,$$

with the output of the (d-c.) proportional-derivative characteristic $(1 + Ujm)$ when $\cos mt$ is applied,

$$\cos mt - mU \sin mt.$$

Thus if the carrier phase is incorrect, then effectively another proportional-derivative characteristic $(1 \pm Ujm)$ is in cascade with the stabilizing characteristic $(1 + T_djm)$, with $U = \tan \phi/2\omega_0$. The gain is reduced by the factor $\cos \phi$, although the actual voltage on the motor (and its heating effect) is not reduced. Also the maximum available torque of the motor, that is, the torque at saturation, is reduced by the factor $\cos \phi$. If gain and maximum torque reduction by a factor of 0.5 is tolerable, then as much as 60° phase shift may be allowed, since $\cos 60° = 0.5$. For errors ϕ in carrier phase up to 45° or 60°, $U\omega_0$ is less than 0.5 or 0.87, so for $T_d\omega_0 > 5$, the effect on stability of $(1 \pm Ujm)$ is comparatively quite insignificant.

The high values of k_v attainable with the parallel "T," and the flat frequency characteristic, have been verified experimentally in a servo setup using 5*G* synchros and the Diehl FPE-49-2 motor. (We had k_v = 5000 sec.$^{-1}$ with satisfactory freedom from chatter; the only limitation on k_v seems to be backlash in the gear train.)

Curves (*e*) and (*f*) in Fig. 9 are decibel-phase margin plots for the servo controller consisting of the motor of section 5, and the parallel

"T" approximate proportional-derivative characteristic, the characteristic however being tuned to frequencies $\omega_1 = 2\pi 70$ and $\omega_1 = 2\pi 50$, instead of to the carrier frequency $\omega_0 = 2\pi 60$. There is then respectively a phase lag of 64.5°, or a phase lead of 62.1°, at carrier frequency ω_0. Including this phase shift, the phase of the carrier at the control winding of the motor is assumed to be correct (if not, the previous remark on the small effect of incorrect carrier phase still applies, and for phase errors up to 65° the stability is not appreciably different from that indicated by curves (*e*) and (*f*)). It is evident from the diagrams that when the parallel "T" network is tuned to a frequency as much as 10 cycles *lower* than the 60-cycle carrier, the stability still is very satisfactory at high values of k_v. However, when the parallel "T" network is tuned to a frequency 10 cycles *higher* than the 60-cycle carrier, even with the optimal setting of k_v (about 3000 sec.$^{-1}$), there will be a peak in the region of 15 to 25 cps. of height greater than 2. (For deviations of ± 8 cycles, these conclusions also have been verified experimentally.)

8. EFFECT OF PHASE-LAG NETWORKS ON STABILITY.

Experimentally, it has been found satisfactory to shift the phase of the carrier to obtain the proper phase at the control winding, by inserting into the servo controller a one- or two-section low pass *R*-*C* filter preceding the proportional-derivative parallel "T." (Phase-lag networks will be discussed in Part III of this paper.) When this is done, the proportional-derivative characteristic is multiplied by $1/(Up + 1)$ or by $1/(U_1p + 1)(U_2p + 1)$. The effect on stability to be expected is indicated by curves (*g*) and (*h*) in Fig. 9. Curve (*h*) will be approached as U becomes very large, or as one of U_1, U_2 approaches infinity and the other approaches zero. Curves (*g*) and (*h*) indicate, for a two-section filter to obtain say 100° phase lag, that better results, as regards keeping the amplification of noise or jitter and backlash chatter small, will be obtained by having the roots U_1^{-1} and U_2^{-1} as unequal as possible, than by having them nearly equal. Also, the high rate of decrease of phase margin with respect to error frequency in the range 15 to 45 cps., in combination with other time lags in the system not included in curve (*g*), may help to explain an observed tendency of the servos to oscillation at the subharmonic frequency of 30 cps.

REFERENCES.

(1) A. C. HALL, "Application of Circuit Theory to the Design of Servomechanisms," JOURNAL FRANKLIN INST., Vol. 242, October, 1946, pp. 279–307.

(2) J. L. BARNES AND M. F. GARDNER, "Transients in Linear Systems," Vol. I, New York, John Wiley & Sons, Inc. (1942).

(3) H. W. BODE, "Network Analysis and Feedback Amplifier Design," New York, Van Nostrand & Co. (1946).

(4) L. A. MACCOLL, "Fundamental Theory of Servomechanisms," New York, Van Nostrand & Co. (1946).

Radioactive Chemical Precautions.—In experimenting with radioactive chemicals from the atomic piles at Oak Ridge, plant scientists of the U. S. Department of Agriculture have stopped using the familiar clay pots and jars in their greenhouse experiments. Instead they are using inexpensive tin cans coated with enamel. After each experiment they discard the cans and soil that contains the radioactive material, and bury them deep in the ground to get rid of them.

The reason is simple. In checking the action of the radioactive chemicals moving out of the soil and into and through the plant, the scientists make use of the delicate "tracer" method. Counting instruments detect and record the passage of charged molecules as they pass through the plant tissues. It is necessary to have exact knowledge as to the radiant energy present at the start of the experiment. This is possible by using fresh soil, a new container, and and exactly measured quantities of radioactive chemicals. But if a clay container had been used previously, the pot would have become at least slightly radioactive, and so would add some unmeasured radioactive energy that would complicate the experiment. It would disturb what the scientists term the "control" of conditions.

R. H. O.

X-Ray Gage Controls Sheet Thickness.—In answer to the thickness gaging problems in modern high speed rolling mills, there has been developed an X-ray thickness gage offering unique advantages of (1) no contact with the material being gaged, (2) ready access to areas never before gaged, and (3) high sensitivity and accuracy.

Present applications to sheet metals indicate that the gage will increase output of prime sheet, solve some operating problems, and eliminate marring of the surface of the materials being gaged. As a quality control instrument, this equipment is expected to find application in the thickness control of hot metal and glass sheet, aluminum foil, plastics, paper, and other sheet and strip materials.

Employing two X-ray sources at right angles with one photomultiplier-fluorescent screen pick-up unit at the apex, the gage compares the controlled material to a correct sample. The two X-ray sources emit X-rays 180° out of phase.

Positive and negative readings showing the thickness differential above or below the standard sample are shown on an instrument within view of the operator. The gage automatically rejects off-gage material in flying shear applications or controls the screw downs to maintain uniform thickness in material coming from a cold mill. One model now installed on a tinplate line is designed for thickness of sheet from 0.005 to 0.050 in., but the basic equipment is the same for other applications. The indicator needle on the gage instrument will react to a variation of 0.0001 in. in a 0.1-in. thick sheet.

It is unaffected by fluttering of the sheet, surface coatings such as lacquer, or by atmospheres heavy with palm oil, steam, and water. Normally mounted on a slide so that it can be removed from the pass line, it can also be arranged to rove across the sheet.

R. H. O.

THE FULL RESPONSIBILITY OF MANAGEMENT.*

BY

WILLIAM J. MEINEL.[1]

We are assembled here tonight to pay honor to Mr. Charles Day, who did much for business and industry as one of the outstanding pioneers in modern industrial management. He was a great engineer and leader in his field.

I think we should also pay honor to The Franklin Institute and its staff for the great job they are doing in research and development, for which they recently received a gold medal. They are aiding our government in improving its defense weapons in the interest of keeping America strong.

I have been asked to speak to you tonight on the subject of management in business and industry. I must confess that this embarrasses me, since many of you in this audience are better qualified to talk on this subject.

In discussing the responsibilities of management, it is my motive to impress upon you the importance of being modern, alert and everlastingly on the look-out for new business opportunities.

Keeping our factories and our products up-to-date; making every effort to be well informed; keeping abreast of the times, especially with respect to the many external influences which affect our business affairs; having the courage to accept new responsibilities; and taking time out for honest appraisal of ourselves and the effort which we are devoting to our jobs, are all required of good management and should be foremost among the policies of modern executives.

Wisdom in management also demands that modern executives have a definite program of training those who are to succeed them as leaders. Charlie Schwab, for example, selected Eugene Grace to serve his great institution when he no longer was able to carry the burden of management. He sensed that a younger man was required to plan for the future of that business. At the present time, Mr. Eugene Grace is training a younger man by the name of Homer to accept the full responsibility of top management in the Bethlehem organization.

Whenever possible, these future leaders should be chosen from among the young men in your organization.

They should be the men who have shown initiative and some capacity to perform the lighter tasks of management, and who have the physical and mental vigor demanded of those in high executive positions. Our training programs should, if possible, follow a regular prescribed form, arranged so that nothing of importance is overlooked in the business education of these future executives. A well-thought-out plan for training our successors is the best insurance with which we can endow our company. It is the means by which we pass along to others a store of knowledge useful to the business long after we have retired from active participation. This is our contribution to the future.

---

* Presented under the Charles Day Lecture Foundation of The Franklin Institute of the State of Pennsylvania, given in the Hall of The Franklin Institute at the Stated Meeting held May 19, 1948.

[1] President, Heintz Mfg. Co., Philadelphia, Pa.

Our responsibility to the present is a much more pressing matter. If we are to be regarded as good managers, we must find or create opportunities for the entire business to prosper, to progress, expand and improve.

It is only through constant improvement that we can maintain our position in the world's markets and so remain in business. If we fail to make improvements, we are certain to lose ground. We must advance, or go out of business as failures.

On this point I know that many of you take great pride in a particular product of your company, which you consider the last word in improvement and I have no doubt there is justification for your satisfaction.

My only suggestion is, do not become too complacent. Philco, Frigidaire, Buick, Ford, Dodge, Luckies, Camels, Wheaties and a host of other products have been made a part of our everyday lives by good businessmen with a plan of constant adherence to the policy of product improvement. Who would be so foolhardy as to insist that there has not been a profound improvement in these products? Even tobacco smokers receive an incredible amount of attention from the producers of smokers' requirements. Each year the makers of cigarettes spend millions of dollars to urge upon us the information that their products have been improved by being "moisturized," "toasted," "cooled," or by some betterment not found in competitive brands. The improvement which has been made in these products is undoubtedly the reason why they have retained their popularity for so many years. I think it goes without question that keeping up-to-date should be the first concern of good management.

While on this point, I should like to discuss another side of this problem of keeping abreast of the times.

In the past several years I have visited a large number of factories in the eastern part of the United States and I have found an almost unbelievable amount of neglect of the machinery, equipment, buildings and other facilities, upon which the very existence of some of these businesses depends. I have always been inclined to think the external appearance of a factory and the amount of attention paid to cleanliness and maintenance is a good guide as to the calibre of management of a business.

Let me suggest that you include plant conservation in your keeping-up-to-date program. You will find it one of the most gratifying accomplishments of management for the reason that everyone associated with the business is visibly benefitted. Employees have a better place in which to work and as a result, the output of the plant is usually increased; the assets of the business are properly conserved; the appearance of the plant is improved to the point where our neighbors become proud of us (I shall have more to say on the subject of neighbors later in this message).

We have looked carefully at our products and our properties, so now let us take a careful scrutiny at ourselves.

What about our attitude toward our jobs? How much enthusiasm do we bring to the office every day? How many divisions of the business do we find genuinely interesting? How well informed are we in the case of those divisions of the business which we find a bit dull or commonplace? How much civic pride do we take in the community where our plant is located? These are only a few of the matters which test our talent as executives.

Enthusiasm for our jobs is usually reflected in those who work with and for us and is a tremendous factor in creating employee interest and loyalty. No other single factor is as important to good labor relations as a workingman's sound confidence in the managers of his company. Right here is what I consider the most important part of my message. The one way management can build confidence and earn the loyalty of its workingmen is to be on the job every day. The man who runs a lathe in the shop is always proud (and therefore gratified) to know the company executives. Let us not delude ourselves about the man who works for us. He is an intelligent human being and is quick to learn whether or not the management of his company is putting real effort into making a success of the enterprise. If he has assurance that the executives are going to be on the job every day, he learns to group himself with them as a team.

Therefore, if the executives are enthusiastic, then so is the remainder of the organization.

Contrast this situation with the case where there is absentee management, where a company maintains its home office in Philadelphia and its factory in Boston. No one among the workingmen ever sees the executives of his company, and too often the same thing applies to many of the supervisors of the factory. They, and the home office executives, are complete strangers.

I am about to make a statement which may surprise you. The management of big business is, in many cases, responsible for the numerous difficulties with which business is confronted in dealing with today's labor. It would seem to me that there was every reason for businesses in America to expand their operations and increase their output, so that low-cost production could be made available to more people at prices within the range of their ability to pay. The automobile, refrigerator and many other necessities and luxuries would not be available to the average workingman if big business had not made the tremendous investment in tooling that was not possible on the part of the smaller company.

I am trying to point out to you that the merging of small business and the building up of large companies was fully justified. However, the employee no longer had the comfort of knowing top management. The executives believed that the appointment of a personnel manager would take care of the human relation problems that are so important to every business.

There is no doubt that the personnel manager has his rightful place. However, he cannot take the place of top management where the important business decisions are made, particularly those that affect the security of the workingman.

One other practice of big business that contributes some to our worker difficulties is the shifting of top management from one division of the business to another. Every time this is done, the entire organization is obliged to make a new start, and until the employees learn what new duties are assigned to them by the incoming manager, the operations of the branch are unsatisfactory.

Many of the lessons learned and policies established by the departing manager must be discovered anew by his successor.

If the departing manager had been actively interested in local civic matters, or had been highly regarded by his workers and the community as a whole, there is certain to be some antagonism toward his successor, who cannot possibly know how far he must go in filling the void created by the departure of his predecessor.

I know of one large company which was as much as 15 years ahead in its labor policy. This company pioneered collective bargaining. The Wagner Act made little or no change in their industrial relations set-up, since they had followed the mandates of this Act for many years before it became a law. They had a very happy employee situation until they adopted a policy of shifting top management. The difficulties they have encountered over the past 5 or 6 years are, in my opinion, a direct outcome of the many changes they made in the managers of their various plants located in many cities throughout the country.

Another disturbing action of absentee management is the stopping or changing of production for reasons which are seldom made clear to the people who are most affected—the workers of the factory. Imagine the resentment that is created.

No other single action of management is as likely to cause the workers to follow a professional organizer (whether he presents a good union or a bad one, and whether he is capable of helping them, or is just an adroit twister of the truth).

This brings to mind a subject discussed at some length by Mr. Ralph Kelly when he addressed this group on May 21, 1947.

His statistics covering large industries indicated there has been a trend of diminishing output per man hour. I think Mr. Kelly's question of how are we to obtain a fair day's work for the high hourly rates we pay can be answered fairly by saying management should set the pace by putting more effort into its own executive positions and should encourage its supervisory and foremen groups to do the same. We must remember we are always happier at our jobs when the man next to us is enjoying his. Supervisors and foremen who display enthusiams for the job at hand will constantly create worker enthusiasms for the same job. Worker enthusiasms raise output and our goal is achieved pleasantly.

An honest analysis of our own attitude toward our associates and employees is something we should not neglect when faced with the problem of obtaining a fair day's work for the rates we pay.

Modern management should learn to accept a wide variety of new responsibilities. For example, there are a great many cases where business executives fail to create either good will or good public relations solely because they do not put something of themselves into the solution of their problems. I think you will agree with me when I say that public relations are a responsibility of top management, for it is the conduct and policies of top-bracket management that come to the attention of the public.

With this thought in mind, I feel community activities should be given more consideration. Beginning with the young people in our neighborhood, we can create good will by taking the local elementary school classes on a conducted tour of our factories. This should be a natural thing for us to do, since it gets us acquainted with these young folks and we are no longer the big stranger in their midst.

Among our neighbors, who are a bit older than the grade school group, we can take an interest in individuals who are making better than average progress in their schooling. By learning who these particular individuals are, we may make it a point to be helpful to them in many ways.

Later on, when they start business careers, we could expect to receive a number of ambitious and friendly neighbors into our organization. Any company with a policy of this type is certain to have the good will of the community in which its plant is located, and if a company has a policy of promoting employee welfare, there will be small doubt about the success of its operations.

An example of our interest in the employee is a practice we recently initiated that serves to create new opportunities for working men of our organization. For several years we have had an educational program under which promising young college graduates are brought into our organization for industrial training. These men are given a chance to spend some time in all of the major departments of our plant. I am proud to tell you that the results have been most gratifying to us and we presume satisfactory to the young men, for not one of them has left our employ and they all show considerable interest and enthusiasm in the knowledge which one acquires through association with the working-men of our plants.

I am also proud to tell you that the workers who have been entrusted with the training of these young men all exhibit a tremendous amount of pride in the progress that is being made.

Within the last few weeks we have begun a new program, which is intended to create new opportunities for our workmen. What we are doing is this: We select from our employees a group of men who have been denied the advantages of advanced education for one reason or another. The individuals comprising this group will be sent to college for a term of intensive courses. They will be paid their regular salaries or wages, just the same as if they were at work every day. When the courses are completed, they return to their former jobs, where it is presumed they will be equipped to make progress that would otherwise have been impossible.

I can tell you frankly it is the avowed purpose of our company to maintain good will and understanding between men and management. Acts of good will, fairness and a genuine interest in our employees are all considered to be part of the routine conduct of our business.

I trust I might say a little to you about the management of our city. We all know that not too many years ago our municipality was highly regarded and was considered the workshop of the world.

While I have covered briefly a few of management's responsibilities in business, I think it only proper that the management of our city and county departments receive the same careful consideration that seems to be necessary if our businesses are to be successful.

We cannot have a community that is looked upon as a dying city and expect the businesses in such a city to prosper. Industrial leaders must take a wholesome interest in the management of our community so that the community and its industries prosper as a team.

During the past week I had a discussion with an outstanding business executive, who stated that our city has lost it standing as a progressive community. He tells me that one of the largest trust funds in the country, operating with its main office in Philadelphia, makes practically all its investments in companies outside Philadelphia. They prefer to supply funds to wide-awake municipalities teamed up with business and industry.

The Committee of Fifteen are finding the various city and county departments poorly equipped, inefficient and in no sense modern in their scope.

I am sure that an improvement in our city could be had if our business leaders were more mindful of the assistance they can give to better the administration of this city.

I am hopeful that a greater interest in civic matters on the part of our business leaders might serve to bring Philadelphia back to its rightful place as a great city.

I trust I have not tired you with too serious a talk on the subject of management's responsibilities.

Thank you very much for your patience.

Paint Defies Flame. (*Science Illustrated*, May, 1948.)—A new fire retardant paint, the only one listed by the National Board of Fire Underwriters, is called Albi-R and is made by the Albi Manufacturing Co. It can be used on wood, plywood, fiber board and fabric. The Albi coating retards fire because, when heated, it puffs up, providing good insulation. After the fire has been extinguished, the coating can be scraped off, leaving undamaged wood underneath. In a two-room test bungalow, one room of which was painted with Albi-R and the other with ordinary paint, a fire completely destroyed the one half and left the Albi-painted half hardly warm.

Business Teams With Research.—The aerosol method has proved so successful and profitable in controlling several of the worst greenhouse pests that commercial application is running neck and neck with an active research program. Commercial growers are so anxious to apply the newest improvements that several consult regularly with the Federal workers to keep informed of the progress of their tests. Some of these growers then risk applying on commercial scale the treatments the Bureau of Entomology and Plant Quarantine investigators regard as most promising. The Bureau's research program benefits in turn from the practical experience reported by the commercial greenhouse men. Thus the differential between research results and practical benefit is readily and quickly determined and adjustments made.

The aerosol method is most familiar to the public in the form of an aerosol "bomb" loaded with an insecticide dissolved in a gas liquified under pressure—usually DDT or pyrethrum or a combination of the two in Freon gas. When the nozzle is opened the gas escapes and carries the insecticide in a highly effective atomized mist or cloud.

DDT aerosols have been used widely in greenhouses, and other of the new insecticides are under test. The aerosol equipment at Beltsville treats a greenhouse at the rate of 1,000 cu. ft. in 4 sec. Using the DDT aerosol, experimenters have treated more than 150 varieties of plants with injury to only two—cucumbers and kalanchoes.

A newer chemical, hexaethyl tetraphosphate, when used in aerosols, has given strikingly good control of several of the most troublesome greenhouse pests—red spider, several aphids, whitefly, and thrips. Last year several greenhouse operators applied this chemical in large-scale tests using the dosage the Federal experimenters developed. It proved highly effective, particularly against the red spider. Greenhousemen were able to stop hand "syringing" of roses. Freed of the pest, the roses yielded 30 to 50 per cent more flowers from the same bench space. Red spider and aphids have also been serious pests of carnations. Under some conditions this hexaethyl tetraphosphate aerosol may injure tomatoes and some chrysanthemum varieties.

Newest of the insecticides to be tried in aerosols is a complex organic chemical named parathion. Although this material has proved highly effective against many pests and has not injured any of the more than 50 greenhouse crops on which it had been tested, nevertheless parathion is very poisonous to warm blooded animals and the Bureau is not advising its use. If it is used extreme caution is advised especially in closed spaces.

R. H. O.

THE STORY OF THE NEW BENJAMIN FRANKLIN HALF-DOLLAR.[1]

BY

MRS. NELLIE TAYLOE ROSS.[2]

I am greatly complimented and most appreciative to have been invited here tonight to join in a celebration of the latest honor that has come to the great American patriot, Benjamin Franklin. It is most fitting that such a celebration should take place in this historic city, which is rich in traditions and with which the fame of Franklin was so closely linked. It is also fitting that it should be sponsored by an institution of such outstanding distinction as this which bears his name.

Dr. Allen and his associates, through whose grace I am here, did not intend, I believe, when first they approached me about coming, that I should be asked to talk for my supper. The idea seemed to develop, along with the planning, that you might be interested in hearing from one close to the Mint, something of what is involved in the changing of a design on a coin; and a few observations, perhaps, on related subjects.

You might think, if you have given no thought to the subject, that it would be a small matter to change the design of a coin, it being such a small object. There is much more to it, I assure you, than first meets the eye.

By what authority, you may ask, can a change be made? It is by authority of the Congress of the United States. The Coinage Act empowers the Director of the Mint to change the design of any coin, with the approval of the Secretary of the Treasury, but specifies that a change may not be made more often than once in 25 years; and then it is not mandatory.

Any alteration in the physical aspect of one of our coins is quite a serious responsibility. Most people probably pay little attention to the details of any of them, but let an innovation be proposed and it becomes, at once, a matter of intimate, personal interest to almost everybody. And well it may be—for after a new design is impressed upon a coin and it is once launched, it goes on and on into circulation until it is too worn for use.

In a recent radio talk I ventured to suggest this idea, in connection with the new Franklin piece, saying in effect, that when a coin is once released for circulation the public must accept it—like it or not! I had in mind the legal restraint against frequent changes of design. With all speed a letter came to my office, written in high dudgeon, by somebody in a distant state, charging me with being a typical Federal bureaucrat, arrogantly snapping my fingers in the face of the public.

---

[1] Full text of a talk given in Franklin Hall on the evening of April 29, 1948, when the new Franklin half-dollar was first seen. After the talk Mrs. Ross presented to The Institute Museum an exhibit showing how the coin was designed and produced.

[2] Director of the U. S. Mint, Washington, D. C.

The first concern of us all—the public—and certainly of officials directly concerned when a change is determined upon, is that the design to be adopted shall be worthy of a place on a United States coin. It must have artistic merit; it must have meaning. That is, it must tell a worth-while story in its symbolism and historic suggestion.

We all know that the history of ancient civilizations is, in considerable degree, traceable through coins that have been handed down; handed down by people who had lived and died centuries before the advent of the printing press. Needless to say, it is imperative that the devices, characters, and lettering on a coin's design be so developed and arranged as to be amenable to the coinage process. Only to a sculptor of recognized skill should, therefore, be entrusted the task of producing a coin design. The mere fact of one's having attained fame as a sculptor gives no assurance that he can produce a design satisfactory for use on metal, in bas relief. Some famous artists are shown, by Mint records, to have made dismal failures in that direction. Best results, I mention in passing, are to be expected from Mint sculptors who know something of the exigencies of the coinage process. A design may be perfectly beautiful in a clay model, the size of a dinner plate, and may have been executed by an eminent sculptor, but when reduced to the size of a coin, be lacking in distinction, or be unadaptable to coinage.

At best, the artist's product meets with many vicissitudes from the time of its appearance in the clay model until it finds its place on the little round of metal in the guise of a coin. Mint experts feel that they also, along with it, go through many vicissitudes in their effort to bring the artist's concept into harmony with legal requirements with respect to size and weight, and with the mechanics of coinage. That, as a rule, is a difficult task. The "ironing out of the bugs," as they say in the Mint, calls for good head work a-plenty, and for most meticulous hand work also.

The law itself prescribes the weight and content of a coin, as well as certain legends and characters it must bear. The design therefore, must permit such flow of metal as will produce the correct weight, and size. Thickness must be taken into account. The relief of the dominant details must be high enough to produce a handsome, clear-cut effect, but if slightly too high here or there, in relation to the outside rim, or in relation of one side to the other, rubbing and rocking will result, which prevents perfect stacking, and perfect packaging.

As a rule it is not until the design is reduced and impressed upon the working dies and not until they are subjected to experimentation, that flaws in the design (from the minting standpoint) reveal themselves.

Sometimes the models have to go back to the artist—more often Mint experts, in collaboration with the artist, make necessary alterations in the detail. They are not, as a rule, evident in the finished coin.

The reason is now clear, I hope, why we cannot announce far in advance the exact date a newly-designed coin will be ready to leave the Mint and start on its way into circulation.

You may be wondering how it came about that the portrait of Benjamin Franklin found its place upon the fifty-cent piece together with a replica of the Liberty Bell. For several years I had harbored the hope that during my tenure in the Mint these two features might be incorporated in a design for one of our coins. From time to time, letters have come in to my office urging the use of Franklin's portrait on the one-cent piece; probably because of his identity with the maxim "A penny saved is two-pence clear"—often misquoted "A penny saved is a penny made." You will agree, I believe, that the fifty-cent piece, being larger, and of silver, lends itself much better to the production of an impressive effect.

During the war it appeared for a time that the addition of one or more denominations might be required to ease the demand for one-cent pieces—the enormous production of which, under pressure of demand, was straining the coinage capacity of the Mint. The necessity did not materialize. In order to be ready, however, for any contingency, we assigned to our highly skilled sculptor in the Mint, Mr. Sinnock, the task of evolving a new design, to embody a likeness of Franklin and a representation of the Liberty Bell. That design was in readiness for the fifty-cent piece. I mention with sorrow that Mr. Sinnock's death occurred one year ago.

When, after the war, the extraordinary pressure of work upon the Mint had eased, we determined that the half-dollar should be given a "new look"—it having worn its old attire since the year 1916. We resolved that the figure of Liberty with her flowing robes on the obverse, and the eagle on the reverse, should give place to Franklin and the Liberty Bell; assuming of course, the approval of the Secretary of the Treasury. This, happily, was readily forthcoming with his expression of pleasure in the choice of the design.

I point out that the new fifty-cent piece is unique in one important respect. On it, the eagle is for the first time given a subordinate position on any of the silver coins of denomination above the dime. The coinage law under which we operate and which was enacted in 1873 provides that each of these silver coins must bear the figure of an eagle or representation thereof; ambiguous language that, you will agree. What could be a representation of an eagle but an eagle itself! We tried to persuade ourselves that the eagle was supposed to represent strength and freedom and that the Liberty Bell would suffice as a representation thereof. But in the end we decided that discretion was the better part of valor and we had better put it on to assure conformity with the law. Accordingly, Mr. Roberts, our capable sculptor at the Mint, very cleverly, and with pleasing effect, we think, added a small eagle at the right of the bell to balance with *E Pluribus Unum* on the left of it.

The subordination of the eagle on this particular coin is an interesting coincidence inasmuch as Benjamin Franklin, as Dr. Allen reminded me, condemned, in strong and picturesque language, the use of the eagle as a United States emblem. He called it a bird of prey, of bad moral character—a coward, lazy. The turkey, he contended, was more worthy and being a distinctly American bird would be a more suitable emblem.

Benjamin Franklin is also on record as disapproving the use of portraits on coins. Royal heads he had in mind apparently. He advocated the substitution of proverbs profitable for human reflection. Renowned for wisdom as Dr. Franklin was, he no doubt had some of the weaknesses to which most human flesh is prone; even a little vanity perhaps. Had he known in his day, that 150 years hence his image would be placed on a coin of this, his native land, to whose service his life was consecrated, we may assume, I believe, that he would not have been seriously displeased.

You of this company will agree, I am confident, that it is a distinction, well-merited, in view of his outstanding service to this country in its infant years, when it was struggling to establish itself as a free nation, and in view of his contribution to human progress in general.

Benjamin Franklin was truly an amazing person, as is evidenced in the eminence he achieved in the various roles he filled—statesman, scientist, philosopher.

It is difficult to define the attributes that make for greatness. It is certain that Benjamin Franklin had, in a high degree, such qualities of heart and mind as enabled him to command the confidence, even affection, of people in all walks of life who came within the orbit of his influence. They were, moreover, qualities that enabled him to win support to his objectives.

He was a man of unpretentious bearing, we are told. His attire was simple compared with that of most gentlemen of his stature, in a day when velvet and lace frills contributed much to masculine pulchritude. And yet in any company he entered, at home or abroad, he was at once an arresting, commanding figure. Scholars, statesmen, authors listened to him with great respect. Sophisticates at the court of Louis Sixteenth vied with one another for his attention. Marie Antoinette herself, influenced by his persuasive utterances, according to historians, took up the cudgels in support of the Revolutionary cause of the colonies.

Franklin's faculty for establishing strong friendships stood this country in good stead. The friendship that he won from the people of France during his long sojourn there as our envoy, proved to be, as we know, of priceless value to this country. His one great objective was to win a binding alliance between that country and ours. This he accomplished by dint of great patience and consummate tact. The signing and sealing of that pact was doubtless the greatest triumph of his life. It meant arms for our soldiers with which to fight. It meant money for food and clothes for them and shoes for their feet in a time of direst need.

As we ponder the value of this aid, the question arises, what, without it, would have been the destiny of this country? What would have been the destiny of us, in this generation? Who can reckon our indebtedness to Benjamin Franklin for that service?

As for his contribution to science, be assured I would not presume to exalt its importance in this erudite company, composed largely of persons who are themselves scientists of recognized eminence. I regret to say I can not talk the language of scientists and I fear I could not understand if they tried to explain it to me.

Monumental as were the deeds of this versatile genius in the fields of science and statesmanship, I doubt if they have served as much to keep his name alive down through the generations, even to this day, as has his homely, down-to-earth philosophy, expressed in language that everybody can understand, even the most unlettered person. The fact that his maxims have lived, are still familiar household words, and almost any day appear somewhere in public print, testifies to their general acceptance as sound guides for human conduct, whether or not they are consistently practiced.

Franklin is widely renowned as the apostle of thrift. At this time when a good part of the world is hungry and when a beefsteak here at home costs almost a king's ransom, it seems only good common sense to heed his precepts on thrift. All of us, I am sure, would like to hold on to a few shekels for future needs. Poor Richard says that "He that goes a borrowing, goes a sorrowing."

Thrift I count among the cardinal virtues, quite convinced that it has vital bearing upon the average person's chances of rising to high achievement, and even more important, vital bearing upon character.

It is only by the practice of thrift that a young person starting his career can expect, unless he has wealth bestowed upon him, to meet his financial obligations as he goes through life. That he must do if he is to preserve his self-respect and command the respect of others. How else can he provide, except by exercise of thrift, for the rainy day, if he is dependent upon his own resources, which is sure to come. How else can he hope to realize cherished ambitions that call for material assets.

That young person who starts out adult life spending all that he makes and accumulating nothing but debts is in a poor position to seize choice opportunities coming his way which might lead to outstanding success, and great usefulness.

Furthermore, Poor Richard says "'Tis hard for an empty bag to stand upright." We have all seen that truth exemplified in the person who indulges himself in an extravagant scale of living—to keep up, perhaps, with his neighbors the Joneses—which he knows his income cannot support. That person is following a dishonest course, for he is appropriating to himself the substance of others to which he has no right as though he were reaching his hand into their pockets.

Benjamin Franklin's philosophy I point out, was by no means limited to this one subject of thrift. It covered a wide range of human interests. Young men he admonished: "Keep your eyes wide open before the marriage ceremony, half shut afterwards." Good advice that! In exalting the simple life he opines "A little land well-tilled, a little house well-filled, a little wife well-willed, are true riches." Many of Dr. Franklin's most impressive utterances are in verse too long for comment here.

About the Liberty Bell on the reverse of the coin. I am deeply impressed with its significance to us as a country, as a symbol, at this juncture of the world's history. To me it is a soul-stirring thought that on this coin it will go out to the four corners of our country into the hands of practically every American. As one largely responsible for its being on the coin I express the ardent hope that it may be a constant reminder to us all how blest we are to live in this land of liberty; how blest we are that our forefathers had it in their minds and souls to establish in this new country, a free government, though it should cost them their lives, and to hand it down as such to their children of succeeding generations. So brilliant in mentality were those early statesmen who laid the foundation of this government, so forceful of character were they, and sound in integrity, it seems as if Heaven must have endowed them for that especial purpose.

We do well to remember that this precious freedom, bequeathed us by them, is ours, to have and to hold, if we jealously safeguard it and stand ever ready to defend it at any cost.

No condition in this world can be assumed to be static; not even this blessed state of freedom we enjoy, unless it is safeguarded. We know that right now there are forces on the march which are bent upon undermining the order under which we live. They would put an end, if they could, to this system of ours that exalts the dignity of the individual citizen and respects his right to make for himself a place in the sun, according as his talents and industry make possible; that permits every person to choose his own religion, and manner of worship, and to speak aloud his views on any subject, so long as he does no harm to his fellow men.

Enemies of this system would, through open and insidious tactics (mostly insidious), deprive us of such liberties as these and see us in the same mold that they are cast. Seeing the goings-on in the world right now, as we do, we should realize that this is no time for complacency on the part of any literate citizen. It is a time for all of us to show a keen awareness of any propagandist activity, wherever it asserts itself, which seeks to undermine our free institutions, and to speak out against it with spirit.

Three weeks ago in Philadelphia I stepped into a little shop. The proprietor told me of a woman who had been there before me. She came asking him to contribute money for an organized group, which gives evidence of being unfriendly to our system of government. Not only did he indignantly refuse and tell her that she was not welcome in his shop making such a proposal, but he proceeded in strong language to tell her what she was doing to herself, her husband, and their children. He pointed out to her that her husband's thriving little business was providing a good living to them all and education for their children, such as would be impossible to them in any other country in the world. She left that shop roundly reprimanded for her disloyalty. If all of us were as alive to our responsibility as citizens as was that man, we could, through the force of public opinion alone, beat down and crush any subversive "ism," foreign or otherwise inspired, that dares to invade this country.

Again referring to the coin. On its face it bears, as do our other coins, the words "In God We Trust." Thus do we as a nation proclaim to the world our recognition that human wisdom has its limitations. Right now our statesmen are grappling with problems thrust upon them by post-war conditions, which are challenging their powers of brain and soul. There are many facets to those problems which make decisions so difficult they seem almost to defy human wisdom. The world's woe cannot be ignored by this strong, highly-favored nation. The general recognition of that fact, which is evident, is a tribute to the character of the American people.

There is one thing we know—this government of ours is not seeking to exploit any nation on the earth, through economic or any other advantage. Our statesmen have iterated and re-iterated that truth. All it wants is to help; to help those nations that are stricken or tottering back on their feet again. It wants to put them in a position where they can work out their economic rehabilitation and save, or achieve their freedom, while we keep our own secure. It is plain that the overthrow of freedom, in one country after another, if it is permitted to go on, cannot fail to menace our own.

The concern and compassion which this country is showing for the double tragedy of economic distress and loss of freedom, which multiplied millions of people over the earth are suffering, are in accord with the teachings of Christ. In the untiring labor of our national leaders seeking to reach out a helping hand and in the material treasures which we the people are pouring out, we obey the scriptural injunction "Bear ye one another's burdens and so fulfill the law of Christ." That being true, we have reason to take heart, though the sky is dark over the world at the moment, and to trust that through Divine guidance the mighty effort which we are making will be effective in helping bring back peace and order to the world, and speed the time when liberty-loving people in any part of it can achieve and maintain a civilization based upon the principle of human freedom.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

LARGE PHOTOMETRIC SPHERE.

Measurement of the total light output of lighting equipment is made with the integrating sphere, a device by which the light is so thoroughly diffused that the total flux may be determined by a single observation at any selected spot on the sphere. With the increase in physical size of light sources, the National Bureau of Standards found it necessary to construct a sphere large enough to accommodate the largest commercial and research fixtures. The Bureau's new 15-ft. integrating sphere—believed to be the largest in the world—has extended the photometric study of light sources to include those beyond the capacity of the Bureau's other spheres, which range up to 7 ft. in diameter.

The most useful measure of the value of a light source is the rate at which it supplies light. In the measurement of this luminous flux, all light emitted, regardless of direction, is counted because present lighting practice utilizes all such flux by means of suitable reflectors. Essentially the method of test is to place a light source or lighting fixture in the center of a standard white-lined spherical room, the integrating sphere, and to measure the photometric brightness of a spot on the wall of the room. If the spot observed is screened from the direct rays of the fixture, the photometric brightness of the spot in the ideal case varies in direct proportion to the total luminous flux emitted by the fixture.

It would, of course, be much easier to build the room in the form of a cube instead of a sphere. But any light from a fixture directed initially into one of the corners of the cube would fail to contribute its full share to the brightness of the inner wall; so the result obtained with a cube would depend on how the fixture was oriented with respect to the corners of the cube. The spherical shape, on the other hand, has the unique property that any flux reflected with perfect diffusion from any part of the inner wall illuminates all other parts of the wall uniformly. Therefore in an ideally constructed integrating sphere the reflected light flux falling on any part of the sphere is exactly equal to that falling on any other part. It follows that the photometric brightness of any portion screened from the direct rays of the fixture is exactly equal to that of any other such portion. Thus, only one spot need be measured.

All parts of the inner surface of an ideal integrating sphere must have the same reflection factor, and each spot must diffuse perfectly the incident light flux. Furthermore the ideal sphere must be perfectly empty. In the sphere recently installed at the National Bureau of Standards, great care was taken in applying the sphere paint to the inner wall to insure a coating of uniform reflection factor. The sphere paint was selected especially to give as close an approach as possible to perfect diffusion of reflected light. The reflection factor was kept high so that even if the first component of the reflected flux were to fall somewhat short of the ideal perfect diffusion, the components added by multiple reflection would materially improve the approximation. In addition, the effect of an imperfectly diffusing inner coating may often be minimized by the use of a test method in which the fixture is substituted for a carefully measured standard fixture of the same type.

---

* Communicated by the Director.

Obviously, however, the sphere cannot be perfectly empty. The source has finite dimension, and there must be in the sphere the wires and supports for the fixture. There must also be an observing hole in the sphere and a screen between the fixture and the spot on the wall to be observed. If these obstructions are large compared to the area of the sphere, they prevent an appreciable amount of light flux from reaching the observation spot so that the photometric brightness is erroneously low. To minimize this error, the screen between the fixture and observation spot is made as small as possible compared to the area of the sphere wall. Also, the screen, the fixture, the conduits for wires, and the supports are painted with sphere paint, though the light source, itself, cannot be so treated.

The development of large lighting fixtures such as those utilizing the fluorescent lamp emphasizes the difficulties. The 5-ft. 100-w. fluorescent lamp, for example, has a superficial area of nearly 400 sq. in. For this area to remain below the desired upper limit of 0.5 per cent, the sphere area must be 200 times that of the source. This consideration alone would require a sphere of about 13-ft. diameter. There is an added factor that enters into the question. When a full analysis of the details of using the sphere is made it can be shown that no part of the source should approach too closely the inner wall of the sphere lest appreciable nonuniformities in inner-wall brightness be produced. All these factors lead to the conclusion that the sphere diameter should be at least 50 per cent longer than a thin cylindrical source.

The inside dimensions of the Bureau's 15-ft. sphere, weighing more than a ton as installed in a specially designed high-ceilinged room of the new Materials Testing Laboratory, is accurate to within a quarter of an inch along any diameter. Made of molded mahogany plywood, the sphere is composed of 24 identical segments with two polar caps to permit the lowering (or supporting) of large fixtures in the sphere. One half is movable and opens on hinges, permitting easy access to lamps and fixtures placed in the sphere for test. The movable half may be completely detached so that the fixed hemisphere may be used as a hemispherical integrator for special tests. The increased size of the new sphere permits the measurement of cylindrical sources up to 10 ft. long.

CHROMATED PROTEIN FILMS INHIBIT CORROSION.

Chromated protein coatings[1] offering a convenient, inexpensive means of protecting metals—especially zinc, iron, brass, and aluminum—during outdoor storage in mildly corrosive atmospheres have been developed by Abner Brenner, Grace Riddell, and Robert Seegmiller of the National Bureau of Standards. The protective value of such films is somewhat better than that afforded by chemical surface treatments and is much superior to that of corrosion-inhibited oils and waxes. The metallic surface to be coated is first dipped in casein, albumin, or gelatin; the resultant film is then impregnated with chromate, which both hardens the film and inhibits corrosion.

The wartime scarcity of many nonferrous metals led to the extensive substitution of ordinary carbon steel for copper, brass, aluminum, and stainless steel in both military and civilian uses. While the steel was generally protected with zinc coatings, the tendency of the zinc to form objectionable corrosion products brought about a widespread use of chromate films on the zinc. These films were formed on the surface by chemical attack in aqueous chromate solutions and, though only a few hundred-thousandths of an inch thick, were useful in inhibiting corrosion. However, as this was apparently due to the presence of minute amounts of soluble chromate, slowly liberated by the action of moisture, it seemed possible that an enhanced corrosion resistance might be obtained by applying a large quantity of chromate to the metal surface. The National Bureau of Standards therefore undertook an investigation in which proteins were found to offer a good medium for increasing the amount of chromate on the metal surface since they are basic and readily absorb or combine with chromic acid. Although initially developed at the Bureau as a substitute for chemical surface treatments of zinc, the chromated proteins are also of value in protecting aluminum, steel, brass, and other metals.

The principal constituents of the chromated protein films are a corrosion inhibitor for the metal, a protein which acts as a vehicle for the inhibitor, a hardening agent, and a bactericide to prevent putrefaction of the protein. The chromate always serves as the inhibitor and may act as the hardening agent and the bactericide as well. However, other materials such as formaldehyde, tannins, or certain salts of the heavy metals can be used as the hardening agent.

The four functioning constituents of the film can be applied in a single step or in several steps, according to the compatibility of the agents and the degree of protection desired. In the "two-step" process, which is usually the most satisfactory, the metal if first dipped into an aqueous protein solution. When the resulting protein film is dry, it is immersed in an acidified chromate solution for $\frac{1}{2}$ to 3 min. and allowed to dry without rinsing. The preparation of the protein solution varies somewhat with each protein. Gelatin is dissolved in water by heating gently; albumin may be dissolved in water at room temperature by stirring; casein is dispersed in water and brought into solution with the addition of 25 ml. of ammonium hydroxide per liter of solution. The chromate solution contains chromic acid (0.5 to 2 per cent) or a dichromate of zinc, iron, or nickel (1 to 10 per cent).

---

[1] For further technical details see "Chromated Protein Films for the Protection of Metals," by Abner Brenner, Grace Riddell, and Robert Seegmiller, in *J. Electrochem. Soc.* for March, 1948.

A simpler "one-step" method of forming the inhibiting film by a single immersion in a solution containing both the protein and the chromate has the disadvantage that the solution slowly deteriorates. A solution of this type is made by adding ammoniacal zinc chromate to a solution of casein. The resulting preparation should contain about 10 per cent of casein and the equivalent of about 5 per cent of zinc chromate.

The chromated protein films are yellow and, unless opaque pigments have been added, are transparent. Their flexibility and adhesion are sufficient to prevent cracking or separation when the metal is bent. They are not injured by heating to 150° C., whereas most chromate films produced chemically on zinc lose much of their protective value at 100° C. Thickness depends principally on the concentration and viscosity of the protein solutions, varying with the type of protein up to about 0.0002 in. This is in contrast to the proprietary chromate films, which usually are not more than 0.00003 in. thick.

When freshly prepared and hardened, the films are almost insoluble in water and are so hard that they cannot be scratched with the finger nail. Exposure to light further hardens some films and renders them still less soluble. However, the films may be quickly removed from metal parts by application of an alkaline solution such as 5-per cent sodium hydroxide. Their removal is thus more convenient and less hazardous than the removal of oil films with toxic or inflammable organic solvents.

The protein solutions, unlike the baths used for direct phosphate or chromate treatments, do not become exhausted and require replenishment, but are fairly stable for long periods. While chromated protein films are superior to direct phosphate or chromate coatings or to oil films for corrosion prevention, their life is less than that of suitable paint coatings. They are more effective on large, regular surfaces than on the sharp edges and corners of small objects like nuts and bolts, where the film tends to pull away.

CONDUCTIVITY CELL FOR VISCOUS MATERIALS.

A new type of conductivity cell developed by Dr. Earl Otto of the National Bureau of Standards now makes it possible to determine with precision the resistivities of viscous liquids, pastes, suspensions, and wetted powders. The apparatus, which also possesses several advantages for use with free-flowing liquids, is simple and rugged in construction, made of inexpensive materials, and so easily and quickly disassembled that its inside surfaces are entirely accessible.

As commercially available types of conductivity cells have been limited to use with nonviscous liquids, studies of the resistivity of the gelatinous pastes and carbon-manganese dioxide mixtures used in dry cells have been confined in the past to investigations of the constituent electrolytes. In recent years dry cells capable of functioning at the low temperatures encountered by aircraft at high altitudes have been urgently needed by the Army and Navy. In connection with the design of several types of dry cells at the Bureau for this purpose, the new conductivity cell was developed to determine separately the resistivities of dry-cell components at the required temperatures. With some alteration of cell dimensions it is expected to be very useful in measuring the conductivities of pastes, glues, viscous aqueous solutions, aqueous cement suspensions, pigment slurries, clay suspensions, moistened soil and other materials. Other suggested applications lie in the textile, plastic, and soap industries and in the manufacture of adhesives, batteries and ceramics.

In the apparatus designed at the Bureau, the electrodes are made of graphite impregnated with paraffin oil, instead of the more usual platinum. The corrosive action of the ammonium chloride electrolyte used in dry cells and the sturdiness and durability of the carbon electrodes make this desirable. The reservoirs of electrolytic material surrounding each electrode are separated by a small-diameter tube that also contains some of the electrolytic material in order to obtain a cell with a satisfactory constant for use with strong concentrations of highly conductive salt solutions.

The two graphite electrodes, like those ordinarily used in D-size dry cells but shortened to about $1\frac{3}{16}$ in., are inserted in glass tubes about $2\frac{3}{4}$ in. long and $1\frac{1}{8}$ in. in inside diameter. These larger tubes are joined by a 3-in. tube having an inside diameter of $\frac{1}{4}$ in., the connections being made with rubber stoppers. The extreme end of each electrode has an electrolytically deposited coat of copper, to which is soldered a lead wire. The leads are protected by glass or plastic tubing, the ends of which are embedded in the top and bottom rubber stoppers. The large glass tubes are graduated to permit insertion of stoppers to the same depth each time the cell is assembled, and the stoppers are connected by springs to prevent leakage of material.

Loading the conductivity cell is not difficult. Free-flowing materials are first poured into the lower electrode compartment. The small-bore middle portion, with top section attached, is then fitted into the bottom section, forcing air and excess liquid through the connecting tube into the top compartment. Additional liquid is poured in to fill the top section, and finally the topmost stopper is inserted, expelling any excess through an overflow tube. With an electrolytic paste the procedure is the same. However, care should be taken to get no paste in the overflow tube, and the paste must be promptly gelatinized by immersing the assembled cell in hot water. A slurry is loaded in the same way as a liquid, except that considerable force may be required to join the two upper sections to the bottom section. A wetted powder requires tamping in all three tubes.

Although the new cell is not unbreakable, it will withstand considerable rough handling. Replacement of any one part requires only a few moments, and some parts may even be replaced without a redetermination of the cell constant. On the other hand, an ordinary conductivity cell requires expert glass blowing to repair a break, and the repair must be followed by replatinizing of the electrodes and a recalibration of the cell.

While designed primarily for use with viscous materials, the cell also has some advantages for the study of free-flowing liquids. Since it may be wiped dry before loading, there is no need of rinsing with the liquid under test, as is done before a commercial conductivity cell is used. Following a determination the solution may be diluted or otherwise modified and a further resistivity determination made, or it may be returned to storage with very little loss in volume.

The apparatus may be used in conjunction with any of the usual methods of measuring electrolytic resistances. Reproducibility has apparently been limited mainly by the bridge that is used. However, duplicate sets of data have been found to check within 0.2 per cent.

ELECTRON MICROSCOPY OF WOOL FIBERS BY A SHADOWED REPLICA METHOD.

The National Bureau of Standards is engaged in a broad research program on the causes and nature of wear in textiles. Among the phases under investigation are the degradation effects of various physical and chemical agents. A fundamental approach to the problem is being made through the study of organic fibers by means of electron microscopy. In cooperation with other members of the electron microscopy laboratory and the textile laboratory, Max Swerdlow and Gloria S. Seeman have developed a simplified shadowed replica technique that is comparatively simple and rapid. Through this technique, fine details of the morphology of the wool fiber, both in its natural state and after various treatments, have been revealed.

The electron microscope has been a useful tool for observing objects which are thin enough or which are sufficiently small and adequately dispersed to allow the proper scattering and transmission of electrons in the conventional 50 KV. microscope. The high resolving and magnifying powers of the electron microscope have thus been used with considerable success in the study of the pigments, smokes, viruses, bacteria, and a variety of other colloidal particles. On the other hand, when efforts are made to study organic fibers, ordinary techniques of direct preparation fail.

Partial solutions to the problem have been found by those concerned with the study of wool fibers. Fragments of triturated fibers of reduced and methylated wool are not adversely affected by the intense beam of electrons and are in part transparent to it. Such techniques, however, are very limited. Not only do they preclude the practical prospect of obtaining information about the surface structure of the fiber, but the possibility of giving the same fiber repeated treatment is eliminated because of the mechanical disorganization and chemical alteration produced.

The limitations of direct methods of surface observation by electron optical means has led to the development of ingenious and devious techniques for preparing thin film replicas of the surfaces of electron-optically "opaque" materials. The improvements and variations of the basic replica methods reported by a number of resourceful workers has been successfully applied to the study of many bulk materials. None of these techniques, however, is directly applicable to degraded wool fibers, since the use of excessive heat or crushing pressure in molding the replica introduces undesirable conditions. Moreover, most replica methods involve destruction of the specimen. This too, eliminates the possibility of performing additional treatments on the same fiber.

In order, therefore, to augment the data on the structure of the wool fiber and to correlate these observations with results previously obtained by the direct disintegration of the fiber as well as by other methods, a simple, rapid, faithful replica technique was developed. This system of specimen preparation successfully combines a method useful for optical and electron microscopy. It involves making a negative replica of the wool fiber in a thermoplastic film which is thin enough to examine directly in the electron microscope and which may be shadow-cast with a suitable metal for added contrast.

Two thermoplastic materials, ethyl cellulose and polystyrene, can be used with about equal success. When a suitable solution of the resin is poured onto a standard glass microscope slide a thin film (about 0.4 microns) is formed upon evaporation of the solvent. The separated fibers, free of grease and dirt, are then laid upon the thermoplastic lacquer, and another glass microscope slide placed on top of the fibers to make a sandwich with the plastic film. The sandwich can be held together with the aid of a C-clamp. Reasonable care must be exercised in order not to twist the assembly. The clamped glass sandwich of film and fibers is then heated in an electric oven for about 20 min. at 90° C. The thermoplastic is thus softened to the point where it will flow and form a faithful replica of the surfaces in contact with it. The factors which affect the quality and resolution of the plastic replica are time, temperature, pressure, thickness of the film and diameter of the fiber. To achieve the proper balance of all these elements is not a difficult problem inasmuch as the limits of control are not too critical. One or two trials ordinarily provide sufficient experience to make numerous acceptable replicas in a comparatively short time.

After the sandwich has been cooled and separated the wool fibers usually come away with the uncoated glass slide, or else they can be brushed out. These fibers having once been "fingerprinted" can be saved for further chemical or physical treatment and subsequent replication.

Shadow casting provides added contrast as well as a three-dimensional aspect to the surface details of the replica. A very thin coating of evaporated chromium is deposited obliquely upon the plastic surface. The elevations and depressions in the surface replica cast characteristic shielded radiation shadows, just as their larger counterparts would cast with oblique illumination, for example, the light of the setting sun. Where no metal is condensed transmission of the incident energy is a maximum. Where thick elevations and ridges occur transmission is a minimum.

The chromium shadowed thermoplastic replicas are then examined in an ordinary light microscope. Photomicrographs may now be taken of them. The glass slide bearing the negative replicas may also be inserted in a photo-enlarger, or desired areas may be selected for further observation in the electron microscope. In the latter case $\frac{1}{8}$-in. No. 200 stainless steel specimen screens are placed over the desired areas with the aid of a low powered microscope. The glass slide bearing the replica and the specimen screens is slowly immersed into a dish of water The film with the screens still in place can thus be floated free of the glass and retrieved from the surface of the water by turning it over and out with a dry glass slide or a piece of paper. After drying, the thin shadowed plastic replica supported on the specimen screen is ready for examination in the electron microscope.

Thus, with this method, the structural changes in textile fibers resulting from various chemical and physical factors can be studied. Alterations in surface structure brought about by such factors as abrasion, flexing, repeated loading, acids, alkalies, chlorination, heat, sunlight, moisture, and other elements of degradation may be studied. The manifestations of fiber structure may in turn be correlated with the morphological modifications resulting from actual wear in service. The fundamental problem of the molecular structure of organic fibers cannot be solved by electron microscopy alone, but considerable information may be determined that is of interest to the textile technologist and to those concerned with the subject of wear resistance.

With this in view attention was concentrated on the study of wool. The wool fiber as shorn from the sheep generally consists of three principal concentric layers—a thin outer covering of scales (cuticle), a middle region (cortex), and a central core or pith (medulla). It has been suggested that the ability of wool fibers to felt, tangle, and curl is dependent upon the direction and shape of the imbricated scale structure, and that the cuticle serves to protect the fiber from chemical and mechanical degradation. It has also been suggested recently that the three dominant cellular structures are encased in an amorphous material which serves both as a matrix and as an intercellular cementum, and that the cuticle layer is in reality covered by this same matrix material.

The present study confirms, at least partially, this theory of the architecture of the wool fiber. Electron micrographs of the surface replicas of mechanically abraded and unabraded wool fibers reveal that the scale is inhomogeneous and consists of a double layer. The outer layer is a smooth covering for the under structure and shows no specific surface feature. Other investigators have shown this "amorphous plastic layer" to be readily digested in an enzyme as is the featureless intercellular cementum which exists between the scale and the cortex as well as between the cortical cells themselves. The similar reaction to enzymic treatment suggests that these smooth featureless layers are of the same composition.

When this lacquer-like covering is removed by abrasion a rigid striated structure is revealed. The corrugations (ridges and furrows) run more or less parallel to the longitudinal axis of the fiber. They are a feature of the scale itself and are not dependent upon, or a reflection of, the formation and size of the cortical cells. This is borne out by the fact that this corrugated structure is seen in the overhanging portions of the scales under which there is no cortex. Upon additional abrasion the scales are polished away leaving a smooth filamentary structure with slight vestiges of the former imbricated surface. More rubbing removes the next amorphous plastic layer under the scales and exposes the cortical cells themselves inbedded in the cementum. By this time the fiber no longer manifests its characteristic filamentary structure and, as such, is destroyed.

The fabrics studied were mechanically abraded by means of an apparatus [1] designed by Dr. H. F. Schiefer of the Bureau's textiles laboratory. A rotary disc of parallel, flat metallic strips is applied to the fabric. During the abrasion process fragments of the cloth fall off. This debris as well as the fibers bordering the hole worn into the fabric were examined. Hundreds of specimens were obtained and studied. Electron micrographs, taken at magnifications of 716 and 3600 diameters for example, when subsequently "reversed" and enlarged optically about 2.5 times give total magnifications ranging from 1400 to 9000 diameters.

---

[1] H. F. Schiefer "Solution of Problem of Producing Uniform Abrasion and Its Application to the Testing of Textiles," *J. Research* NBC, 39, 1 (1947), RP 1807.

THE FRANKLIN INSTITUTE.

ANNUAL REPORT OF THE BOARD OF MANAGERS FOR THE YEAR 1947.

It was announced at the Annual Meeting in January that the report of the Board of Managers on the condition of the affairs of The Institute for the year 1947, as provided for in the By-laws, would be deferred until a later meeting in the spring. The President now presents such report herewith on behalf of the Board of Managers.

Membership.

Our Membership at the end of the year 1947 totaled 5,425, a net loss of 137 for the year, the majority of whom were student members. During the year there were elected 867 new members.

Your officers, membership committee and membership department are conscious of the importance of a stabilized membership. After a careful study it was decided to increase our efforts in general and to place emphasis on two major programs for the purpose of retaining the interest and support of our membership:

1. The series of popular talks under the general topic of "Science is Fun," which have proved so popular that on several occasions the seating capacity of the Lecture Hall has been exhausted. Five or six such talks of this nature are planned for 1948.
2. Active cooperation was sought by our members during the past year in suggesting prospective members. This has proved most helpful. It is planned to cultivate further this close cooperation during 1948 for the purpose of creating in our membership a warmer feeling of interest in their membership directly and in the affairs of the Institute generally.

Meetings.

During the year a total of sixteen stated and joint meetings with other technical societies were held at which the average attendance was 214. The dinners preceding these meetings were apparently enjoyed by the members and their guests who attended. An outstanding meeting was the first public demonstration of color television by one of our medallists, Dr. V. K. Zworykin, of the Radio Corporation of America.

The tenth series of piano lecture-recitals by Mr. Guy Marriner, Director of Music, continued during the spring of 1947 with an extra request recital given in April. The eleventh series opened in October. The increase in attendance during 1947 amounted to 13.8 per cent over 1947, indicating continued appreciation by the public.

It has always been our practice to allow organizations whose aims parallel those of The Institute to use our Lecture Hall and other rooms for their meetings at a nominal charge covering our costs. During the year 1947, thirty-two organizations used these facilities for a total of 252 meetings.

Committee on Science and The Arts.

The Science and Arts Committee met nine times during the year. Twenty-six subcommittee meetings were held. Seventeen medals and two Certificates of Merit were awarded at the annual Medal Day exercises including two Franklin Medals—one to Dr. Enrico Fermi and one to Sir Robert Robinson. Two George A. Hoadley Certificates for long service on this Committee were presented during the year. These went to Mr. W. H. Fulweiler and Mr. C. D. Galloway for service of thirty-five and twenty-five years, respectively.

Journal of The Franklin Institute.

Our Journal, as the oldest scientific magazine continuously issued in the United States, continued to maintain its preeminence among scientific and technical periodicals by publishing authoritative articles by specialists in many branches of science. The twelve numbers for 1947 contained thirty-five technical papers and a large amount of other material. During the year seventy papers were submitted for consideration, of which eighteen were published and thirty-seven added to the lengthy list of papers awaiting publication. Each month approximately 4,250 copies were distributed—3,050 to members, 550 to domestic subscribers, 350 to foreign subscribers, and 300 in exchange for other publications.

The esteem in which the Journal is held is evidenced by the number of papers submitted and the desire of their contributors to have their work appear in the Journal, even with considerable delay in publication.

Library.

The Library contents were increased during the year by a total of 3,604 volumes by purchase and gifts from the many friends of The Institute.

These additions were as follows:

	Bound Volumes	*Unbound Volumes*	*Pamphlets*	*Maps*
By Gift.	340	344	851	171
Binding..	1,210			
Purchase	634	54		
	2,184	398	851	171

The total contents of the Library as of December 31, 1947 were:

128,961 Volumes
46,717 Pamphlets
4,443 Maps

Our members and staff as well as outside research workers continued to make constant use of the Library. A total of 13,364 readers consulted the Library staff, through whose efforts they received valuable assistance in their research work. The Library staff was also called upon to give a vast amount of information in answer to numerous letters and over 6,500 telephone calls. Circulation increased over that of last year. In response to requests, 13,000 photostat prints were furnished from references in the Library collections.

During the year 3,406 volumes and 466 pamphlets of a non-technical nature not connected with our field were withdrawn from the Library shelves and given to or exchanged with other libraries.

The publication of a number of foreign periodicals, suspended during the war, was resumed during the year, and copies were received by the Library together with several new ones added to the collection through exchange.

The Library continued to be the only source in this city for copies of the United States, British, and Swiss patent specifications. These are received regularly as published.

An important event of the year was the creation of the Library Fund in Memory of Alfred Rigling, who was with The Franklin Institute Library from 1883–1940. Started with a small anonymous gift and increased by additional small gifts, the Fund is as yet unimpressive in size, but the faith of that first donor should in turn inspire many more to give either large or small amounts.

Museum.

The Museum staff suffered a great loss from the death in March, 1947 of Mr. James D. Lee, Jr., who had been Associate Director in charge of Lighting and Illumination from the opening of the present building. Dr. D. M. Ludlum and Mr. James S. Collins joined the staff—the former as Associate Director in charge of Meteorology and the latter in charge of Graphic Arts, Lighting and Illumination, and the Photographic Salon. Continuing plans inaugurated in 1946, a special group of demonstrators has been employed further to interpret the purpose and contents of the Museum to visitors. Experiments are being conducted to ascertain how the services of this group can be utilized to the best advantage.

The following tabulation shows the attendance at the Museum and Planetarium:

	Paid	*Free*	*Total*
1945..	186,559	82,593	269,152
1946.	232,547	75,147	307,694
1947	.178,724	72,247	250,971

In making comparisons with other years, it should be borne in mind that, until December 1, 1946, a separate charge of twenty-five cents was made for admission to the Museum and to the Planetarium; after that date, a combined charge of 50 cents was made (in each case exclusive of Federal taxes). Following this rate change, there was an increase in income of 12.3 per cent, but a decrease in paid attendance of 23 per cent.

As in prior years all public and parochial school children from Philadelphia continued to be admitted to the Museum without charge when attending in groups accompanied by a teacher. When attending individually, such children paid only the Federal admissions tax.

The shortage of labor and materials during the war years inevitably led to the cessation of many operating exhibits which fell into disrepair. The members of the staff succeeded during the course of the year in restoring these exhibits.

Many exhibits continue to be offered to the Museum, but pressure on exhibition space and inadequate storage facilities have made a careful screening of such offers necessary before any are accepted. During the year a total of 50 exhibits were, however, accepted for permanent or temporary display, among the more noteworthy of the former being a P51 (Mustang) airplane from the U. S. Army; a model electric generating station and a meter exhibit from the Philadelphia Electric Co.; a recording Pyrometer from the Brown Instrument Co.; a replica of the Wright Brothers original wind tunnel from the late Dr. Orville Wright; a collection of firearms from Mr. Marion Eppley; and a railroad crossing signal from the Union Switch and Signal Co. Outstanding among temporary exhibits were an old-time apothecary's shop, lent by Smith, Kline, and French; a helicopter, lent by the Bell Aircraft Corp.; a turbo-jet engine, lent by the Westinghouse Electric Corp.; a diorama of cold and warm weather fronts, lent by Airology Inc.; a model of an electric power plant, lent by the General Electric Co.; a watch rate recorder, lent by American Time Products, Inc.; a memorial of Frederick Goudy, lent by Mr. Sol Hess; large Japanese binoculars, lent by Mr. H. J. Adicks; a crank shaft for a Diesel locomotive, lent by the Park Drop Forge Co.; a postage stamp collection, lent by the Austro-Hungarian Philatelic Society; and an atomic energy exhibit, from the Enoch Pratt Free Library.

The popular "Exhibits of the Month" were continued, and in this connection an opportunity was given visitors to see the working of the electron microscope purchased for the Research Laboratories. The Junior Engineers group of the Engineers' Club of Philadelphia sponsored an exhibition of Engineering Progress in Franklin Hall from April 5 to 13, which will be repeated in 1948. The display of photographs illustrating industry and industrial processes was continued and an intensified effort to secure the cooperation of exhibitors has produced highly satisfactory results.

A series of television broadcasts from the Museum was inaugurated in August by station WPTZ, with the program under the direction of Dr. Roy K. Marshall. This series included the first telecast of the moon through a telescope. In addition to these regular broadcasts, members of the staff have given occasional studio demonstrations of Museum activities. The fourth series of educational radio programs prepared with the cooperation of station WFIL and the public, parochial, and private schools of this area opened in October. Mr. Armand Spitz again planned the program and supervised the script writing.

The Museum attendance of school, clubs, and institutions groups has mounted continuously since 1943. The figures covering such visits (contained in totals above) including individual students as well are as follows:

	1943	1944	1945	1946	1947
Paid	17,472	27,225	38,368	64,396	65,527
Free	15,000	33,633	42,675	52,140	57,207
Total	32,472	60,858	81,543	116,536	122,734

Work has been done with the Schools of Education of the University of Pennsylvania and Temple University and with the State Teachers' Colleges. This cooperation has taken the form of lectures and conferences in the colleges and also of informal visits to the Museum by groups of student teachers. In-service classes, affiliated with the Museum's activities, have been held, and the Department of Museum Education has been consulted in connection with curriculum planning.

There has been close cooperation with the Philadelphia Board of Education through its Curriculum Office, Visual Aids Department, Adult Education, and other functions. There has also been close association with the parochial and private schools in the area as well as with the Boy Scouts, Girl Scouts, Y.M.C.A., Y.W.C.A., church, and other non-school groups.

The work of the Philadelphia Science Council has been followed, attention being devoted to its activities in relation to the encouragement and development of science talent among students in the schools of this area.

A total of 981 demonstrations was presented in the Fels Planetarium during the year, of which 170 were on special topics chosen in consultation with curriculum officials and teachers as most suitable for school groups. As in the past, Planetarium demonstrations were of two categories—the conventional demonstration in which fundamental information on astronomy is conveyed in non-technical language, and a more spectacular type of demonstration in which special effects are used to arouse unusual interest and to promote attendance.

Research Laboratories.

Bartol Research Foundation.

The Bartol Research Foundation has been working under an extension of its Navy contract for work in nuclear physics and cosmic rays. In addition, it has received a very substantial grant for the development of a high energy particle accelerator.

Following the work reported last year, the Foundation carried on successful cosmic ray observations in B-29 bombers during the National Geographic Society's expedition to Brazil, made in connection with the eclipse of the sun. It has also done considerable high-altitude work with radiosonde balloons, and it has participated in the Navy's expedition to regions in the vicinity of the north magnetic pole.

The Foundation's Van de Graaff generator has been considerably improved, and is operating exceptionally satisfactorily at voltages up to two million. It is probably the most compact generator for its range in the world.

Biochemical Research Foundation.

The Biochemical Research Foundation, of which The Franklin Institute is Trustee, has increased its ties with the University of Delaware. By September 1, 1948, there will be eight post-graduate students working for their Ph.D's in the laboratories of the Foundation. Professor William A. Mosher, Director of Chemistry at the University, has been released from a considerable amount of his work and will give half his time to the direction of these students, who will work upon problems chosen by the Foundation.

The work of the laboratories has gone along well and the year has been a satisfactory one, in spite of losing five of our chief workers. They will be replaced by September.

The Foundation's book, "Neutron Effects on Animals," has had a good sale and has received a number of favorable editorial comments. It promises to become a classic in its field.

Franklin Institute Laboratories for Research and Development.

It is gratifying to be able to report that these laboratories have made progress during the year. By December 31, 1947 the staff, exclusive of shop personnel and consultants, had increased to a figure of approximately 260, with the distribution of scientific functions among three Divisions still maintained.

Conversion, for laboratory use, of the remainder of the old Court Building on our property was started in October, 1947. The additional area furnished by the ground floor and two upper floors of this building into which we moved in the spring of 1948 will go far toward alleviating the space problems with which the laboratories have been confronted in the past.

Acquisition of scientific equipment has, in general, kept pace with the requirements for our various projects. As a scientific and educational institution, we have been able to obtain from the War Assets Administration scientific equipment worth many thousands of dollars at a price within our means. This equipment has been put to immediate use on our problems.

Special acquisitions that provide us with facilities for the investigation of new fields of research are a combination electron microscope and electron diffraction unit; an ultramicrotome capable of producing slices of material less than 1/50,000 of an inch in thickness; and an X-ray diffraction unit.

Other important equipment, such as precision electrical measuring apparatus, oscilloscopes, oscillators, frequency meters, galvanometers, and an optical comparator, has also been obtained. These and other types of equipment have been and will be valuable in our effort toward expanding our industrial research projects.

The electron microscope is capable of magnifying directly 20,000 times. This can be increased to 200,000 magnification by photographic means. The apparatus is being used in the study of metal surfaces and various powdered materials as well as for measurement of particle sizes too small to be otherwise estimated.

The X-ray diffraction unit produces a high speed, accurate X-ray diffraction pattern. This unit will shortly be augmented by certain specialized equipment. It is being used for basic and applied research in various fields, including those of metals, polymers, catalysts, and certain ceramics.

On December 16, 1947 the laboratory for making photoelastic stress analyses, donated by Mrs. David Halstead in memory of her late husband, was duly dedicated. It is now being used in the study of stress problems in both glass and metal sections. It is a memorial of great value to The Institute.

Apparatus has been constructed for the production and measurement of ultrasonic waves in liquids. With these waves as a tool, instruments are being developed to control the commercial processing of rubber and other materials. Special sonic apparatus has also been successfully developed and is undergoing commercial test of principle for the process control of materials. Also in the field of endeavor relating to sound, special techniques and instrumentation have been devised for magnetic wire and disc recording, including a special instrument for graphically recording the magnetic properties of materials. Special techniques have been developed in the field of computers and simulators. Although much of this work has, in the past, applied to Government problems, these techniques are now being studied for industrial application.

The volume of work for various Government agencies has continued to increase, but our aim to make these laboratories primarily a center for industrial research in this area is slowly materializing with twenty sponsors already among our clients and every indication of a steady increase in this number.

Articles have been written and published by members of the staff, descriptive of new developments arising from their basic research. Other developments have been reported at meetings of the different scientific societies.

Hostess Committee.

During the year 1947, the Hostess Committee considered three projects—the establishment of a Children's Room in the Museum, the planting of the lot at 21st and Race Streets, and the furnishing of a new rest room for women in the Laboratory Building and the improvement of the old rest room in the main building. After careful consideration, it was decided that the project for a Children's Room was too large an undertaking for the present, and it was laid aside for action at a future date. Committees have been formed to study each of the other projects. The Rest Room Committee has already decided what is to be done in the rooms, and some of the furnishings have already been ordered. The Garden Committee has considered possible methods of planting, and will soon decide specifically what plan shall be followed. The Institute is indeed greatly indebted to the Hostess Committee for the services it has rendered.

General.

In addition to its normal departmental routine and its work in connection with various special events which The Institute has sponsored, our Public Relations Department has been active in the continued dissemination of knowledge of Benjamin Franklin. During the year two new leaflets were added to the series which we continue to distribute throughout the country, and numerous requests for odd information about Franklin were answered. *The Institute News* continued to be issued monthly. Consideration is now being given to possibly enlarging and changing the scope of this publication. The Department is already engaged in considering future plans for the proper celebration in 1949 of the 125th anniversary of the founding of The Institute.

Finances.

The financial position of the Institute at December 31, 1947 as compared with December 31, 1946 is indicated by the following comparative summarized statements of assets and liabilities:

Assets	*December 31, 1946*	*December 31, 1947*
Current assets. . .	$ 461,293	$ 662,753
Fixed assets. . . .	4,209,402	4,366,250
Less: Reserve for depreciation .		(139,137)
Endowment funds (including Notes Payable contra)	1,631,433	1,681,215
	$6,302,128	$6,571,081
Liabilities		
Notes payable (included in Endowment funds contra)	$ 80,000	$ 319,000
Liabilities, deferred income and reserves.	90,039	260,548
Endowment. . . .	1,631,433	1,681,215
Capital stock and contributed funds. .	3,995,633	3,954,141
General appropriation fund (surplus) . .	505,023	356,177
	$6,302,128	$6,571,081

The Institute's endowment at the end of 1947 as shown above may be analyzed as follows:

Unrestricted as to principal and income (including $319,000 advanced to The Institute for operating purposes). . . .	$ 970,549
Unrestricted as to income. . . .	534,014
Restricted as to both principal and income.	156,695
Unexpended income. . . .	19,957
	$1,681,215

The results of operations for the year 1947 are shown in the following statement of income and expenses, it being not possible to compare these figures directly with those of 1946 because of the change made in The Institute's accounting system during the year. This change involved the installation of a cost system as of July 1, 1947 to provide proper control of the costs of the large amount of research work being carried on in The Institute's laboratories.

Operating Revenue:		
Memberships	$ 59,091.78	
Museum	101,410.49	
Other Institute Activities	30,526.74	
Laboratory research projects	1,436,902.61	
		$1,627,931.62
Operating Costs and Expenses:		
Museum	279,302.14	
Library, less amounts transferred	46,748.90	
Other Institute activities	46,997.70	
Laboratory research projects	1,426,864.20	
Administrative and general expense, less amounts transferred	115,586 07	
		1,915,499.01
Net operating loss		287,567.39
Other Income:		
Donations—Friends of Franklin	24,071.00	
Service fees from Bartol and Biochemical Foundations	14,868.57	
Appropriation—State of Pennsylvania	31,583.33	
Income from trust funds (including interest below from The Institute)	70,309.90	
		140,832.80
		146,734.59
Other Charges:		
Friends of Franklin	3,967.01	
National Franklin Committee	2,443.29	
Cafeteria expense (net)	4,763.94	
Interest	4,132.42	
Bad debts	211.33	
		15,517.99
Net deficit for the year		$ 162,252.58

The operations for the year, after including in costs a charge for depreciation of buildings and equipment amounting to $139,137, resulted in a deficit, as indicated, of $162,253, or $23,116 without such depreciation. Compared with a surplus of $86,087 for the year 1946, this deficit of $23,116 reflects the further restoration of activities suspended during the war, the reduction in government overhead allowances on research projects since the war, and increased costs in general operations. The budget for the current year indicates a deficit of $153,000 after depreciation of $173,000.

The group of about 250 individuals and companies comprising the Friends of Franklin has continued its generous support, contributing as indicated above the sum of $24,071 to the funds of The Institute during the year. Both the moral and material help of this important group is of such inestimable value that we should like to see it grow in numbers; and its financial assistance is of such aid to The Institute and its work with the community that we hope its contributions may even be increased this coming year.

The financial statements above bring into clear light the unfortunate fact that the crying need of The Institute now, just as before the war, is for largely increased unrestricted endowment to enable it to carry on the work to which it committed itself when its new building was erected in 1933. We are not unique in this respect; it is well known that scores and hundreds of similar institutions throughout the country, each serving the community in its own particular field, must have more income to meet rising costs and the expanding demands on their facilities or else restrict the services they are performing. We are hoping that a formal campaign can be started in the near future for this purpose. Even before its inauguration, however, perhaps there are those who have sufficient belief in our work to step forward and start the ball rolling. There would then be no doubt of The Institute's ability to continue its ramified scientific activities, so much a part of this present day and age. It would give renewed energy to the staff who have so well served us this past year as in earlier years and to whom the Board can be nothing but grateful.

Surely the 125 years of service which The Franklin Institute has rendered to this local community of Philadelphia and to the state and country at large in the "promotion of the mechanic arts" and in spreading the teachings of Benjamin Franklin—its patron saint and the country's great practical philosopher—would seem to merit support. Surely the early vision of our founders will be supported by the present generation in carrying forward the work which they so well began.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, June 9, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 9, 1948.

DR. JULIAN W. HILL in the Chair.

The following report was presented for final action:

No. 3160: Development of Phospho-Asbestos.

This report recommended the award of an Edward Longstreth Medal each to Nicholas F. Arone, of Philadelphia, Pennsylvania, and Edwin H. Brink, of Laurel, Mississippi, "In consideration of their persistent search and ingenuity, resulting in the discovery of Phospho-Asbestos and in the chemical and productional development of this uniquely useful product."

JOHN FRAZER,
Secretary to Committee.

NEW MEMBERS OF THE FRANKLIN INSTITUTE ELECTED DECEMBER 1, 1947 TO JUNE 1, 1948.

NON-RESIDENT LIFE MEMBER.

Harry Sidney Newcomer, M.D.

ACTIVE MEMBERS WITH FAMILY PRIVILEGES.

Mr. John Barber, Jr.
Mr. Arthur F. Bolger
Mr. Paul Brosz
Mr. George A. Butler
Mario A. Castallo, M.D.
Mr. Herbert K. Clofine
Mr. Bert W. Conn
Anthony A. Donato, M.D.
Mr. William A. Erhardt, Jr.
Columbus Rudolph Gangemi, M.D.
Mr. James G. Hall

Mr. Bertrand W. Hayward
Mr. Albert M. Hunter
Leon Kacher, M.D.
Miss Jane Kenworthy
Mr. Scott Maddux
Captain Robert A. Maguire
Mr. Clarence E. McCormick
Mr. John A. Naegele, Jr.
Mr. Edward F. O'Brien
Darius Gray Ornston, M.D.
Mr. Bertrand Schneeberg
Mr. Myron A. Shoffner
Mr. Clarence R. Shook
Mr. H. A. Sosnin
Mr. Lombard Squires
Mr. William Dean Stapp
Mr. Frederick A. Weisenbach
Miss Louise B. Weston

ACTIVE MEMBERS.

Mr. Yervant V. Amadouni
Mr. Charles J. Andrews
Mr. F. Wallis Armstrong, Jr.
Mr. Jules Axelrod
Mr. J. L. Baker
Mr. Robert E. Barney
Mr. William J. Barry, Jr.
Mr. Jacob Becker
Mr. Harry M. Bell
Mr. Robert A. Benedetto
Mr. Alexander Bickstein
Mr. M. Black
Mr. Joseph F. Burke
Mr. Maurice C. Burton
Mr. Paul C. Calissi
Miss Catharine Fow Casnet
Mr. Elwood M. Childs
Mr. Joseph Cohen
Mr. Charles S. Cole, Jr.
Mr. J. A. Corsey
Mr. Radley Cross
Mr. Leopoldo De Cristofaro
Charles Q. DeLuca, M.D.
Mr. Richard A. Denton
Anthony F. DePalma, M.D.
Mr. Earle W. Deppich
Mr. John Di Sangro
Patricia Drant, M.D.
Mr. Sol Duchovnay
Mr. Robert B. Durst
Mr. Thor I. Eklund
Mr. Frank P. Florentine, Jr.
Mr. Robert M. Focht
Mr. Samuel E. Foultz
Mr. James J. Freney
Mr. John J. Fuhrman
Mr. Samuel J. Galespie
Mr. Daniel J. Gallagher
Mr. Robert E. Gallagher
Mr. Edward W. Geis
Mr. Edward J. Gentsch
Chester Ginieczki, M.D.
H. M. Ginsberg, M.D.
Mr. A. Franklin Hahn
Mr. John T. Hall
Leonard C. Hamblock, M.D.
Mr. C. Lawrence Hamilton
Mr. George J. Harding, III
Mr. R. S. Hayes
Mr. Virginius A. Hermann
Mr. Henry I. Hoot
Mr. Harry Russell Horn
Robert E. Huttenlock, M.D.
Mr. William A. Johnson
Roosevelt Juele, M.D.
Mr. Fred J. Junkermann, Jr.
Charles S. Kambe, M.D.
Mr. J. Edwin Kay
Mr. Harry H. Keller
R. A. Kimbrough, Jr., M.D.
Mr. Isadore L. Kirschner
Mr. William H. Knowles, Jr.
Igho H. Kornbluch, M.D.
Mr. John W. Kroeger
Mr. Maurice B. Lederer
Mr. Harry D. Lehman
Mr. John V. Leinmiller
Mr. Louis Leonetti
Mr. John B. Lewis
Mr. Thomas K. Liversidge
Mr. Harry H. Longenecker
Mr. Joseph Lopacki
Pascal Lucchesi, M.D.
Mr. Rinaldo A. Lukens
Mr. Edward A. Lynch
Mr. William MacNeill
Mr. John J. Maguire
Mr. Kenneth J. Marlatt
Mr. George H. Maurer
Mr. John T. McCabe
Mr. John R. McConnell
Mr. John McNutt
Mr. Wolcott W. Merrow
Joseph V. Missett, Jr., M.D.
Mr. T. Morris
Mr. Julius Nadel
Anthony A. Nardone, M.D.
Albert A. Order, M.D.
Mr. J. C. Osten
Mr. William J. Paolini
Mr. George M. Patterson
Mr. William Pechenick
Mr. John D. Peterson
Peter C. Petropoulos, M.D.
Mr. A. R. Pezzillo
Mr. George A. Porreca
Mr. Thomas J. Power
Mr. Edward D. Pritchett
Mr. A. Raymond Raff, Jr.
Mr. W. F. Randolph
Mr. N. Rathblott
Leonard E. Ravich, Ph.D.
Mr. Norman Reinboth
Mr. Cornelius J. Ridgley
Mr. Louis Robbins
Mr. Gerald P. Roeser
Mr. Fred J Rosenau
Mr. Barney Roth
Mr. Martin Saligman
Mr. August F. Sander
Mr. Bertram E. Schatz
Mr. Charles Howard Schlecht
Mr. Jacob Schlessinger
Mr. Alan M. Shapiro
Mr. T. Kite Sharpless
Mr. Harry D. Smith
Mr. J. Gordon Smith
Mrs. Reba Soffe
Mr. Michael Subotich
Mr. Francis W. Sullivan
Mr. Eugene Sunshine
John B. Szall, M.D.
Mr. Carrow Thibault, Jr.
Mr. Alfred M. Townson
Mr. Harold M. Van Horn
Mr. C. Laurence Warwick
Mr. William V. Wence
Mr. Harry K. Wilcox, Jr.
Mr. Eugene Ziegler, Jr.

ACTIVE NON-RESIDENT MEMBERS.

General George L. Bliss
Mr. H. H. Buttner
Mr. Leonard J. Edwards
Mr. Edgar E. Isaacs
Mr. K. T. Kuck
Mr. William S. McGilvray
Mr. J. B. Monk
Mr. Morton Simon
Mr. Georg Stabenow
Mr. Hoy J. Walls

NECROLOGY.

Mr. Charles J. Beck '37
Mr. Irwin A. Becker '96
Dr. E. H. Bedrossian '44
Mr. James G. Biddle '09
Professor S. Lawrence Bigelow '13
Mr. Joseph H. Bromley, Jr. '34
Mrs. Philip S. Collins '34
Mr. Thomas G. Conklin '46
Mr. Philemon Dickinson '44
Mr. George S. Eddy '41
Professor A. S. Eve '34
Mr. Edward P. Flannery '36
Dr. Thomas S. Gates '35
Mr. John R. Geary '42
Mr. Chester H. Gibbons '38
Mr. Frederick C. Gladeck '42
Dr. J. Calvin Hartman '44
Mr. John V. Hastings, Jr. '34
Miss Katharine P. Hutchinson '35
Mr. Harry L. Jefferys '36
Mr. George Joyce '43
Dr. Vladimir Karapetoff '27
Mr. George B. Koch '17
Mr. C. Hartman Kuhn '09
Mr. Charles Henry Lachman '47
Mrs. Pearl Lachman '45
Mr. Charles Longstreth '88
Mr. D. J. Loughery '46
Mr. W. F. Lueckel '43
Mr. W. Logan MacCoy '39
Mr. J. D. McIntyre '45
J. Bernhard Mencke, M.D. '45
Mr. George W. Norris '36
Dr. William Pepper '36
Mr. A. Raymond Raff '36
Mr. William B. Read '36
Mr. Theodore W. Reath '17
Mrs. George S. Robbins '36
Mr. Clarence V. Roberts '33
Mrs. George D. Rosengarten '36
Mr. Benjamin Rush '34
Mr. Edgar M. Scattergood '43
Mr. Edwin J. Schoettle '36
Mr. Carl A. Selzer '42
Mr. Cecil F. Shallcross '36
Mr. Lee Sowden '36
Mrs. Albert G. Thatcher '36
Mrs. Marino Tomacelli '44
Mr. William T. Tonner '38
Mr. Edwin B. H. Tower '40
Mrs. Samuel B. Van Dusen '36
Mrs. J. Atwood White '36
Dr. W. E. Wickenden '38
Mr. Orville Wright '33
Mr. Leroy E. Zabel '42

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from 9 A.M. until 5 P.M., Thursdays from 2 P.M. until 10 P.M., and Saturdays from 9 A.M. until 12 noon.

RECENT ADDITIONS.

AERONAUTICS.

Moe, Harris G. Flight Engineering and Cruise Control. 1947.
Northrup Aeronautical Institute. Aircraft Basic Science. 1948.

ASTRONOMY.

Olcott, William Tyler and Edmund W. Putnam. Field Book of the Skies. 1936.

ARCHITECTURE AND BUILDING.

Chase Brass and Copper Co. Chase Radiant Heating Manual. 1947.
Crane, Theodore. Architectural Construction. 1947.

BACTERIOLOGY.

BREED, R. S., E. G. D. MURRAY AND A. P. HITCHENS. Bergey's Manual of Determinative Bacteriology. 1948.

BIBLIOGRAPHY.

GREAT BRITAIN. BRITISH INTELLIGENCE OBJECTIVES. Technical Index of Reports on German Industry. 1946.

BIOGRAPHY.

BERMAN, ELEANOR D. Thomas Jefferson Among the Arts. 1947.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

ALFREY, TURNER, JR. Mechanical Behavior of High Polymers. 1948.
BIKERMAN, J. J. Surface Chemistry for Industrial Research. 1947.
CONANT, JAMES B. AND A. H. BLATT. The Chemistry of Organic Compounds. Second Edition. 1947.
DEAN, ROBERT B. Modern Colloids. 1948.
EILERS, H., R. SALL, AND M. WAARDEN. Chemical and Physical Investigations on Dairy Products. 1947.
GUENTHER, ERNEST. The Essential Oils. Volume 1. 1948.
KENT-JONES, D. W. AND A. J. AMOS. Modern Cereal Chemistry. 1947.
KIRKBRIDE, CHALMER G. Chemical Engineering Fundamentals. 1947.
KUHN, WERNER. Physikalische Chemie. 1947.
MOELWYN-HUGHES, E. A. The Kinetics of Reactions in Solution. 1947.
NORTHEY, ELMORE H. The Sulfonamides and Allied Compounds. 1948.
OLSSON, E. The Band Spectra of Sulphur, Selenium and Tellurium. 1938.
SHRINER, R. L. AND R. C. FUSON. The Systematic Identification of Organic Compounds. Third Edition. 1948.
VOGEL, ARTHUR I. A Textbook of Practical Organic Chemistry. 1948.
WHITTAKER, R. M. AND A. P. MARION. Laboratory Handbook for General Chemistry. 1947.

DICTIONARIES.

KERCHOVE, RENÉ DE. International Maritime Dictionary. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

HALL, ALBERT C. The Analysis and Synthesis of Linear Servomechanisms. 1943.
MUELLER, GEORGE V. Introduction to Electrical Engineering. 1948.
TIMBIE, WILLIAM HENRY. Elements of Electricity. 1947.

ELECTRONICS.

BACHMAN, C. H. Techniques of Experimental Electronics. 1948.

ENGINEERING.

STAGER, HANS. Allgemeine Werkstoffkunde. 1947.

FOOD.

SHERMAN, HENRY CLAPP. Calcium and Phosphorus in Foods and Nutrition. 1947.

GEOLOGY.

HAYNES, B. C. Techniques of Observing the Weather. 1948.
KRAUS, E. H. AND C. B. SLAWSON. Gems and Gem Materials. 1947.
MACELWANE, JAMES B. When the Earth Quakes. 1947.

HISTORY.

BJORK, KENNETH. Saga in Steel and Concrete. 1947.
VAN DOREN, CARL. The Great Rehearsal. 1948.

MANUFACTURE.

FLETCHER, D. W. Watch Repairing as a Hobby. 1947.
JAMES, ARTHUR E. Chester County Clocks and Their Makers. 1947.

MATHEMATICS.

COLUMBIA UNIVERSITY. STATISTICAL RESEARCH GROUP. Selected Techniques of Statistical Analysis. 1947.
LEVENS, A. S. Nomography. 1948.
SAKS, STANISLAW. Theory of the Integral. No date.
SVOBODA, ANTONIN. Computing Mechanisms and Linkages. 1948.

METALLURGY.

BAEYERTZ, M. Nonmetallic Inclusions in Steel. 1947.
BOYLES, ALFRED. The Structure of Cast Iron. 1947.
BUDGEN, N. F. Aluminum and its Alloys. 1947.
CARRINGTON, E. Aluminum Alloy Castings. 1948.
HAMBURG, H. F. AND OTHERS. Electronic Methods of Inspection of Metals. 1947.
MASON, CLYDE WALTER. Introductory Physical Metallurgy. 1947.
MOORE, HERBERT F. AND OTHERS. Surface Stressing of Metals. 1947.
UHLIG, HERBERT H. Corrosion Handbook. 1948.

PATENTS.

BURLINGAME, WILLIAM R. Inventors Behind the Inventor. 1947.
KRAMER, ALBERT J. Inventions at Your Service. 1946.

PHARMACY AND PUBLIC HEALTH.

BRANDT, ALLEN D. Industrial Health Engineering. 1948.

PHOTOGRAPHY.

QUIGLEY, MARTIN. Magic Shadows. 1948.
WALL, E. J. AND F. I. JORDAN, Photographic Facts and Figures. 1947.

PHYSICS.

BRITISH RHEOLOGISTS CLUB. Essays in Rheology. 1947.
CARLETON, R. H. AND H. H. WILLIAMS. Physics for the New Age. 1947.
DALLAVALLE, J. M. Micromeritics. 1948.
DIRAC, PAUL ADRIAN M. The Principles of Quantum Mechanics. 1947.
DUBS, ROBERT. Angewandte Hydraulik. 1947.
ELDRIDGE, JOHN ADAMS. College Physics. Third Edition. 1947.
EWING, JAMES ALFRED. Thermodynamics for Engineers. 1946.
GIRVIN, HARVEY F. A Historical Appraisal of Mechanics. 1948.
HARRISON, V. G. W. Definition and Measurement of Gloss. 1945.
HOAG, J. BARTON. Electron and Nuclear Physics. Third Edition. 1948.
HUNSAKER, JEROME AND B. G. RIGHTMIRE. Engineering Applications of Fluid Mechanics. 1947.
MICHENER, WILLIAM H. Physics for Students of Science and Engineering. 1947.
POWELL, C. F. AND P. S. OCCHIALINE. Nuclear Physics in Photographs. 1947.
WORTHINGTON, A. G. AND D. HALLIDAY. Heat. 1948.

PLASTICS.

SMITH, PAUL IGNATIUS. Plastics for Production. 1946.

RADIO.

AMERICAN RADIO RELAY LEAGUE. Radio Amateur's Handbook. 1948.
BEATTY, R. J. Radio Data Charts. 1947.

BIGUENET, CHARLES. Les Cathodes Chaudes. 1947.
ENNES, HAROLD E. Broadcast Operator's Handbook. 1947.
PHYSICAL SOCIETY OF LONDON AND ROYAL METEOROLOGICAL SOCIETY. Meteorological Factors in Radio Wave Propagation. 1946.
POUND, ROBERT. MICROWAVE MIXERS. 1948.
TORREY, HENRY C. AND C. A. WHITMER. Crystal Rectifiers. 1948.

SANITARY ENGINEERING.

ROGERS, D. PAUL. Laboratory Guide for Sewage Works Operators. 1946.

SCIENCE.

FURNAS, CLIFFORD COOK. (ed.). Research in Industry. 1948.
JEANS, JAMES HOPWOOD. The Growth of Physical Science. 1948.

TEXTILES.

MARSH, J. T. An Introduction to Textile Finishing. 1947.
TEXTILE BRAND NAMES DICTIONARY. 1947.
WEINDLING, LUDWIG. Long Vegetable Fibers. 1947.

PUBLICATIONS RECEIVED.

Electronic Instruments, edited by Ivan A. Greenwood and others. 721 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $9.00.

Pulse Generators, edited by G. N. Glasoe and J. V. Lebacaz. 741 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $8.00.

Supercharging the Internal Combustion Engine, by E. T. Vincent. 315 pages, 16 × 23 cm., plates, tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.00.

Fatty Acids and Their Derivatives, by A. W. Ralston. 986 pages, 14 × 22 cm., tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $10.00.

The Modern Metallurgy of Alloys, by R. H. Harrington. 209 pages, 14 × 22 cm., plates, tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.50.

Dairy Bacteriology, by Bernard W. Hammer. Third edition, 593 pages, 15 × 23 cm., illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $6.00.

Physics of Metals and Alloys, by W. Boas. 193 pages, 16 × 25 cm., plates, tables, drawings and illustrations. New York, John Wiley & Sons, Inc., 1947. Price, $3.50.

Journal of the Society of Motion Picture Engineers, Index 1936–1945. 156 pages, 15 × 23 cm. New York, Society of Motion Picture Engineers, 1948.

Animal Breeding, by Laurence M. Winters. Fourth edition, 404 pages, 15 × 23 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $4.50.

Eruptive Rocks, by S. James Shand. Third edition, 488 pages, 14 × 22 cm., drawings, illustrations and plates. New York, John Wiley & Sons, Inc.; London, Thomas Murby & Co.; 1947. Price, $7.50.

Fundamentals of Soil Mechanics, by Donald W. Taylor. 700 pages, 14 × 22 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $6.00.

Quantitative Analysis, by Willis Conway Pierce and Edward Lauth Haenisch. Third edition, 520 pages, 15 × 23 cm., tables, plates and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.75.

Petroleum Production, by Park J. Jones. Volume 4, 238 pages, 15 × 23 cm., tables and drawings. New York, Reinhold Publishing Corp., 1948. Price, $5.00.

Encyclopedia of Chemical Reactions, by C. A. Jacobson. Volume 2, 917 pages, 16 × 23 cm. New York, Reinhold Publishing Corp., 1948. Price, $12.00.

Two-Dimensional Fields in Electrical Engineering, by L. V. Bewley. 204 pages, 16 × 24 cm., drawings and illustrations. New York, The Macmillan Co., 1948. Price, $5.50.

Aircraft Engines, by Colonel Rollen H. Drake. 348 pages, 16 × 24 cm., drawings and illustrations. New York, The Macmillan Co., 1948. Price, $5.90.

Chymia, edited by Tenney L. Davis. 190 pages, 16 × 24 cm., portraits and plates. Philadelphia, University of Pennsylvania Press, 1948. Price, $3.50.

Tables of Bessel Functions of Fractional Order, prepared by The Computation Laboratory of the National Applied Mathematics Laboratories, National Bureau of Standards. 413 pages, 20 × 27 cm., tables. New York, Columbia University Press, 1948. Price, $7.50.

Microwave Magnetrons, edited by George B. Collins. First edition, 806 pages, 15 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $9.00.

BOOK REVIEWS.

DIE DISSERTATIONEN DER EIDGENÖSSISCHEN TECHNISCHEN HOCHSCULE, 1909–1946, EINE SYSTEMATISCHE BIBLIOGRAPHIE, by W. Mikulaschek. 142 pages, 15 × 21 cm. Zürich, AG. Gebr. Leemann & Co., 1948. Price, 10 Fr.

Representative of the useful bibliographical works published by many universities and other higher institutions of learning are lists of dissertations. The present volume is such a list and includes all the dissertations presented at the Eidgenössische Technische Hochschule from the first in 1909 to the end of 1946.

The dissertations have been arranged in a classified order, an outline of which is given. An alphabetical index to the outline is also included. Under each subject the entries are arranged alphabetically by author and include further the titles of the dissertation, the number of pages, place, printer, and date. Of the 1458 dissertations listed, 697 are in the field of chemistry. The remainder are divided among the various other branches of science and technology.

As the first volume of a new series "Schriftenreihe der Bibliothek der Eidgenössischen Technischen Hochschule," this volume offers an auspicious beginning.

G. E. PETTENGILL.

AMERICAN WOOL HANDBOOK, by Werner von Bergen and Herbert R. Mauersberger. Second edition, 1055 pages, illustrations, 13 × 19 cm. New York, Textile Book Publishers, Inc., 1948. Price, $8.00.

Originally published in 1938, this *Handbook* now appears in a revised and enlarged edition. The authors, both experienced in textile work, have been aided by twenty specialists in various phases of the wool industry.

The text covers the complete subject of wool from sheep raising to the finishing of the cloth. New material has been added in several sections, particularly on the chemical properties of wool, worsted yarn manufacture and the testing of wool. The many illustrations and a comprehensive bibliography will prove useful. With the extensive changes in this edition, this work remains an essential and up-to-date handbook for those interested in wool.

G. E. PETTENGILL.

WHAT ELECTRONICS DOES, by Vin Zeluff and John Markus. 306 pages, illustrations, 14 × 20 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $3.00.

Electronics has become a common word since it was first chosen as the title of a periodical in 1930. The applications of electronics have multiplied to such an extent that the present book contains a selection of over four hundred examples.

The text is divided into nineteen chapters which present the subject from various viewpoints. Different types of electronic phenomena are presented in the first five chapters—

electrostatics, dielectric heating, induction heating, ultrasonics, and electronic viewers and phototubes. Then follow chapters on the use of electronics in particular media—in the air, on water, on land and underground. Specific functions that electronics may perform are also considered, such as inspection, production control, measuring, and counting.

The many uses of electronics range from those with which the average individual may come in contact, such as traffic controls, diathermy equipment, or safety devices on machinery, to the unusual application seen only by a few—a device for tracking schools of fish. Directly or indirectly nearly all the uses detailed in this book contribute in some way to increased efficiency or heightened production. Electronics has also made possible the accomplishment of many tasks hitherto thought impossible.

Written in a readable style with many illustrations, this book will be of interest to those who wish to learn about the wonders of electronics. Read carefully, it may suggest applications of electronics suitable to the reader's own business.

G. E. PETTENGILL.

VACUUM TUBES, by Karl R. Spangenberg. 860 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $7.50.

In the United States the number of vacuum tubes in use is several times its population, and comprises nearly half of the world's total. Thus, a textbook dealing with vacuum tubes only would seem highly appropriate.

In his book Dr. Spangenberg is concerned with the determination of vacuum tube characteristics in terms of the electron action within the tube. The development of the physical laws involved is indicated, after which emphasis is placed upon their description and utilization. Attention is centered on the internal factors which contribute to the operating characteristics of tubes over a broad range of types.

No attempt has been made to describe circuits and tube application, except in the case of ultra-high frequencies where the tube cannot be separated from the circuit.

More recent advances are comprehensively covered. A number of items hitherto unpublished in book form are also included. These include space charge flow, noise, specific characteristics of triodes, tetrodes, pentodes, etc

H. N. MICHAEL.

VACUUM-TUBE CIRCUITS, by Lawrence Baker Arguimbau. 668 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948 Price, $6.00.

The material in this textbook, although often referring to matters of current interest, emphasizes general methods by which engineering problems can be attacked. Frequency modulation, basic material on transient response and generation of microwaves are particularly thoroughly covered.'

Although the emphasis has been placed on physical concepts and ideas rather than on discussion of actual circuits, the ideas have been applied to specific numerical examples.

The book is written, in the main, for those who have a background in physics of the vacuum tube. Nevertheless much of this material has been summarized in the initial chapters and, besides, there are adequate references to the basic electronics needed for the pursuit of this study.

The sound expository treatment of the material and the frequent illustrations and problems present a well-knit and comprehensive text which recommends itself to the student specializing in this field.

H. N. MICHAEL.

POWER SYSTEM STABILITY, by Edward Wilson Kimbark. Volume 1, 355 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.00.

This first volume of a three-volume work stresses transient rather than steady-state stability. Intended for the use of power-system engineers and graduate students, it covers the principal factors affecting stability, the ordinary simplified methods of making stability calculations, and illustrations of the application of these methods to actual power systems.

The related topics of a-c. calculating boards, fault studies, circuit breakers, protective relaying, synchronous-machine theory, exciters and voltage regulators, etc., are brought out clearly in the discussion of power-system stability, and thus increase the referential value of the book.

H. N. MICHAEL.

ELEMENTARY THERMODYNAMICS, by Virgil Moring Faires. Revised edition, 268 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1948. Price, $4.40.

The revised edition of this book presents textual material for short courses in thermodynamics. However, its brevity does not imply that the discussions within it are shortened. To the contrary, those topics that are presented are fully discussed. In words of the author, "Completeness in treatment of topics should not be sacrificed in order to include a wide range of material that would be appropriate for a course designed to be full in scope."

Many small changes have been made in this edition and a discussion on the theory of the gas turbine has been added. Most of the problems have been replaced or revised.

Specifically, the thirteen chapters of the book deal with general energy equation, characteristic and energy equations of a perfect gas, reversible nonflow processes of gases, Carnot and Ericsson cycles, cycles for internal combustion engines, the compression and expansion of air. There are three chapters on liquids and vapors, and also chapters on refrigeration, flow in nozzles, and on mixtures.

To most of the chapters are appended selected references and there is also a list of general references on thermodynamics.

H. N. MICHAEL.

TECHNIQUES IN EXPERIMENTAL ELECTRONICS, by C. H. Bachman. 252 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

Electronics, as discussed in this book, is defined as "that brand of science and technology which relates to the conduction of electricity through gases or in vacuo."

Starting with the description of the necessary apparatus, the book proceeds to the active production and measurement of vacua, instructs in the glass blowing fundamentals such as tubulation, glass seals, preparation of electrodes, etc., and also discusses the sources, control, and utilization of charged particles. There is also a chapter on the assembly and processing of electronic devices.

The author covers the basic techniques employed in electronics research and his book should prove valuable to physicists, electrical engineers, and others about to tackle research in this field.

H. N. MICHAEL.

FREQUENCY MODULATION, Vol. I, collected reprints by R. C. A. Engineers. 515 pages, illustrations, nomograms and charts, 14 × 23 cm. Princeton, *R. C. A. Review*, 1948. Price, $2.50.

In 1940 the R. C. A. Institutes introduced the plan of reprinting in a single volume the leading papers presented by its staff of engineers on such subjects as U. H. F. radio, facsimile and television. *Frequency Modulation*, Vol. I follows that pattern by reprinting 24 outstanding papers on general topics, transmission and reception, and miscellaneous FM subjects from the *Proceedings of the I. R. E.*, *Electronics*, *FM*, and *R. C. A. Review*. In addition, there are 21 summaries and abstracts which provide the necessary background material in the fields that lie within or near the scope of frequency modulation broadcasting. A bibliography of all the papers and articles on FM by R. C. A. engineers is given in the back of the volume, according to date.

Generally the review editors have made what appear to be rather good choices from quite an extensive selection. Some readers and engineers may, however, find the reprinting of the so-called "characteristic" series by Crosby to be out of date; while including such excellent articles as "The Input Impedance of Pentodes" by F. Mural which is not commonly available is highly commendable.

O. P. FERRELL.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

A Method of Ultraviolet Photomicrography.—J. O. Ely and M. H. Ross. Photomicrographs of unstained cells and tissue sections made with ultraviolet light near 2600 A wave length, because of selective absorption of the light, reveal the location of purine and pyrimidine nucleotides and nucleic acids. Absorption of ultraviolet light by these compounds makes possible a comparison of the amounts in different tissues or in the same tissue under different circumstances, and with refined technic makes possible quantitative determinations of nucleic acids and other substances in cells.

A quartz optical system microscope, described in 1904 by Köhler and von Rohr of the Zeiss Optical Co. staff, which had attachments for photomicrography, has been employed by many investigators—Cas-

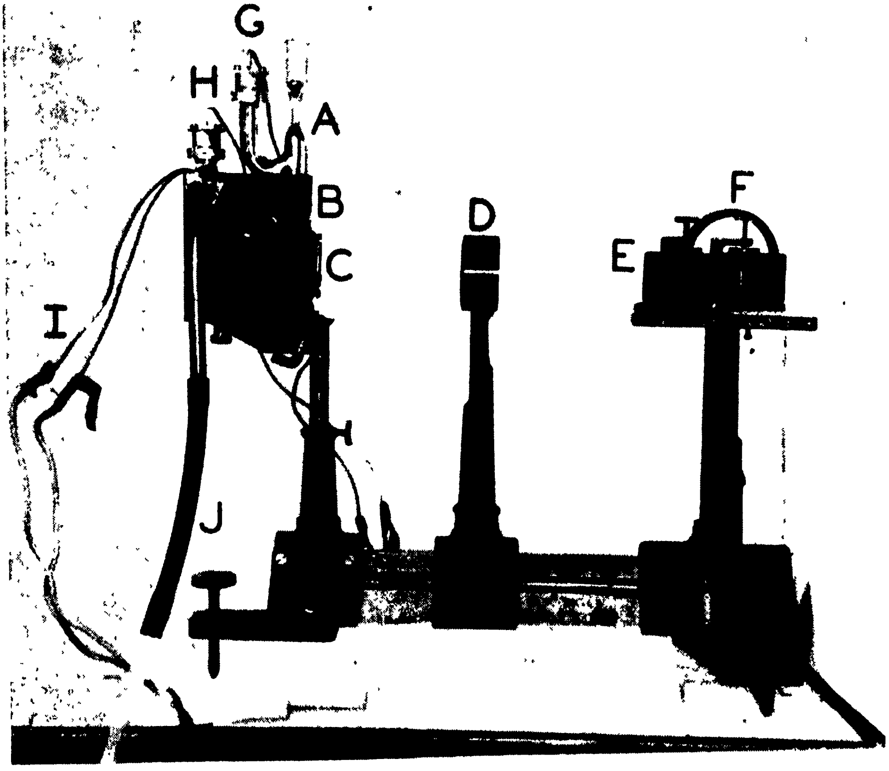


Fig. 1. Photograph of lamp-monochromator assembly. (*A*) Quartz-mercury lamp; (*B*) water-tight metal box holder for lamp with water inlet and outlet and quartz window (*C*); (*D*) collimator lens of quartz; (*E*) monochromator prisms of quartz; (*F*) collecting lens of quartz; (*G*) positive electrode; (*H*) negative electrode; (*I*) lead wires to heater; (*J*) water outlet.

persson and Schultz; Wyckoff; Lucas; Allen, Franklin and McDonald; Cole and Sutton; Ris and Pollister; Lavin; Mitchell; Bromburg; Loofbourow; and others. The difficulties of operation and the uncertainty

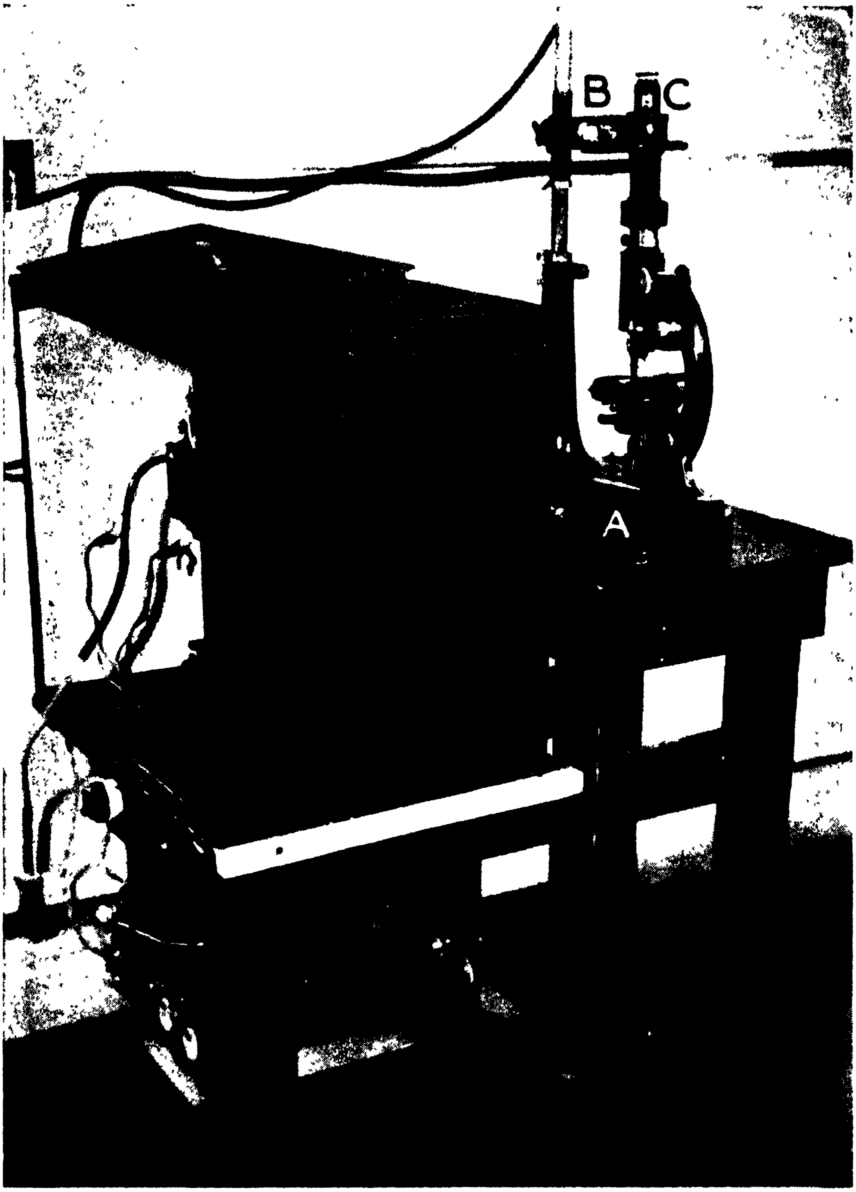


FIG. 2. Ultraviolet light photomicrograph apparatus. (*A*) Hole in base of stand through which monochromatic light passes to and is reflected upward by a totally reflecting prism in the optical axis of the microscope; (*B*) Leica camera in position for focussing; (*C*) magnifier used to aid in focussing.

of obtaining good quality photomicrographs with this apparatus were great.

The improved apparatus described here utilizes the Zeiss Optical Co. quartz microscope and monochromator. The various pieces of apparatus and their location in the assembly are shown in Figs. 1 and

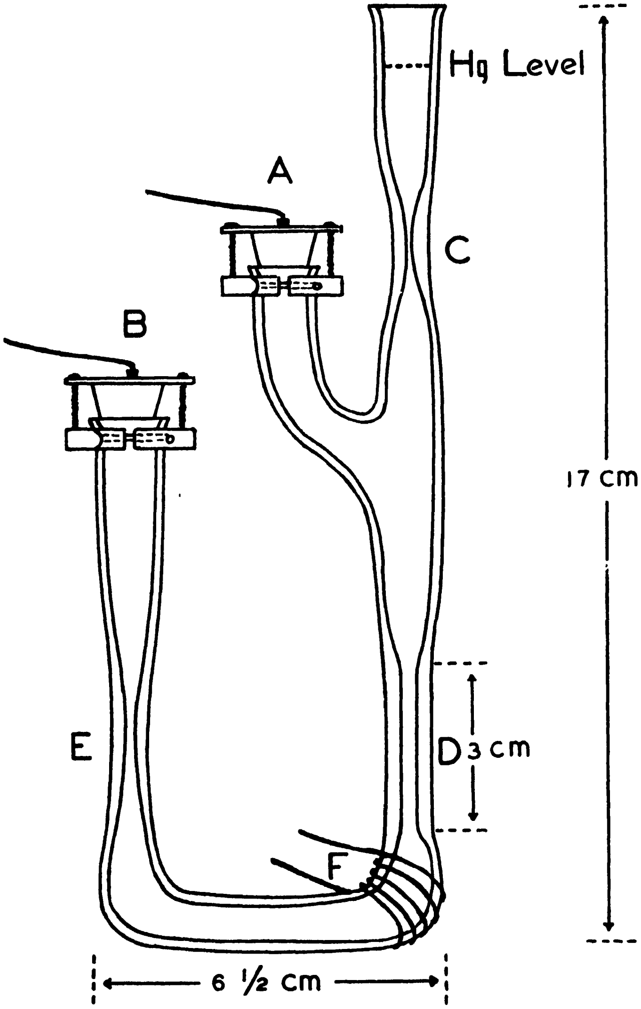


FIG. 3. Diagram of quartz-mercury lamp.

2. The essential changes from the original apparatus are substitution of a quartz-mercury lamp for the original cadmium electrodes and a 35-mm. camera for the original plate camera.

A circular disc of metal 33 mm. in diameter and 1 mm. thick with a slit 1 mm. wide and 20 mm. long across the center is placed in the substage of the condenser. The bottom surface of the disc is photosensitized with Conti-Glo Pearl White Lacquer enamel 604, Continental Lithograph Co., Cleveland, Ohio.

The lamp is a modification of, and differs only slightly from the one described by A. J. Allen (*Science*, **83**: 336, 1936). A schematic diagram of the lamp is shown in Fig. 3. Approximately 15 in. of quartz tubing with a wall thickness of 2 mm. are required for each lamp. Tubes of 8 to 12-mm. bore have been found satisfactory. The tube is drawn out to a diameter of 3 to 4 mm. at *D* where the arc forms, to approximately 0.5 mm. at *C*, and to 1 to 2 mm. at *E*. The ends at *A* and *B* are flared slightly. Steel sewing needles make satisfactory electrodes. Cork stoppers at *A* and *B* are held in place by collar and plate assemblies constructed of aluminum. A heater, necessary in starting the lamp, is composed of a 6-in. length of nicrome wire, 0.025 in. in diameter. The wire is wound about the tubing at *F* below the arc forming area on the diagram. Alternating current with a fixed resister in series is used to operate the heater coil. Direct current at 70 v. and 4.8 amp. is used to operate the lamp. These values vary somewhat with different lamps.

The lamp, partly filled with clean mercury, must be heated to remove air bubbles from the mercury; the operation is repeated several times until the lamp is filled. Cork stoppers with electrodes are inserted and held securely in place with collar and plate assemblies. The lamp is then placed upright into the metal holding box.

To put the lamp into operation, the heater switch is turned on. When the mercury bubbles in the capillary above the heater, the direct current is turned on and allowed to remain until the lamp lights. The heater switch is then turned off, and water is allowed to circulate through the box containing the lamp.

The lamp end of the optical bench holding the lamp-monochromator assembly is moved in an arc until the desired light band strikes the slit of the photosensitized disc in the substage of the microscope. The collimator lens is then used to focus the band on the slit.

A Leica camera with the ground glass focussing screen photosensitized to ultraviolet light (Eastman Kodak Co. Ultraviolet Sensitizing Solution No. 3177) has been substituted for the original plate camera.

The object may be focussed by either of two procedures: (*a*) The Zeiss uranium fluorescent screen searcher may be used above the ocular of the microscope. The object is easily focussed, especially when the strong mercury band 2654 Å or 2536 Å is used. The camera is then put into place. With the aid of a small hand magnifier, the image is focussed on the photosensitized focussing screen of the camera. (*b*) The image may be focussed directly on the focussing screen of the camera without preliminary use of the uranium screen searcher.

Twelve microscopic fields can be photographed, three exposures per field, on one roll of 35-mm. film, in a short time. Almost invariably a well focussed negative is obtained from three exposures at slightly different settings of the fine adjustment of the microscope.

Summary.

A specially constructed quartz-mercury lamp and a 35-mm. camera have been adapted for use with a Zeiss Optical Co. quartz optical system microscope and monochromator. With this improved apparatus ultra-violet light photomicrography may be easily and rapidly accomplished.

CURRENT TOPICS.

New Technique Facilitates Brazing Stainless Steel Parts.—The coming era of jet engines and rockets is being accelerated by the development of a new brazing technique which makes possible the joining of precision, high-temperature, stainless-steel parts.

Furnace brazing, the uniting of several metal parts by melting a metal such as copper into the joints, has been extremely difficult in the case of the heat-resistant alloys containing chromium necessary in the construction of jet engines and rockets. Or, when flux, a chemical for preventing oxidation, was used, the cost of removing flux deposits afterwards has been excessive and time-consuming.

The new process succeeds with tricky stainless-steels by brazing them in a hydrogen atmosphere more pure than any previously used. When moisture or oxygen is present in a furnace atmosphere without flux, the chromium will oxidize.

The parts to be brazed are sealed in a box, which is then filled with pure hydrogen. As in regular furnace brazing operations, copper is placed adjacent to the joints to be bonded, so that when molten it will creep into the joints. The sealed box is introduced into the heating chamber of an electric furnace for 20 min. to an hour. It is then allowed to cool. Before removing the box from the cooling chamber, the hydrogen, which is explosive when mixed with air, is exhausted and replaced by nitrogen, which is not.

The furnace brazing of assemblies by this new technique is sometimes used as an alternative to making parts by the more common methods of forging, casting, machining from solid stock, or even other fabricating methods such as torch brazing. Machining time and waste metals are often reduced in such instances. Intricate assemblies sometimes can be fabricated which otherwise could not be made any other way and still obtain the quality and results needed. The perfectly clean, bright surfaces, after furnace brazing, are a particular advantage.

R. H. O.

Germ-Less Air (Marjorie Fisher, *Science Illustrated*, May, 1948).—A new machine called a glycol vaporizer may revolutionize the long fight against air-borne germs. Respiratory diseases, caused by these germs, annually cost the United States 220,000,000 days and $1,500,000,000. The battle against these costs has progressed through the ventilation, dust-removal, and ultraviolet ray stages, all of which have their drawbacks. In 1941, Dr. Ostwald Robertson, searching for a good air germicide, discovered that triethylene glycol was practical, since it is odorless, fireproof, and hygroscopic. When heated, triethylene glycol gives off a vapor which surrounds the germs in the air and kills most of them. One part of glycol vapor in 400-million parts of air can clear that air of almost everything harmful, including influenza virus, pneumococcus, and streptococcus. The vaporizer was designed by Frederick Schmitz and has been used in pharmaceutical plants, hospitals, theaters, and the White House. Industrial plants may purchase a large 45-lb. machine for about $385 for use with air conditioning systems. A small 10-lb. machine suitable for one room—office, home, or hospital—costs about $50, with operating expense of about $1.50 a month.

Soil Organisms in Vital Contests.—What scientists bring to light on the life that teems about the roots of crop plants indicates complexes of great variety based on physical and chemical conditions, on fungi, bacteria, insects, nematodes, to mention a few.

A recent study of factors influencing the growth of the fungus that causes foot rot of wheat, the so-called "take-all" disease, emphasizes those complicated interrelations which slow down the solution of many crop problems. This requires that scientists of different kinds look at them from different directions.

The scientists—in this case working in Kansas—found that a number of common soil fungi are antagonistic in various ways to the one that causes the "take-all;" while others at the same time produced stimulants that made the "take-all" take more. Some were found to be direct parasites on wheat roots, others produced substances disturbing to growth of the grain. But some of the beneficial ones were only fair weather friends, shifting their roles with conditions, a disturbing situation suggesting a kind of biological treason.

In view of this very mixed situation, say the experimenters, it may be that adding certain amendments to a field will enable some common soil fungi to produce byproducts (hormone-like substances or what not, biotics or antibiotics) that work either for or against what the farmer is trying to do.

After unearthing these difficulties and possible advantages in the biological and other complexes of the root zone, the scientists say there is some hope of ultimate practical use of such knowledge. Although results as yet are not very consistent, and tests have been on a small scale, sometimes the severity of "take-all" has been reduced by increasing the population of certain fungi.

R. H. O.

Elastic Truths Measure Stretch in Fabrics.—Familiar as knit fabrics are, in such things as sweaters, hose, close-fitting underwear, and gloves, the textile and clothing industry has had relatively little scientific information to go on in producing best materials for elasticity—the stretch and recovery or come-back —that makes possible a close yet comfortable fit.

Pioneering in some lines of this research, textile specialists have had to start by devising their own apparatus and procedure to measure elastic recovery of yarns and fabrics under various stresses and strains. Tests thus far completed have yielded technical data on the stretch and come-back of yarns and plain knit fabrics.

Elastic come-back of fabrics cannot be predicted by knowing that of yarns. This is one point thus far learned. Nylon yarn, for example, is far more elastic, than yarn of wool or silk. But when the same yarns have been made into plain knit fabrics, the wool or silk fabric is more elastic than the nylon fabric, say Dr. Hazel M. Fletcher and Pearl A. Gilmore, who have been conducting these tests in the Bureau of Human Nutrition and Home Economics.

Yarns of cotton, linen, wool, silk, nylon, and the three types of rayon have been compared, and also 19 plain knit fabrics, some loosely knit with 40 courses to the inch, some tightly knit with 64 courses.

The two types of fabrics, loosely and tightly knit, point up another problem in elasticity, the research scientists have found. A yarn that has greater elastic recovery when knit into fabric of a certain number of courses may have much less in other fabrics. Plain knit fabrics of wool have more elastic recovery when loosely knit. Those of nylon, silk, and rayon have more recovery when tightly knit.

In further research, Dr. Fletcher and Miss Gilmore are measuring elastic recovery of other constructions of knitwear, such as ribbed fabrics, and plated fabrics made of two layers of different kinds of yarns.

R. H. O.

Manufacturing Advances in Wartime Germany (John L. Kent, *Scientific American*, April, 1948).—American manufacturers can improve their products, add profitable lines and even start new businesses by making use of technical information on German processes and equipment which can be obtained from the Office of Technical Services, Department of Commerce, in Washington. Thousands of these reports, prepared by industrial intelligence "detectives" during surveys of German industry, are being sold—some for less than a dollar. They include descriptions of processes and equipment, formulas, plant layout and other technical data, selected especially by the investigators as being useful to American industry. Examples of the sort of information available are: a process to extrude cold steel, which greatly increases production in sheet metal stamping; salt-bath heat-treatment methods for case hardening, decarbonizing, neutral surface treatments, and for quenching; description of a high-speed trip magnet to afford proper electrical contact in resistance welding; the development of economical and efficient brushes and commutators from powdered copper and iron.

Dehydrofreezing.—Dehydrofreezing is the name of a new process of fruit and vegetable preservation that has been developed in California.

This new method combines the advantages of dehydration with those of quick freezing. It carries the product through the first cycle of dehydration, which removes a considerable amount of the weighty moisture, and then freezes it. Vitamins and flavor are virtually undamaged; so the treated product not only retains the fresh quality, but in some instances its original shape also.

This new method seems to be better than either dehydration or freezing, partly because there is less destruction of tissues by the larger ice crystals that tend to be formed when a greater amount of water is present. The chief advantage of this process is a considerable saving in weight and bulk, which in turn creates a saving all down the line, including packaging materials, storage space, and transportation facilities.

R. H. O.

Journal
of
The Franklin Institute

Devoted to Science and the Mechanic Arts

Vol. 246 AUGUST, 1948 **No. 2**

STABILIZATION OF CARRIER-FREQUENCY SERVOMECHANISMS. II. DESIGN FORMULAE FOR PROPORTIONAL-DERIVATIVE NETWORKS.*

BY

ANDREW SOBCZYK, Ph.D.[1]

ABSTRACT.

In an alternating current servomechanism, the error is proportional to the modulation envelope of a modulated-carrier error signal. It is shown in part I that for stability and fidelity of the servo, it is highly desirable that the effect of the controller includes a proportional-derivative action on the modulation envelope. This action may be obtained with various forms of RC networks, including the parallel "T," bridge "T," and Wien Bridge forms.

This part contains detailed design procedures and tables of values for the various types of proportional-derivative networks. Several forms of parallel "T" networks arise from the fact that there are five independent time constants in the network, while in order to realize the desired transfer characteristic it is necessary to impose only four conditions. It is indicated how the remaining degree of freedom may be used to obtain the most suitable input and output impedances for the source and load impedances with which the parallel "T" is to be used. The derivations for the parallel "T" formulae are given in an Appendix.

Tolerance requirements on the components of parallel "T" and bridge "T" networks are derived. If ±1 per cent components are used at 60 cycles, the resonant frequency will lie between 56.4 and 63.6 cycles, and the notch width (rejection bandwidth) will be within ±0.99 cps. of the correct value. In order to guarantee that the phase shift at 60 cycles is within ±10°, the percentage deviation of each part must be less than $(9.0/T_d\omega_0)$, where ω_0 is the carrier angular frequency, T_d the derivative time constant.

1. INTRODUCTION.

In a servo controller characteristic, including the servo motor and load, time lags inevitably are present. The stability of the servo-

---

* This paper is a continuation of part I, published in the JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 246, July, 1948, p. 21. Part III will follow in the September issue.

[1] Formerly with Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.; now at Watson Laboratories, Cambridge Field Station, Cambridge, Mass.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute
New Delhi.

mechanism is greatly improved by adding to the voltage proportional to the error, which is amplified and applied to the control winding of the servo motor, a component proportional to the derivative of the error. The latter component compensates for the presence of the time lags; effectively it endows the controller with a quality of being able to anticipate and appropriately correct the errors.

In an alternating current servo, the error is the modulation envelope of a modulated-carrier error voltage. Differentiation of the modulation envelope has previously [2] been accomplished approximately by a "resonant" form of parallel "T" network, that is a network in which there is complete rejection, and a discontinuity [3] in phase shift, a jump from −90° to +90°, at the carrier frequency, the magnitude of the response at other frequencies being approximately proportional to the deviation from the carrier frequency. (A more complete description is given in following sections.) The component with modulation proportional to the error is then obtained by a divider or transformer arrangement and combined with the differentiated component to give the desired proportional-derivative stabilizing effect.

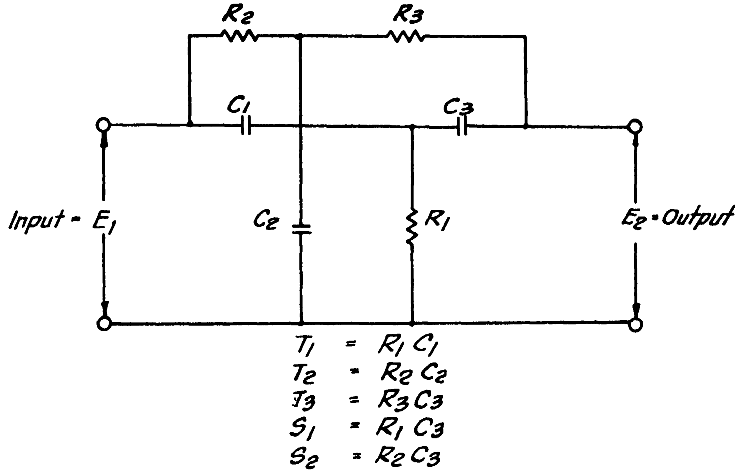


FIG. 1.

In the following sections formulae are presented for a number of forms of parallel "T" networks which to the writer's knowledge have not been previously discussed. These networks give both the proportional and differentiated components directly in the envelope of the modulated output.

Sections 13 through 15 are devoted to derivation of the tolerance requirements on the components of parallel "T" and bridge "T" networks. Formulae are obtained for the variations (1) of parallel "T" resonant frequency ω_0, (2) of derivative time constant T_d (or, equivalently, of notch width $1/\pi T_d$), (3) of phase shift at carrier frequency, (4) of input and output impedance, in terms of the variations of the individual components of the networks from their correct values. From these formulae are derived percentage tolerances for the components. Gain-phase margin diagrams are given to show the effect of varying T_d, and of incorrect resonant frequency ω_0. (The effect of incorrect carrier phase has been discussed in part I.)

---

[2] It is discussed, for example, in an unpublished memorandum by G. T. Coate.

[3] The discontinuity arises from the fact that the magnitude is taken to be non-negative at all frequencies. Change of sign of the amplitude response is equivalent to 180° phase shift.

2. PARALLEL "T" NETWORKS.

The parallel "T" network is shown in Fig. 1. Let time constants be defined as follows:

$$T_1 = R_1C_1, \quad T_2 = R_2C_2, \quad T_3 = R_3C_3, \quad S_1 = R_1C_3, \quad S_2 = R_2C_3.$$

Except for the impedance level, which always may be changed by simple scale transformation of the R's and C's, the parallel "T" is determined by the five constants T_1, T_2, T_3, S_1, S_2. For if these constants are known, and if any one of the components, say for example, C_3, is arbitrarily specified, then the other components are determined:

$$R_1 = S_1/C_3, \quad R_2 = S_2/C_3, \quad R_3 = T_3/C_3, \quad C_1 = T_1/R_1, \quad C_2 = T_2/R_2.$$

3. TRANSFER CHARACTERISTIC; INPUT AND OUTPUT IMPEDANCES.

The transfer characteristic of the parallel "T" is

$$\frac{E_2}{E_1} = \frac{T_1T_2T_3p^3 + T_1(S_2 + T_3)p^2 + (T_1 + S_1)p + 1}{\begin{array}{r}T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + \\ (T_1 + S_1 + T_2 + S_2 + T_3)p + 1\end{array}}. \tag{1}$$

The appropriate form which this is made to assume in order to obtain the approximate proportional-derivative characteristic at carrier frequency ω_0 is

$$\frac{E_2}{E_1} = \frac{(Up + 1).\left(\dfrac{1}{\omega_0{}^2}p^2 + \dfrac{2}{n\omega_0}p + 1\right)}{(Up + 1)\left(\dfrac{1}{\omega_0{}^2}p^2 + \dfrac{2}{m\omega_0}p + 1\right)}$$

$$= \frac{m}{n}\frac{\left[1 + jT_d\dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]}{\left[1 + jl\dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]}, \tag{2}$$

where in the second expression $j\omega$ has been substituted for p, and $T_d = n/\omega_0$ is the derivative time constant, $l = m/\omega_0$ is the time lag. The second expression is obtained by multiplying numerator and denominator of the first by $n\omega_0/2p$, and then factoring n/m out of the denominator. The choice of the time indeterminate U, the coefficient of p in the linear factor which cancels in the numerator and denominator, does not affect the transfer characteristic. (But it does affect the input and output impedances, as will be seen.) If $E_1 = E(t) \sin \omega_0 t$, the approximation $(\omega + \omega_0)/2\omega \cong 1$ is valid when the modulation $E(t)$ does not have high frequency components of large amplitude.

To realize the above transfer characteristic, the determining constants of the parallel "T" must have the following values:

$$\begin{aligned}
T_1 &= \frac{A^2 + B_1U + U^2}{B_2 - B_1} = \frac{g(nu^2 + 2u + n)}{2\omega_0(1 - g)}, \\
T_2 &= \frac{U^2(B_2 - B_1)}{A^2 + B_1U + U^2} = \frac{2(1 - g)u^2}{g\omega_0(nu^2 + 2u + n)}, \\
T_3 &= \frac{A^2}{U} = \frac{1}{u\omega_0},
\end{aligned} \tag{3}$$

$$\begin{aligned}
S_1 &= B_1 + U - T_1 \\
&= \frac{1}{\omega_0}\,\frac{1 - gn^2u^2 + 2n(1 - 2g)u + 4(1 - g) - n^2g}{2n(1 - g)}, \\
S_2 &= B_2 - B_1 - \frac{A^2}{U} - T_2 \\
&= \frac{[4(1 - g) - gn^2]u^2 + 2(1 - 2g)nu - gn^2}{gnu\omega_0(nu^2 + 2u + n)},
\end{aligned}$$

where $A^2 = 1/\omega_0^2$, $B_1 = 2/n\omega_0 = 2/T_d\omega_0^2$, $B_2 = 2/m\omega_0 = 2/l\omega_0^2$, $u = U\omega_0$, and the gain $g = m/n$.

The condition for non-negative S_1 is that u be in the interval with end-points $\frac{1}{m} - \frac{2}{n} \pm \frac{1}{m}\sqrt{1 - m^2}$. The condition for non-negative S_2 is that u be in the interval with end-points $\frac{-n^2 + 2mn \pm n^2\sqrt{1 - m^2}}{4n - 4m - mn^2}$. Therefore for realizability, $m < 1$ and u must lie in the common portion of the two intervals. The possible values of u are indicated in Fig. 2. The left and right hand curves, which lie mainly between $u = 0.8$ and $u = 1.2$, are approached asymptotically by, respectively, the right and left boundaries of the regions for u, as m approaches 1.

For the general parallel "T," the input impedance, with the output open-circuited, is

$$Z_i = \frac{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1}{(C_1T_2T_3 + C_2T_1T_3 + C_3T_1T_2)p^3 + [C_1(T_3 + T_2 + S_2) + C_2(T_3 + T_1 + S_1)]p^2 + (C_1 + C_2 + C_3)p}. \tag{4}$$

The impedance looking back into the output, with the input short-circuited, is:

$$Z_o = \frac{R_3T_2(S_1 + T_1)p^2 + [R_3(S_1 + T_1 + T_2) + R_2(S_1 + T_1)]p + R_2 + R_3}{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1}. \qquad (5)$$

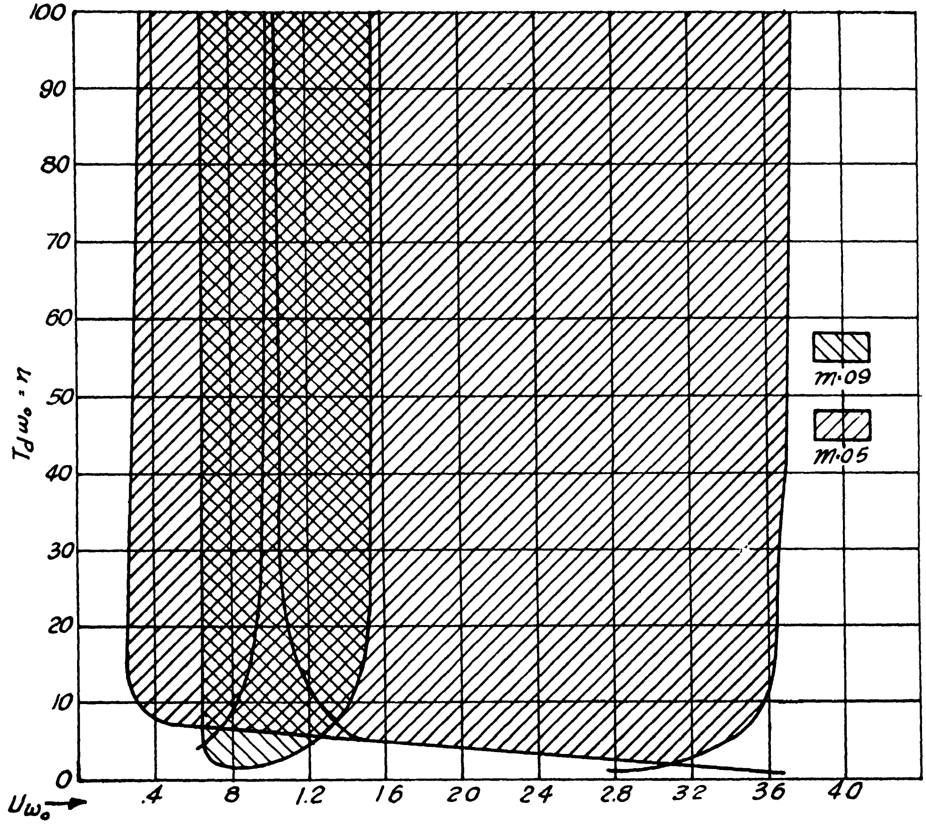


FIG. 2. Possible values of $U\omega_0$. Gain $= \frac{m}{n}$ (m cannot be $\geqq 1$).

The impedances Z_i and Z_o may be expressed in terms of U and one of the parallel "T" constants, say, for example, C_3:

$$Z_i(C_3, U) = \frac{1}{C_3}\,\frac{(Up + 1)\left(\frac{1}{\omega_0^2}p^2 + B_2p + 1\right)}{\left[\left(\frac{1}{S_1}+\frac{1}{S_2}\right)\frac{1}{\omega_0^2}+U\right]Up^3 + (B_2-B_1)\left(\frac{T_1}{S_1}+\frac{U}{S_2}\right)p^2+\left(\frac{T_1}{S_1}+\frac{T_2}{S_2}+1\right)p} \qquad (6)$$

$$Z_o(C_3, U) = \frac{T_2(B_1 + U)p^2 + [(B_1 + U)(1 + S_2Uω_0^2) + T_2]p + (1 + S_2Uω_0^2)}{C_3Uω_0^2(Up + 1)\left(\frac{1}{ω_0^2}p^2 + B_2p + 1\right)}, \qquad (7)$$

where T_1, T_2, S_1, S_2 are given in terms of U by formulae (3) above. From the expression for Z_i, it is evident that as U approaches an endpoint of one of the critical intervals, so that S_1 or S_2 approaches zero, Z_i approaches zero. Thus in order to avoid practically short-circuiting the source of the input voltage, U should not be near one of the endpoints, and also one should not choose a voltage gain g too near the upper bound $1/T_dω_0$.

If it were desired to make a discriminate selection of U among the possible values, the basis might be to best match the angles and ratio of the input and output impedances Z_i and Z_o to the angles and ratio of the source and load impedances to be used with the parallel "T" network. Charts for this purpose could be prepared from the expressions (6), (7) for $Z_i(C_3, U)$ and $Z_o(C_3, U)$.

After T_d, U, and g are selected, the time constants T_1, T_2, T_3, S_1, S_2 are determined by the formulae (3). The formulae in the next sections are obtained from (3) by imposing special conditions which fix the value of u, or the values of u and the gain g.

4. REVERSIBLE PARALLEL "T" NETWORKS.

A parallel "T" proportional-derivative network will be called reversible if it is also a proportional-derivative network when the input and output are interchanged. It is shown in the Appendix that this requirement is equivalent to $C_3/C_1 = R_2/R_3$, or $S_1T_3 = S_2T_1$, and $u = 1$. The derivative time constant T_d necessarily is the same in both directions, but unless the parallel "T" is symmetric, the gains and input and output impedances are different.

The design formulae for the reversible parallel "T" proportional-derivative network are as follows:

$$T_1 = \frac{g}{1 - g}\frac{(n + 1)}{ω_0},$$

$$T_2 = \frac{1 - g}{g}\frac{1}{ω_0}\frac{1}{(n + 1)},$$

$$T_3 = \frac{1}{ω_0},$$

$$S_1 = \frac{1}{ω_0}\left[\frac{(2 + n)(1 - g) - gn(n + 1)}{n(1 - g)}\right],$$

$$S_2 = \frac{1}{ω_0}\left[\frac{2(1 - g)(n + 1) - n(gn + 1)}{gn(n + 1)}\right], \qquad (8)$$

where $T_d\omega_0 = n$ and g is the gain in the forward direction. The gain g and n may be selected as desired; the gain g' in the reverse direction then may be seen to be

$$g' = \frac{g(n^2 + 2n + 2) - (n + 2)}{g(2n^2 + 3n + 2) - (n^2 + 2n + 2)}. \tag{9}$$

To express g in terms of g', it is necessary merely to interchange g and g'. Figure 3 shows the values and relationship of the forward and reverse

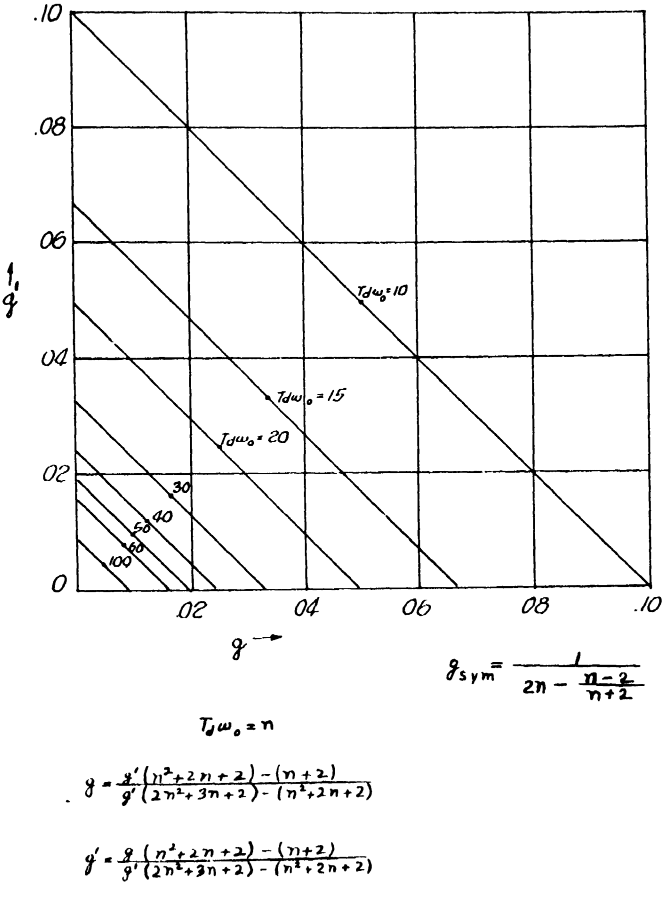


FIG. 3. Forward and reverse gains for reversible parallel "T."

gains g and g', for several values of $T_d\omega_0 = n$.

If the parallel "T" is symmetric, that is $C_3 = C_1$ and $R_2 = R_3$, or $S_1 = T_1$ and $S_2 = T_3$, the gain is

$$g = g' = g_{\text{sym.}} = \frac{1}{2n - \dfrac{n - 2}{n + 2}}. \tag{10}$$

5. PARALLEL "T" WITH ARBITRARY CONDENSERS.

Let us assume that the values of the condensers C_1, C_2, C_3 in the parallel "T" are known or specified, and determine the values of R_1, R_2, R_3 to realize any desired $T_d = n/\omega_0$. Eliminating g and u from Eqs. 3, we find

$$
\begin{aligned}
R_1 &= \frac{1}{\omega_0}\frac{\left(\frac{2}{n}+u\right)}{(C_1+C_3)},\\
R_2 &= \frac{r}{\omega_0}\frac{u^2}{\left(\frac{2}{n}+u\right)C_2}, \qquad\qquad (11)\\
R_3 &= \frac{1}{u\omega_0C_3},
\end{aligned}
$$

where $r = (C_1 + C_3)/C_1$, and u is a solution of

$$ru^3 - \frac{2}{n}\frac{C_2}{C_3}ru^2 + \frac{C_2}{C_3}(1-r)u + \frac{2}{n}\frac{C_2}{C_3} = 0. \tag{12}$$

The gain g is given by

$$\frac{2}{g} = \frac{rn}{\left(\frac{2}{n}+u\right)} + rnu + 2. \tag{13}$$

For $C_1 = C_2 = C_3$, the positive solutions for u are $u = 2^{-\frac{1}{2}}$ and $u = 2/n$.

6. PARALLEL "T" WITH ARBITRARY RESISTORS.

If the resistors in the parallel "T" are arbitrarily specified, then to realize the proportional-derivative characteristic with $T_d\omega_0 = n$, the value of condenser C_3 must be a solution of the following equation:

$$R_1R_3(R_2+R_3)C_3^3 - \frac{2}{n\omega_0}R_3(R_2+R_3)C_3^2 - \frac{1}{\omega_0^2}R_2C_3 + \frac{2}{n\omega_0^3} = 0.$$

The values of condensers C_1 and C_2, then, are given by

$$C_1 = \frac{1}{R_1R_3C_3\omega_0^2} + \frac{2}{R_1n\omega_0} - C_3$$

and

$$C_1C_2 = \frac{1}{R_1R_2R_3^2C_3^2\omega_0^4}.$$

The gain is given by

$$\frac{2}{gn} = \frac{1}{\omega_0R_1C_1}\left(1 + \frac{2}{n\omega_0R_3C_3} + \frac{1}{\omega_0^2R_3^2C_3^2}\right) + \frac{2}{n}.$$

For the case of equal resistors, $R_1 = R_2 = R_3 = R$, the two positive solutions of the cubic are $RC_3 = 2^{-\frac{1}{2}}\omega_0^{-1}$ and $RC_3 = 2/n\omega_0$. The values of the condensers corresponding to the first solution are:

$$C_3 = \frac{1}{2^{\frac{1}{2}}R\omega_0},$$

$$C_1 = \frac{1}{R\omega_0}\left(2^{-\frac{1}{2}} + \frac{2}{n}\right)$$

$$C_2 = \frac{1}{R\omega_0}\frac{2}{\left(2^{-\frac{1}{2}} + \frac{2}{n}\right)}.$$

The indeterminate U in the general transfer characteristic has the value $U = 2^{\frac{1}{2}}/\omega_0$. The gain is:

$$g = \frac{2^{\frac{1}{2}}n + 4}{3n^2 + 3\cdot 2^{\frac{1}{2}}n + 4}.$$

The input impedance is:

$$Z_i = \frac{(3n^2 + 3\sqrt{2}\,n + 4)jR}{(-\,5.25\sqrt{2}\,n^2 - 18n - 2\sqrt{2}) + j(2n^2 + \sqrt{2}\,n + 4)}.$$

For the case of $n = T_d\omega_0 = 15$, the value of the input impedance is $Z_i = (0.0882 - 0.3606j)R$.

7. DIVIDER CIRCUITS TO INCREASE AND DECREASE T_d.

Given a proportional derivative characteristic $B(p)$ with derivative time constant T_d', a characteristic with any lower value of T_d may be obtained with the circuit of Fig. 4, assuming a perfect voltage divider,

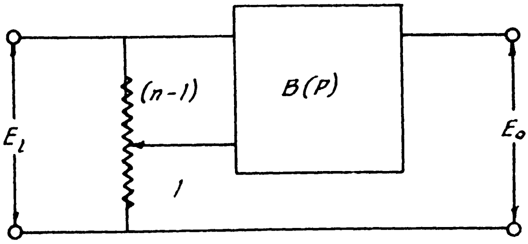


FIG. 4.

that is, neglecting the loading effect of the input impedance of $B(p)$ on the divider. (Actually the frequencies of minimum output and zero phase of $B(p)$ will change; the amount of this shift may be readily calculated; or for fixed divider values $B(p)$ may be designed to give exactly the desired characteristic in combination with the divider.) In particular, $T_d\omega_0 = n$ may be taken to be ∞ in the formulae of the preceding sections, to obtain a "resonant" parallel "T," and then with suitable divider values any desired T_d may be obtained.

The transfer characteristic of a resonant parallel "T" is:

$$\frac{jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1+jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}.$$

If $B(j\omega)$ in Fig. 4 is this transfer characteristic, assuming a perfect divider of ratio $1/n$, the transfer characteristic of Fig. 4 is

$$\frac{E_o}{E_i} = \frac{1}{n}\left[1+\frac{j(n-1)l\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1+jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}\right]$$

$$= \frac{l}{T_d}\left[\frac{1+jT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1+jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}\right],$$

where $T_d = n/\omega_0$, which is the same as (2). Although the gain for the combination of resonant "T" and divider is the same as for the proportional-derivative parallel "T," the former has the disadvantage that the input impedance is lower for equivalent output impedances.

Given a proportional-derivative characteristic $B(p)$, a characteristic with any higher value of T_d may be obtained with the circuit of Fig. 5.

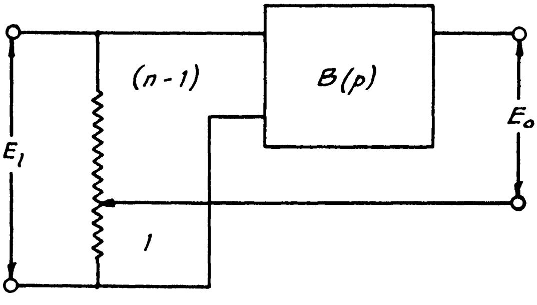


FIG. 5.

In this case there is no loading effect on the divider, so it is unnecessary to make corrective calculations as it is in the case of Fig. 4. If

$$B(p) = \left[1+jT_d'\frac{\omega+\omega_0}{2\omega}(\omega-\omega_0)\right]\Big/\left[1+jl\frac{\omega+\omega_0}{2\omega}(\omega-\omega_0)\right],$$

then for Fig. 5,

$$\frac{E_o}{E_i} = \frac{n-1}{n}\frac{l}{T_d}\frac{1 + jT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1 + jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)},$$

where

$$T_d = (n-1)l/(nl/T_d' - 1), \quad n = (T_d/l - 1)/(T_d/T_d' - 1).$$

This is the same as (2) except that the gain is lower by the factor $(n-1)/n$. If $n < T_d'/l$, T_d is negative, and a servo using the circuit of Fig. 5 then naturally would be less stable than a proportional servo.

A scheme for increasing T_d, in which input and output have a common terminal (which may be grounded), is shown in Fig. 6.

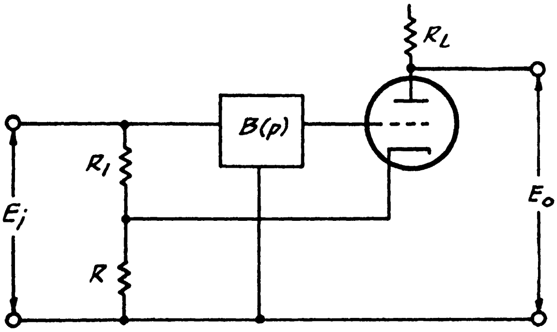


FIG. 6.

If T_d' is the derivative time constant of $B(p)$, it may be shown that the derivative time constant T_d of E_o/E_i is given by

$$T_d = \frac{T_d'}{1 - \dfrac{(\mu+1)a}{\mu(1-b)}} \cong \frac{R+R_1}{R_1}T_d',$$

where μ is the amplification factor of the tube, r_p is its plate resistance, and

$$a = R(r_p + R_L)/[R(R_1 + r_p + R_L) + R_1(r_p + R_L)],$$

$$(1-b) = (R + R_1)(r_p + R_L)/[R(R_1 + r_p + R_L) + R_1(r_p + R_L)].$$

8. NUMERICAL VALUES FOR 60-CYCLE PARALLEL "T" 's.

Following is a table of values of R's and C's for various parallel "T" networks of the types described in the preceding sections, for a carrier frequency of 60 cycles. The R's and C's are numbered as in Fig. 1.

The "notch width" of a band-rejection network is defined as the

TABLE I.—*Symmetric Parallel "T" Networks*, $f_0 = 60$ *cps.*
$R_2 = R_3 = 0.002652$, $C_1 = C_3 = 1.000$.

$T_d\omega_0$	Notch Interval, cps.	$R_1 \times 10^3$	C_2	Forward Gain	Reverse Gain	Input Impedance, $Z_i \times 10^3$	Output Impedance, $Z_o \times 10^3$
2.5	±24	2.387	1.1111	0.2045	0.2045	$1.667(1-j)$	$1.869(1-j)$
5	±12	1.856	1.4286	0.1045	0.1045	$1.481(1-j)$	$1.465(1-j)$
7.5	±9	1.680	1.5790	0.06934	0.06934	$1.425(1-j)$	$1.418(1-j)$
10	±6	1.591	1.6667	0.05172	0.05172	$1.399(1-j)$	$1.395(1-j)$
15	±4	1.503	1.7647	0.03421	0.03421	$1.373(1-j)$	$1.372(1-j)$
20	±3	1.459	1.8182	0.02552	0.02552	$1.361(1-j)$	$1.360(1-j)$
30	±2	1.414	1.8750	0.01691	0.01691	$1.349(1-j)$	$1.349(1-j)$
40	±1.5	1.392	1.9048	0.01264	0.01264	$1.343(1-j)$	$1.343(1-j)$
50	±1.2	1.379	1.9231	0.01009	0.01009	$1.340(1-j)$	$1.340(1-j)$
60	±1.0	1.370	1.9355	0.008400	0.008400	$1.337(1-j)$	$1.337(1-j)$
∞	(Resonant)	1.326	2.0000			$1.326(1-j)$	$1.326(1-j)$

TABLE II.—*Reversible Parallel "T" Networks.*
$R_2 = 0.01326$, $R_3 = 0.002652$, $C_1 = 0.2000$, $C_3 = 1.000$.

$T_d\omega_0$	Notch Interval, cps.	$R_1 \times 10^3$	C_2	Forward Gain	Reverse Gain	Input Impedance, $Z_i \times 10^3$	Output Impedance, $Z_o \times 10^3$
2.5	±24	3.978	0.6667	0.07895	0.3000	$7.200(1-j)$	$1.808(1-j)$
5	±12	3.094	0.8571	0.03743	0.1628	$6.889(1-j)$	$1.574(1-j)$
7.5	±9	2.799	0.9474	0.02423	0.1105	$6.796(1-j)$	$1.487(1-j)$
10	±6	2.652	1.0000	0.01786	0.06417	$6.752(1-j)$	$1.445(1-j)$
15	±4	2.505	1.0588	0.01167	0.05574	$6.710(1-j)$	$1.404(1-j)$
20	±3	2.431	1.0909	0.008654	0.04183	$6.689(1-j)$	$1.384(1-j)$
30	±2	2.357	1.1250	0.005702	0.02787	$6.669(1-j)$	$1.364(1-j)$
40	±1.5	2.321	1.1429	0.004250	0.02090	$6.660(1-j)$	$1.354(1-j)$
50	±1.2	2.298	1.1539	0.003387	0.01671	$6.654(1-j)$	$1.349(1-j)$
60	±1.0	2.284	1.1613	0.002815	0.01392	$6.650(1-j)$	$1.345(1-j)$
∞	(Resonant)	2.210	1.2000			$6.631(1-j)$	$1.326(1-j)$

TABLE III.—*Parallel "T" Networks With Equal Condensers*, $u = 1/\sqrt{2}$.
$R_3 = 0.003751$, $C_1 = C_2 = C_3 = 1.000$.

$T_d\omega_0$	Notch Interval, cps.	$R_1 \times 10^3$	$R_2 \times 10^3$	Forward Gain	Reverse Gain	Input Impedance, $Z_i \times 10^3$	Output Impedance, $Z_o \times 10^3$	Reverse Phase Shift at ω_0
2.5	±24	1.998	1.760	0.2259	0.17419	$1.324-1.545j$	$1.798-1.768j$	−5.00°
5	±12	1.468	2.395	0.1105	0.09455	$1.303-1.573j$	$1.565-1.768j$	−1.84°
7.5	±9	1.291	2.723	0.07690	0.06467	$1.303-1.620j$	$1.469-1.768j$	−0.95°
10	±6	1.203	2.934	0.05237	0.04807	$1.289-1.640j$	$1.418-1.768j$	−0.57°
15	±4	1.114	3.155	0.03395	0.03200	$1.280-1.675j$	$1.364-1.768j$	−0.27°
20	±3	1.070	3.286	0.02505	0.02394	$1.274-1.695j$	$1.337-1.768j$	−0.16°
30	±2	1.026	3.427	0.01640	0.01590	$1.267-1.717j$	$1.306-1.768j$	−0.06°
40	±1.5	1.004	3.503	0.01218	0.01202	$1.264-1.729j$	$1.293-1.768j$	−0.04°
50	±1.2	0.9907	3.550	0.009682	0.009500	$1.261-1.737j$	$1.285-1.768j$	−0.02°
60	±1.0	0.9818	3.582	0.008034	0.007910	$1.260-1.742j$	$1.279-1.768j$	−0.01°
∞	(Resonant)	0.9378	3.751			$1.250-1.768j$	$1.250-1.768j$	

length of the frequency interval between the points at which the output is $\sqrt{2}$ times the minimum output. A proportional-derivative network may be characterized as well by the notch width, as by the derivative time constant T_d. The notch widths in the following table are for the ideal proportional-derivative characteristic $[1 + jT_d(\omega - \omega_0)]$; they are only slightly different from the actual ones for the parallel "T" networks. Evidently for the ideal characteristic at 60 cycles, the end points of the notch interval are at distances $\pm \frac{1}{2\pi T_d} = \pm \frac{60}{n}$ cps. from 60 cps., where $T_d\omega_0 = n$.

The R's and C's in Tables I, II, and III are in corresponding units, as for example megohms and microfarads. As mentioned previously, a parallel "T" with any desired different impedance level may be obtained from the values in the table by multiplying R's by any constant and dividing C's by the same constant. For a carrier frequency f different from 60 cycles, if $f = 60a$, divide either the R's or C's by a, divide T_d by a, multiply notch width by a; and Z_i/R_3 and Z_o/R_3 at carrier frequency are unchanged.

9. DESIGN FORMULAE FOR BRIDGE "T" NETWORKS.

The two forms of the bridge "T" proportional-derivative networks, Fig. 7(a) and (b), may be obtained as special cases of the parallel "T"

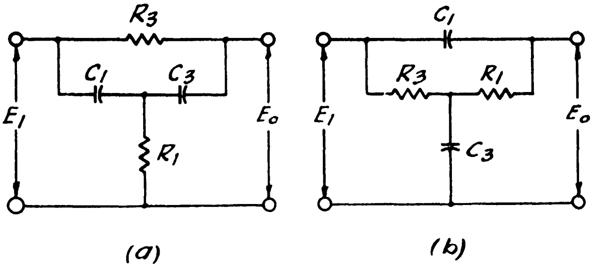


FIG. 7.

in Fig. 1, (a) by taking $R_2 = 0$, and (b) by taking $C_1 = \infty$. In (b), R_2 of Fig. 1 is changed to R_3, R_3 is changed to R_1, C_3 to C_1, C_2 to C_3, for convenience, so that similar formulae will apply to both (a) and (b).

The transfer characteristic is

$$\frac{E_o}{E_1} = \frac{T_1T_3p^2 + (T_1 + S_1)p + 1}{T_1T_3p^2 + (T_1 + S_1 + T_3)p + 1},$$

where $T_1 = R_1C_1$, $T_3 = R_3C_3$; and $S_1 = R_1C_3$ in case (a), $S_1 = R_3C_1$ in case (b).

To realize the proportional derivative characteristic (2), with derivative time constant T_d and lag l, or gain $g = l/T_d$, the time constants

T_1, T_3, S_1 are given by the following formulae:

$$T_3 = \frac{2}{\omega_0^2}\left(\frac{1}{l} - \frac{1}{T_d}\right) = \frac{2}{\omega_0^2 T_d}\left(\frac{1}{g} - 1\right),$$

$$T_1 = \frac{1}{2\left(\frac{1}{l} - \frac{1}{T_d}\right)} = \frac{T_d}{2\left(\frac{1}{g} - 1\right)},$$

$$S_1 = \left[\frac{2}{T_d\omega_0^2} - \frac{1}{2\left(\frac{1}{l} - \frac{1}{T_d}\right)}\right] = \frac{2}{T_d\omega_0^2} - \frac{T_d}{2\left(\frac{1}{g} - 1\right)}.$$

For positive S_1, $g < 4/(4 + n^2)$, where $n = T_d\omega_0$. For the parallel "T," the upper bound of gain is $1/n$. Thus for n greater than 2, the gains attainable with the bridge "T" are considerably smaller than those which can be obtained with the parallel "T." (Also, since the time lag l is smaller, the relative amplification at frequency $3\omega_0$, the predominant frequency of synchro "hash," is greater for the bridge "T.")

The following formulae in terms of arbitrary C_1, C_3 for case (*a*), and in terms of arbitrary R_1, R_3 for case (*b*), may be more convenient. For case (*a*),

$$R_1 = \frac{2}{n\omega_0(C_1 + C_3)},$$

$$R_3 = \frac{n}{2\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_3}\right),$$

$$g = \frac{4}{n^2\left(1 + \frac{C_3}{C_1}\right) + 4}.$$

For case (*b*),

$$C_1 = \frac{2}{n\omega_0(R_1 + R_3)},$$

$$C_3 = \frac{n}{2\omega_0}\left(\frac{1}{R_1} + \frac{1}{R_3}\right),$$

$$g = \frac{4}{n^2\left(1 + \frac{R_3}{R_1}\right) + 4}.$$

The formulae for (*b*) are obtained from those for (*a*) by interchanging R_1 with C_1, and R_3 with C_3.

From the above formulae, it is clear that any bridge "T" is reversible, with the same T_d but different gains in the forward and reverse directions, unless it is symmetric, that is $C_1 = C_3$ in case (*a*) $R_1 = R_3$ in case (*b*).

10. INPUT AND OUTPUT IMPEDANCES.

For bridge "T" (*a*), the input impedance (with the output open-circuited) is

$$Z_i = R_1 + \frac{(T_3p + 1)}{C_1T_3p^2 + (C_1 + C_3)p}.$$

The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_3[(S_1 + T_1)p + 1]}{T_1T_3p^2 + (T_1 + S_1 + T_3)p + 1},$$

where $S_1 = R_1C_3$.

For bridge "T" (*b*), the input impedance (with the output open-circuited) is

$$Z_i = \frac{1}{C_3p} + \frac{R_3(T_1p + 1)}{(T_1 + S_1)p + 1},$$

where $S_1 = R_3C_1$. The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_1T_3p + (R_1 + R_3)}{T_1T_3p^2 + (S_1 + T_1 + T_3)p + 1}.$$

11. FORMULAE FOR WIEN BRIDGE.

The Wien Bridge proportional-derivative circuit is shown in Fig. 8. The transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{R_2}{(R_1 + R_2)}\,\frac{T_3T_4p^2 + [T_3 + T_4 - (R_1/R_2)S_4]p + 1}{T_3T_4p^2 + (T_3 + T_4 + S_4)p + 1},$$

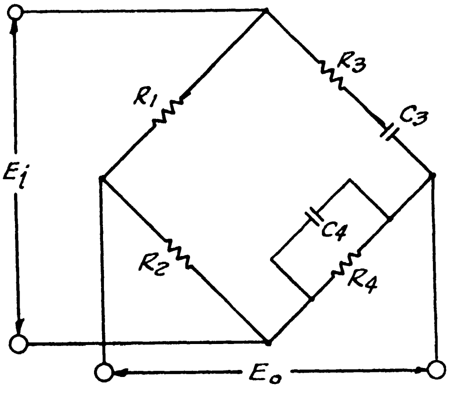


FIG. 8.

where $T_3 = R_3C_3$, $T_4 = R_4C_4$, $S_4 = R_4C_3$. The independent variables of the transfer characteristic may be taken to be T_3, T_4, S_4, and the divider ratio $\rho = R_2/(R_1 + R_2)$. Solving for T_3, T_4, S_4 in terms of

$T_d\omega_0 = n$, $l\omega_0 = m$, and ρ, we obtain:

$$T_3 = \frac{1}{\omega_0}\left[\frac{\rho}{n} + \frac{(1-\rho)}{m}\right] \pm \frac{1}{\omega_0}\left[\left(\frac{\rho}{n} + \frac{(1-\rho)}{m}\right)^2 - 1\right]^{\frac{1}{2}},$$

$$T_4 = \frac{1}{\omega_0}\left[\frac{\rho}{n} + \frac{(1-\rho)}{m}\right] \mp \frac{1}{\omega_0}\left[\left(\frac{\rho}{n} + \frac{(1-\rho)}{m}\right)^2 - 1\right]^{\frac{1}{2}},$$

$$S_4 = \frac{2\rho}{\omega_0}\left(\frac{1}{m} - \frac{1}{n}\right).$$

For a given m and ρ, the condition for realizability is $n \leqq \rho m/(m + \rho - 1)$. From the form of the transfer characteristic, the gain is $\rho m/n$. For maximum gain, $\rho = m = n - [n(n-1)]^{\frac{1}{2}} > n - [(n-0.5)^2]^{\frac{1}{2}} = 0.5$, the radical vanishes, and $T_3 = T_4 = 1/\omega_0$, $S_4 = 2(n-1)^{\frac{1}{2}}/\omega_0 n^{\frac{1}{2}}$. The maximum gain is $2\rho - 1$. For the usual values of $T_d\omega_0 = n$, the maximum obtainable gain is approximately $1/(4n)$, which is one half the gain of the symmetric parallel "T," one fourth the upper bound of gains obtainable with the parallel "T."

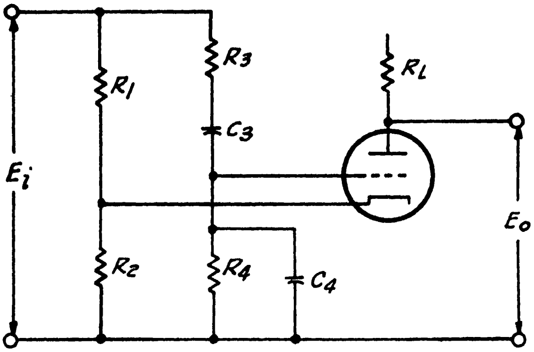


FIG. 9.

The derivative time constant T_d may be varied in the Wien Bridge by changing the divider ratio ρ, without changing R_3, C_3, R_4, C_4. Solving for $T_d\omega_0 = n$, we have

$$n = \frac{2m}{2 - \dfrac{\omega_0 S_4 m}{\rho}}.$$

Thus as ρ decreases to $\frac{1}{2}\omega_0 S_4 m$, $n \to \infty$; if $\rho = \frac{1}{2}\omega_0 S_4 m$, the Wien Bridge is "resonant"; and as ρ increases to 1, by the realizability condition, n decreases to a value not greater than 1. For any R_3, C_3, R_4, C_4 such that $T_3 T_4 \omega_0^2 = 1$, any desired T_d may be obtained by taking ρ to be sufficiently small. Since $2/m\omega_0 = T_3 + T_4 + S_4$, the maximum time lag $l = m/\omega_0$ (but not maximum gain) will be obtained if $T_3 = T_4 = 1/\omega_0$

and R_4 and C_3 are as small as possible. For $T_3 = T_4 = 1/\omega_0$, the realizability condition is $n \leqq 2\rho/[(S_4\omega_0 + 2)\rho - S_4\omega_0]$.

A vacuum tube circuit similar to Fig. 6 may be used with the Wien Bridge in order to have a common input and output terminal (which may be grounded). See Fig. 9. It may be shown that the transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{-R_LR_2(\mu - 1)\left[(T_3p + 1)(T_4p + 1) - \dfrac{(R_2 + R_1\mu)S_4p}{R_2(\mu - 1)}\right]}{[R_2(R_1 + r_p + R_L) + R_1(r_p + R_L)][(T_3p + 1)(T_4p + 1) + S_4p]}.$$

If the amplification factor μ is very large compared to $R_2/R_1 = \rho/(1 - \rho)$, then except for the gain factor, this transfer characteristic is the same as the transfer characteristic for Fig. 8.

12. NUMERICAL VALUES FOR BRIDGE "T" NETWORKS.

The remarks on Tables I, II, and III for parallel "T" networks apply also to Tables IV, V, and VI for 60-cycle Bridge "T" networks.

TABLE IV.—*Symmetrical Bridge "T" Networks (a).*[4]
$C_1 = C_3 = 1.000$.

$T_d\omega_0$	Notch Interval, cps.	$R_1 \times 10^3$	$R_3 \times 10^3$	Gain	Input Impedance, $Z_i \times 10^3$	Output Impedance, $Z_o \times 10^3$
2.5	±24	1.0610	6.63	0.24242	$1.7079 - 2.135j$	$1.6075 - 2.009j$
5	±12	0.5305	13.26	0.07407	$0.9878 - 2.470j$	$0.9824 - 2.456j$
7.5	±9	0.3536	19.89	0.03433	$0.6838 - 2.564j$	$0.6830 - 2.561j$
10	±6	0.2653	26.53	0.01961	$0.5203 - 2.602j$	$0.5201 - 2.601j$
15	±4	0.1768	39.79	0.008811	$0.3505 - 2.629j$	$0.3506 - 2.629j$
20	±3	0.1326	53.05	0.004975	$0.2639 - 2.639j$	$0.2639 - 2.639j$
30	±2	0.08841	79.58	0.002217	$0.1764 - 2.647j$	$0.1764 - 2.647j$
40	±1.5	0.06631	106.1	0.001248	$0.1325 - 2.649j$	$0.1325 - 2.649j$
50	±1.2	0.05305	132.6	0.0007994	$0.1060 - 2.650j$	$0.1060 - 2.650j$
60	±1.0	0.04420	159.2	0.0005552	$0.08836 - 2.651j$	$0.08836 - 2.651j$
100	±0.6	0.02653	265.3	0.0002000	$0.05304 - 2.652j$	$0.05304 - 2.652j$

TABLE V.—*Bridge "T" Networks (a).*[4]
$C_1 = 1.000$, $C_3 = 2.500$.

$T_d\omega_0$	Notch Interval, cps.	$R_1 \times 10^3$	$R_3 \times 10^3$	Forward Gain	Reverse Gain	Input Impedance, $Z_i \times 10^3$	Output Impedance, $Z_o \times 10^3$
2.5	±24	0.6062	4.642	0.15456	0.3137	$1.5305 - 1.914j$	$0.7175 - 0.8979j$
5	±12	0.3031	9.284	0.04372	0.1026	$0.9565 - 2.391j$	$0.4059 - 1.015j$
7.5	±9	0.2020	13.926	0.01991	0.04834	$0.6737 - 2.527j$	$0.2772 - 1.040j$
10	±6	0.1516	18.57	0.01130	0.02778	$0.5159 - 2.580j$	$0.2098 - 1.049j$
15	±4	0.1010	27.85	0.005054	0.01254	$0.3493 - 2.619j$	$0.1408 - 1.056j$
20	±3	0.07579	37.14	0.002849	0.007092	$0.2634 - 2.634j$	$0.1058 - 1.058j$
30	±2	0.05052	55.70	0.001268	0.003165	$0.1763 - 2.644j$	$0.07064 - 1.0597j$
40	±1.5	0.03789	74.27	0.0007138	0.001783	$0.1324 - 2.648j$	$0.05301 - 1.0603j$
50	±1.2	0.03031	92.84	0.0004569	0.001142	$0.1060 - 2.649j$	$0.04242 - 1.06052j$
60	±1.0	0.02526	111.4	0.0003174	0.0007930	$0.08835 - 2.650j$	$0.03536 - 1.0607j$
100	±0.6	0.01516	185.7	0.0001143	0.0002856	$0.05304 - 2.652j$	$0.02122 - 1.0609j$

[4] To obtain values for Bridge "T" (*b*), in Tables IV and V interchange R_1 with C_1, and R_3 with C_3.

TABLE VI.—*Maximum Gain Wien Bridge Networks.*

$R_3 = 0.002653$, $C_3 = 1.000$.

$Z_i = (R_1 + R_2)$ in parallel with Z_i'. $Z_o = R_1R_2/(R_1 + R_2)$ in series with Z_o'.

$T_{d\omega_0}$	Notch Interval, cps.	$R_2/(R_1 + R_2) = \rho = m$	$R_4 \times 10^3$	C_4	Gain	$Z_i' \times 10^3$	$Z_o' \times 10^3$
5	±12	0.5279	4.745	0.5590	0.05572	$5.024(1-j)$	$1.252(1-j)$
10	±6	0.5162	5.033	0.5270	0.03234	$5.168(1-j)$	$1.291(1-j)$
15	±4	0.5086	5.125	0.5175	0.01724	$5.214(1-j)$	$1.303(1-j)$
20	±3	0.5064	5.171	0.5130	0.01282	$5.237(1-j)$	$1.309(1-j)$
30	±2	0.5042	5.216	0.5086	0.008476	$5.259(1-j)$	$1.315(1-j)$
40	±1.5	0.5032	5.238	0.5064	0.006330	$5.271(1-j)$	$1.318(1-j)$
50	±1.2	0.5025	5.252	0.5051	0.005050	$5.277(1-j)$	$1.319(1-j)$
60	±1.0	0.5021	5.260	0.5042	0.004220	$5.282(1-j)$	$1.320(1-j)$
100	±0.6	0.5013	5.278	0.5025	0.002512	$5.291(1-j)$	$1.322(1-j)$
∞	(Resonant)	0.5000	5.305	0.5000		$5.305(1-j)$	$1.326(1-j)$

Wien Bridge, $R_3 = 0.002653 = 2R_4$, $C_4 = 2.0 = 2C_3$, $m = 0.8$.

$T_{d\omega_0}$	$R_2/(R_1 + R_2) = \rho$	Gain
5	0.2381	0.03810
10	0.2174	0.01739
15	0.2113	0.01127
20	0.2083	0.008333
30	0.2055	0.005479
40	0.2041	0.004081
50	0.2033	0.003252
60	0.2027	0.002702
100	0.2016	0.001612
∞	0.2000	

Wien Bridge, $R_4 = 0.005305 = 2R_3$, $C_3 = 1.0 = 2C_4$, $m = 0.5$.

$T_{d\omega_0}$	$R_2/(R_1 + R_2) = \rho$	Gain
5	0.5556	0.05556
10	0.5263	0.02632
15	0.5172	0.01724
20	0.5128	0.01282
30	0.5085	0.008474
40	0.5063	0.006329
50	0.5051	0.005050
60	0.5042	0.004201
100	0.5025	0.002502
∞	0.5000	

13. VARIATION OF RESONANT FREQUENCY ω_0 OF PARALLEL "T" NETWORK.

For any parallel "T," the numerator of the transfer characteristic is of the form:

$$\left(\frac{u}{\omega_1}p + 1\right)\left(\frac{1}{\omega_1^2}p^2 + \frac{2}{n\omega_1}p + 1\right)$$

for some ω_1, u, and n. By comparison with the transfer characteristic (1) in section 3, it may be seen that the relations between these quantities and the parallel "T" constants are:

$$\frac{u}{\omega_1^3} = T_1T_2T_3 = a,$$

$$\frac{1}{\omega_1^2}\left(1 + \frac{2u}{n}\right) = T_1(S_2 + T_3) = b,$$

$$\frac{1}{\omega_1}\left(\frac{2}{n} + u\right) = (T_1 + S_1) = c.$$

Eliminating u and n, we obtain:

$$a^2x^3 - acx^2 + bx - 1 = 0,$$

where $x = \omega_1^2$. If $\Delta_i = dR_i/R_i$, $\epsilon_i = dC_i/C_i$, we have

$$\begin{aligned} da &= a(\Delta_1 + \Delta_2 + \Delta_3 + \epsilon_1 + \epsilon_2 + \epsilon_3), \\ db &= b\Delta_1 + T_1S_2\Delta_2 + T_1T_3\Delta_3 + b(\epsilon_1 + \epsilon_3), \\ dc &= c\Delta_1 + T_1\epsilon_1 + S_1\epsilon_3, \end{aligned}$$

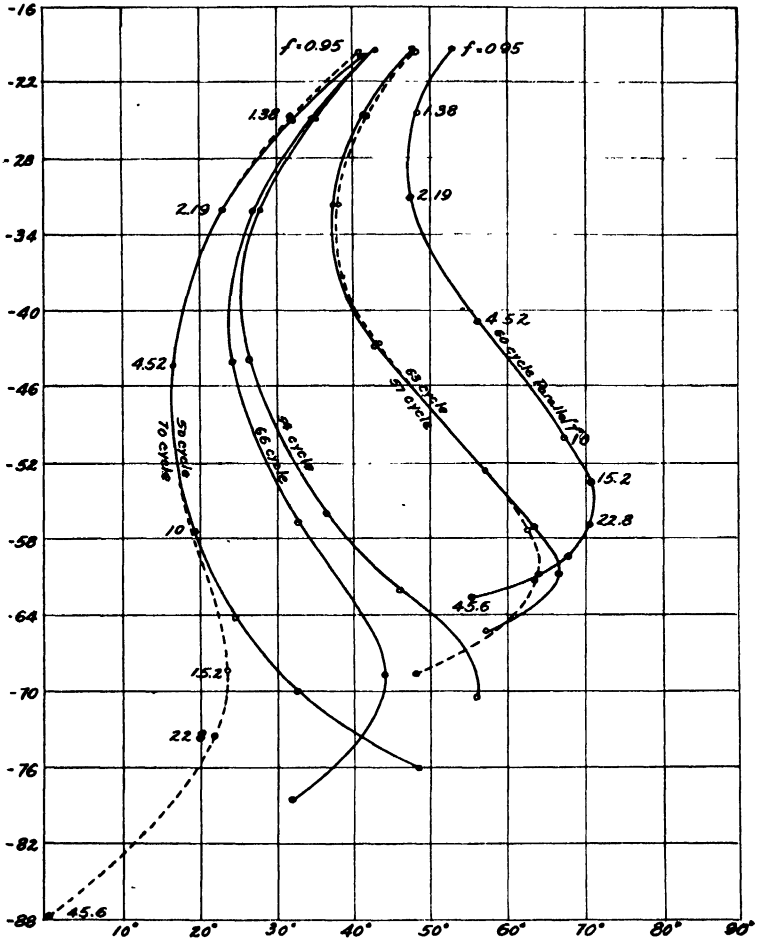


FIG. 10. Effect of incorrect ω_0 on gain-phase diagram for 60-cycle parallel "T" servo. $T_d = 15/2\pi 60$, motor time constant = 0.2 sec. Values of ω_0 from $2\pi 50$ to $2\pi 70$ (ω_0 = parallel "T" frequency).

and

$$\begin{aligned} Qdx = {} & \Delta_1[(cx^2 - 2ax^3)a - bx + acx^2] \\ & + \Delta_2[(cx^2 - 2ax^3)a - xS_2T_1] \\ & + \Delta_3[(cx^2 - 2ax^3)a - xT_1T_3] \\ & + \epsilon_1[(cx^2 - 2ax^3)a - bx + ax^2T_1] \\ & + \epsilon_2[(cx^2 - 2ax^3)a] \\ & + \epsilon_3[(cx^2 - 2ax^3)a - bx + ax^2S_1], \end{aligned}$$

where $Q = (3a^2x^2 - 2acx + b)$.

For the parallel "T" with equal condensers, section 5, we have $T_1 = S_1 = (2/n + u)/2\omega_0$, $T_2 = S_2 = 1/\omega_0(2/n + u)$, $T_3 = 1/u\omega_0$.

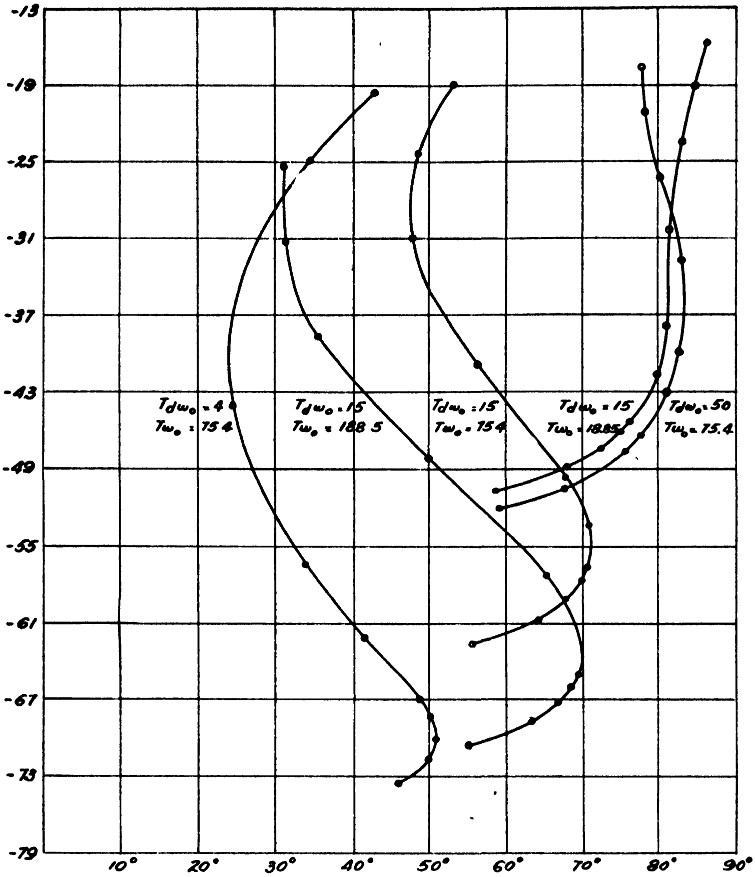


FIG. 11. Effects of change of derivative time constant T_d and of motor time constant T. Parallel "T" proportional-derivative characteristic, $l\omega_0 = 0.5$.

Substituting these values and $\omega_1 = \omega_0$ in the expression for dx, we obtain:

$$\left(1 + \frac{2\sqrt{2}}{n}\right)\frac{d\omega_0}{\omega_0} = \left(1 - \frac{\sqrt{2}}{n}\right)(\Delta_1 + \Delta_2) + \Delta_3$$
$$+ \left(\frac{5}{4} - \frac{1}{\sqrt{2}n}\right)(\epsilon_1 + \epsilon_3) + \left(\frac{1}{2} - \frac{\sqrt{2}}{n}\right)\epsilon_2.$$

For fairly large n, this is approximately:

$$\frac{d\omega_0}{\omega_0} = \Delta_1 + \Delta_2 + \Delta_3 + \frac{5}{4}(\epsilon_1 + \epsilon_3) + \tfrac{1}{2}\epsilon_2.$$

For 5 per cent tolerance on resonant frequency ω_0 of the parallel "T" with equal condensers, taking $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$ and $\epsilon_1 = \epsilon_3 = \epsilon$, $0.05 = 3\Delta + 10\epsilon/4 + \epsilon_2/2$. Therefore, in order to insure that ω_0 will fall in the range $2\pi\ 57$ to $2\pi\ 63$ (see curves in Fig. 10), the appropriate tolerances on the parallel "T" components are ±0.83 per cent for R_1, R_2, R_3, ±0.67 per cent for C_1, C_3, and ±1.67 per cent for C_2. Uniform tolerance of ±1 per cent will insure that the resonant frequency will fall within a band of ±6 per cent from the correct value.

14. VARIATION OF NOTCH WIDTH OF PARALLEL "T" TRANSFER CHARACTERISTIC.

The notch width is $1/\pi T_d = y/2\pi$, where $y = 2/T_d$. Eliminating ω_1 and u from the relations (1), we obtain:

$$y(b - ay)^2 - c(b - ay) + a = 0.$$

Differentiating, we have:

$$\begin{aligned} Rdy = {} & \Delta_1[2ay^2(b - ay) - acy - a + b(2ay^2 - 2by + c) + (b - ay)c] \\ & + \Delta_2[2ay^2(b - ay) - acy - a + T_1S_2(2ay^2 - 2by + c)] \\ & + \Delta_3[2ay^2(b - ay) - acy - a + T_1T_3(2ay^2 - 2by + c)] \\ & + \epsilon_1[2ay^2(b - ay) - acy - a + b(2ay^2 - 2by + c) + (b - ay)T_1] \\ & + \epsilon_2[2ay^2(b - ay) - acy - a] \\ & + \epsilon_3[2ay^2(b - ay) - acy - a + b(2ay^2 - 2by + c) + (b - ay)S_1], \end{aligned}$$

where $R = b^2 + ac - 4aby + 3a^2y^2$.

For the parallel "T" with equal condensers, the expression for Rdy becomes:

$$\begin{aligned} \left(\frac{3}{2} - \frac{\sqrt{2}}{n}\right)\frac{dy}{\omega_0} = {} & \frac{1}{\sqrt{2}}\Delta_1 + \left(\frac{2\sqrt{2}}{n^2} - \frac{2}{n} - \frac{1}{2\sqrt{2}}\right)\Delta_2 - \left(\frac{1}{n} + \frac{1}{2\sqrt{2}}\right)\Delta_3 \\ & + \left(\frac{1}{2\sqrt{2}} - \frac{1}{n}\right)(\epsilon_1 + \epsilon_3) - \left(\frac{1}{2\sqrt{2}} + \frac{2}{n} - \frac{2\sqrt{2}}{n^2}\right)\epsilon_2. \end{aligned}$$

For fairly large n, this is approximately:

$$\frac{3}{2}\frac{dy}{\omega_0} = \frac{1}{\sqrt{2}}\Delta_1 + \frac{1}{2\sqrt{2}}(-\Delta_2 - \Delta_3 + \epsilon_1 - \epsilon_2 + \epsilon_3).$$

Therefore if all components of the parallel "T" are correct within ±1 per cent, dy/ω_0 will not exceed $7\sqrt{2}/6 = 1.65$ per cent in numerical value. That is, the notch width will be within ±1.65 per cent of the carrier frequency of its correct value. For 60-cycle carrier frequency and ±1 per cent tolerances, the variation in notch width cannot exceed $0.0165 \times 60 = 0.99$ cps. (For $T_d\omega_0 = 15$, the notch width then would be (8 ± 0.99) cps.; for $T_d\omega_0 = 30$, (4 ± 0.99) cps.).

Figure 11 shows the effect of variation of T_d (and also the effect of variation of motor time constant T).

15. VARIATION OF PHASE SHIFT AT CARRIER FREQUENCY.

The phase shift at carrier frequency ω_0 is

$$\phi = \text{arc tan}\,\frac{b\omega_0^2 - 1}{c\omega_0 - a\omega_0^3} - \text{arc tan}\,\frac{(b + T_2T_3 + T_2c)\omega_0^2 - 1}{(c + T_2 + S_2 + T_3)\omega_0 - a\omega_0^3}$$

For fairly large n, for small variations of the parts from their correct values, the major part of the variation of ϕ is the variation of the first term; the second term is practically constant and equal to arc tan U. Accordingly for simplicity we calculate only the variation of the first term. Let the first term be denoted by ψ. Then we have:

$$\begin{aligned} A^2 d \tan \psi = {} & \Delta_1[\omega_0(c - a\omega_0^2)] + \Delta_2[\omega_0^3(cT_1S_2 - a + aT_1T_3\omega_0^2)] \\ & + \Delta_3[\omega_0^3(cT_1T_3 - a + aT_1S_2\omega_0^2)] + \epsilon_1[\omega_0(bS_1\omega_0^2 - a\omega_0^2 + T_1)] \\ & + \epsilon_2[a\omega_0^3(b\omega_0^2 - 1)] + \epsilon_3[\omega_0(bT_1\omega_0^2 - a\omega_0^2 + S_1)], \end{aligned}$$

where $A = (c\omega_0 - a\omega_0^3)$. For the parallel "T" with equal condensers,

$$d \tan \psi = \frac{n}{2}(\Delta_1 + \Delta_2) + \left(\frac{n}{2} + \frac{1}{\sqrt{2}}\right)\Delta_3 + \left(\frac{5n}{8} + \frac{1}{2\sqrt{2}}\right)(\epsilon_1 + \epsilon_3) + \frac{n}{4}\epsilon_2,$$

and arc tan U = arc tan $2^{-\frac{1}{2}}$ = 35.26°. Since tan 45.26° = 1.0094 and tan 25.26° = 0.4720, the conditions for phase shift within ±10° are $d \tan \psi < +0.3023$ and $-0.2351 < d \tan \psi$. Thus in case $n = T_d\omega_0 = 15$, the appropriate tolerances are +0.67 per cent, −0.52 per cent for R_1, R_2; +0.61 per cent, −0.48 per cent for R_3; +0.52 per cent, −0.40 per cent for C_1, C_3; +1.34 per cent, −1.05 per cent, for C_2; or a uniform tolerance of +0.65 per cent, −0.51 per cent on all parts.

For the symmetric parallel "T," the corresponding conditions are $-0.0572 < 1.488\Delta_1 + 1.811(\Delta_2 + \Delta_3) + 2.055(\epsilon_1 + \epsilon_3) + \epsilon_2$ for $-10° < \phi$, and $0.66\Delta_1 + 1.397(\Delta_2 + \Delta_3) + 1.227(\epsilon_1 + \epsilon_3) + \epsilon_2 < 0.04$ for $\phi < +10°$, which requires a uniform tolerance of +0.58 per cent, −0.56 per cent on all parts. (For n different from 15, the tolerances should be multiplied by approximately $15/n$.)

For the resonant symmetric parallel "T" with perfect divider, the conditions are:

$$-45.7\Delta_1 - 54.16\Delta_2 - 53.3\Delta_3 - 59.3\epsilon_1 - 33.72\epsilon_2 - 60.2\epsilon_3 < 1.72 \quad \text{for} \quad -10° < \phi,$$

$$22.4\Delta_1 + 40.3\Delta_2 + 40.9\Delta_3 + 36.7\epsilon_1 + 30.8\epsilon_2 + 36.1\epsilon_3 < 1.2 \text{ for } \phi < 10°,$$

which requires the same uniform tolerance of +0.58 per cent, −0.56 per cent on all parts.

For the symmetric bridge "T," the corresponding conditions are:

$$-0.0235 < \Delta_1 + \epsilon_1 + \Delta_3 + \epsilon_3 < 0.0235 \quad \text{for} \quad -10° < \phi < 10°,$$

which requires a uniform tolerance of ±0.59 per cent on all parts. (For n different from 15, multiply tolerances by $15/n$.)

For the general bridge "T," if ϕ is the phase shift at carrier frequency,

$$\tan\phi = -(\Delta_1 + \epsilon_1 + \Delta_3 + \epsilon_3)\frac{nr}{2\left(\frac{4}{n^2} + r\right)},$$

where $r = (C_1 + C_3)/C_1$ (R_1, C_1, R_3, C_3 are as in Fig. 7(a), section 9; C_1 is the condenser on the input side).

For the bridge "T" with values as in Fig. 12, $T_d\omega_0 = 11$, and the conditions for phase shift at 60 cycles within ±10° are:

$$-0.32 < \Delta_1 + \epsilon_1 + \Delta_3 + \epsilon_3 < 0.32,$$

which requires ±0.8 per cent tolerance on all parts, or for example ±1.5 per cent tolerance on C's if the R's are held to within ±0.1 per cent.

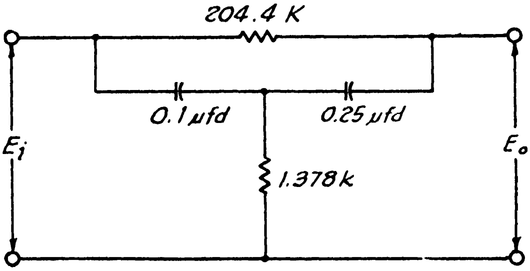


FIG. 12.

The input impedance at 60 cycles in k is $Z_i = 4.24 - 26.16j$. The fractional change of this impedance as a function of the variations of the components is:

$$\frac{\Delta Z_i}{Z_i} = (0.0211 + 0.0895j)\Delta_1 - (0.607 + 0.416j)\Delta_3 - (0.927 + 0.117j)\epsilon_1 - (0.659 + 0.209j)\epsilon_3.$$

This result will be used in a calculation of tolerance on the components of a phase lag network in Part III.

APPENDIX.

Analysis of the Parallel "T" Network.

In this appendix the method will be indicated by means of which the formulae of sections 3 through 6 were derived. In the last section, formulae are obtained to realize a proportional-derivative characteristic with phase shift of the carrier.

1. *Transfer Characteristic and Input and Output Impedances.*

It is convenient to transform each "T" (or "Y") of the parallel "T" to its equivalent "π" (or "Δ") of impedances. The formulae for transformation from Δ to Y, and from Y to Δ (see Fig. 13), are as follows:

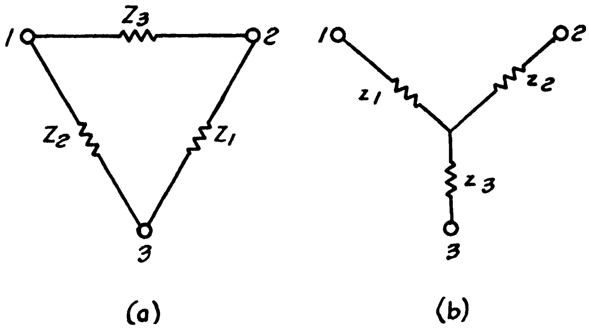


FIG. 13.

$$z_1 = \frac{Z_2Z_3}{Z_1 + Z_2 + Z_3}, \qquad Z_2 = \frac{z_1z_2 + z_1z_3 + z_2z_3}{z_2},$$

$$z_2 = \frac{Z_3Z_1}{Z_1 + Z_2 + Z_3}, \qquad Z_3 = \frac{z_1z_2 + z_1z_3 + z_2z_3}{z_3},$$

$$z_3 = \frac{Z_1Z_2}{Z_1 + Z_2 + Z_3}, \qquad Z_1 = \frac{z_1z_2 + z_1z_3 + z_2z_3}{z_1}.$$

These formulae are well known. If unfamiliar to the reader, they may be established easily by a simple application of Kirchoff's laws.

In the parallel "T" as in Fig. 1, let $z_1' = R_2$, $z_2' = R_3$, $z_3' = 1/C_2p$; $z_1'' = 1/C_1p$, $z_2'' = 1/C_3p$, $z_3'' = R_1$. Then after the transformation of each "T" to its equivalent "π," the parallel "T" network becomes the network shown in Fig. 14(a). For $k = 1, 2, 3$, let Z_k be the impedance

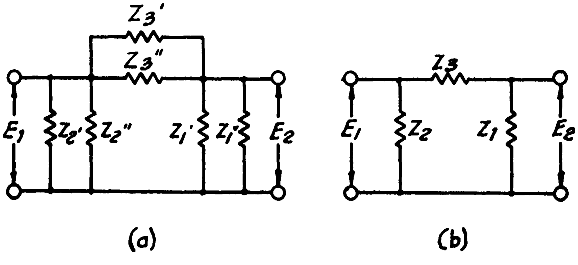


FIG. 14. Networks equivalent to the parallel "T" network.

of the parallel combination of Z_k' and Z_k''. Then it may be seen that

$$Z_2 = \frac{(T_2p + a)[(S_1 + T_1)p + 1]}{(T_2p + a)C_1p + [(S_1 + T_1)p + 1]C_2p},$$

$$Z_3 = \frac{R_3(T_2p + a)[(S_1 + T_1)p + 1]}{T_3T_1(T_2p + a)p^2 + [(S_1 + T_1)p + 1]},$$

$$Z_1 = \frac{(R_3C_2p + b)[(S_1 + T_1)p + 1]}{(R_3C_2p + b)C_3p + [(S_1 + T_1)p + 1]C_2p},$$

where the time constants are as previously defined, and $a = 1 + R_2/R_3$, $b = 1 + R_3/R_2$.

The impedance looking back into the output, with the input short-circuited (formula 5, section 3), is the parallel combination of Z_3 and Z_1:

$$\begin{aligned} Z_o &= \frac{R_3(T_2p + a)[(S_1 + T_1)p + 1]}{T_3p(T_1p + 1)(T_2p + a) + (T_2p + 1)[(S_1 + T_1)p + 1]} \\ &= \frac{R_3\{T_2(S_1 + T_1)p^2 + [a(S_1 + T_1) + T_2]p + a\}}{\begin{array}{r} T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2T_3 + T_2(T_1 + S_1)]p^2 \\ + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1 \end{array}}. \end{aligned}$$

The input impedance with the output open-circuited (formula 4, section 3) is the parallel combination of Z_2 and $(Z_3 + Z_1)$:

$$Z_i = \frac{\begin{array}{r} T_1T_2T_3p^3 + [T_2(T_1 + S_1 + T_3) + T_1(T_3 + S_2)]p^2 \\ + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1 \end{array}}{\begin{array}{r} (C_1T_2T_3 + C_2T_1T_3 + C_3T_1T_2)p^3 + [C_1(T_3 + T_2 + S_2) \\ + C_2(T_3 + T_1 + S_1)]p^2 + (C_1 + C_2 + C_3)p \end{array}}.$$

The input impedance with any load similarly may be calculated as the parallel combination of Z_2 and $(Z_3 + \bar{Z}_1)$ where $\bar{Z}_1$ is the parallel combination of Z_1 and the load.

The transfer characteristic (formula 1, section 3) is:

$$\begin{aligned} \frac{E_2}{E_1} &= \frac{Z_1}{Z_3 + Z_1} \\ &= \frac{T_1T_2T_3p^3 + T_1(S_2 + T_3)p^2 + (T_1 + S_1)p + 1}{\begin{array}{r} T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2T_3 + T_2(T_1 + S_1)]p^2 \\ + (T_1 + S_1 + T_2 + S_2 + T_3)p + 1 \end{array}}. \end{aligned} \tag{1}$$

2. *Determination of Constants to Realize the Proportional-Derivative Characteristic.*

The transfer characteristic may be written in factored form as follows:

$$\frac{E_2}{E_1} = \frac{(U_1p + 1)(A_1^2p^2 + B_1p + 1)}{(U_2p + 1)(A_2^2p^2 + B_2p + 1)}, \tag{2}$$

where $U_1A_1^2 = U_2A_2^2 = T_1T_2T_3$. To obtain a proportional-derivative network at frequency ω_0, we take $U_1 = U_2 = U$, $A_1 = A_2 = A = 1/\omega_0$. Then, if we let $B_1 = 2/T_d\omega_0^2$, $B_2 = 2/l\omega_0^2$, divide numerator and denominator of (2) by B_1p/l, and replace p by $j\omega$, we obtain the characteristic:

$$\frac{E_2}{E_1} = \frac{l}{T_d}\,\frac{1 + jT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1 + jl\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)} = g\,\frac{1 + jT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1 + jgT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}. \tag{3}$$

To solve for the constants in the parallel "T" network which will give a transfer characteristic of the form (3), we compare (2) with (1). We thus obtain:

$$T_1T_2T_3 = A^2U, \tag{a}$$

$$T_1(S_2 + T_3) = A^2 + B_1U, \tag{b}$$

$$T_1 + S_1 = B_1 + U, \tag{c}$$

$$T_2(T_3 + T_1 + S_1) = (B_2 - B_1)U, \tag{d}$$

$$T_2 + S_2 + T_3 = B_2 - B_1. \tag{e}$$

Substituting from (a), (b), (c) into (d) and (e), we obtain $T_1T_2 = U^2$. Then from (a), $T_3 = A^2/U$. Substituting the value of T_3 in (c) and (b), we are able to solve for T_2 and S_2. Then $T_1 = U^2/T_2$, and we solve (c) for S_1. Collecting results, we obtain the group of formulae (3) of section 3.

3. Proportional-Derivative Characteristic with Phase Shift of Carrier.

If $T_d\omega_0 = n$ is quite small (in the vicinity of 2 or 3), the upper bound arc tan $2/n$ will permit a useful amount of intrinsic phase shift. To obtain the proportional-derivative effect on the modulation, with phase shift of the carrier, the parallel "T" transfer characteristic should have the form:

$$\frac{E_2}{E_1} = \frac{Up + 1}{\alpha^{-2}Up + 1}\,\frac{(A^2p^2 + B_1p + 1)}{(A^2\alpha^2p^2 + B_2p + 1)}$$

$$= \left(\frac{l}{T_d}\right)\frac{Uj\omega + 1}{\alpha^{-2}Uj\omega + 1}\,\frac{1 + jT_d\dfrac{\omega+\omega_0}{2\omega}(\omega-\omega_0)}{1 + jl\dfrac{\alpha\omega+\omega_0}{2\omega}(\alpha\omega-\omega_0)}$$

where $\alpha \neq 1$, where as before $A = 1/\omega_0$, $B_1 = 2/T_d\omega_0^2$, $B_2 = 2/l\omega_0^2$.

Comparing the above transfer characteristic with the parallel "T" characteristic (1), we have:

$$
\begin{aligned}
T_1T_2T_3 &= A^2U,\\
T_1(S_2 + T_3) &= A^2 + B_1U,\\
T_1 + S_1 &= B_1 + U,\\
T_2(T_3 + T_1 + S_1) &= (\alpha^{-2}B_2 - B_1)U + A^2(\alpha^2 - 1),\\
T_2 + S_2 + T_3 &= (\alpha^{-2} - 1)U + (B_2 - B_1).
\end{aligned}
$$

Solving for T_1, T_2, T_3, S_1, S_2, we obtain the following formulae for determination of the constants in the parallel "T" network to give the phase-shifting proportional-derivative characteristic:

$$
T_1 = \frac{B_1(A^2 + B_1U + U^2)}{[(B_2 - B_1)B_1 - A^2(\alpha^2 - 1)] + (B_2 - B_1)(1 - \alpha^{-2})U + (\alpha^{-2} - 1)U^2},
$$

$$
T_2 = \frac{A^4(\alpha^2 - 1) + A^2[B_2(\alpha^{-2} - 1) + B_1(\alpha^2 - 1)]U + [B_1(\alpha^{-2}B_2 - B_1) - A^2(\alpha^{-2} - 1)]U^2}{B_1(A^2 + B_1U + U^2)},
$$

$$
T_3 = \frac{A^2U\{[(B_2 - B_1)B_1 - A^2(\alpha^2 - 1)] + (B_2 - B_1)(1 - \alpha^{-2})U + (\alpha^{-2} - 1)U^2\}}{A^4(\alpha^2 - 1) + A^2[B_2(\alpha^{-2} - 1) + B_1(\alpha^2 - 1)]U + [B_1(\alpha^{-2}B_2 - B_1) - A^2(\alpha^{-2} - 1)]U^2},
$$

$$
S_1 = B_1 + U - T_1,
$$

$$
S_2 = (\alpha^{-2} - 1)U + (B_2 - B_1) - T_2 - T_3.
$$

When the input and output of the parallel "T" are reversed, that is when $R_2 \rightleftarrows R_3$ and $C_1 \rightleftarrows C_3$, it may be seen that the coefficients in the numerator of the transfer characteristic are unchanged. Thus when a phase-shifting proportional-derivative parallel "T" is reversed, it is still a phase-shifting proportional-derivative parallel "T" with the same T_d, but in general with a different gain and phase-shift.

Blind Guidance by Ultrasonics (Frank H. Slaymaker and Willard F. Meeker, *Electronics*, May, 1948).—An effort to develop artificial devices for the military blind has led to the invention of a portable radar-like device employing pulsed frequency modulation. This device gives a single audio frequency tone corresponding to any given obstacle distance, and simultaneous echoes from different distances are identified by combinations of individual tones. The complete experimental equipment weighs 5½ lb. and operates at 65 kc. detecting obstacles up to 30 ft. Tests are being planned to determine the best method of using such devices—whether by headphones, tone conduction receiver, or controlled electrical shock.

Increased Uses of Turbojet by Augmentation.—The useful range of the turbojet can be expanded by augmenting the thrust in a number of different ways so that it fulfills more military requirements.

These plans for augmentation serve as a means of gaining more thrust for little additional weight and complication, without resorting to extra large engines or additional engines for most airplane applications.

Methods of augmentation include the reheat cycle or "tailpipe burning," engine overspeed, "bleed-burn" cycle, water injection in the combustion chambers, and water or ammonia or carbon dioxide injection in the compressor inlet.

The most practical augmentation process is the reheat cycle. In this method, additional fuel is burned in the hot air after it has expanded through the turbine, yielding a much higher temperature than could be withstood by the stressed turbine blades. Final expansion then takes place at the discharge end of the exhaust pipes, where the gas is ejected at a much higher velocity than without reheat, increasing the thrust accordingly.

Results show that at 600 mph., and with "tailpipe burning," the net thrust is almost equal to the output of two turbojet engines operating in the normal manner. However, full advantage of the reheat cycle cannot be realized unless adequate space is provided for the tailpipe burner, since a burner of adequate size is essential to obtain stable and efficient combustion over a wide range of fuel flows at all altitudes.

Augmentation is necessary because jet-propelled airplanes require longer take-off runs than a propeller-driven plane. Operation out of small emergency landing fields or airports located at high altitudes or in equatorial regions, and the frequent needs for a plane carrying a heavy pay load require an augmented thrust for a successful take-off.

In addition, poor climbing rates and lack of a high degree of maneuverability result when thrust falls off in the light air found in higher altitudes. The effect would be greater still, were it not for the increase in jet velocity. This shortcoming is overcome by augmenting the thrust at this altitude through the use of the reheat cycle.

Additional thrust is obtained at the expense of high specific liquid consumptions, but with only a slight increase in the weight of the whole propulsion unit. Until more progress is made in the field of high temperature metallurgy and more effective means are found to protect parts that are exposed to high temperatures, thrust augmentation will be used.

R. H. O.

THE MECHANISM OF LIGHTNING DISCHARGE.

BY

LEONARD B. LOEB,[1] Ph.D.

The first great step in liberating mankind from the thralls of superstitious fear at the awesome and dazzling phenomenon of the lightning stroke was made by Benjamin Franklin in his famous kite experiment when he drew some of the lightning from the clouds and stored it in a Leyden jar just as one did static electricity. Much further advance into an understanding of the very intricate mechanisms at work required two important developments: first, an understanding of the mechanism of the ordinary sparks at atmospheric pressure proceeding from impulsively applied potentials; and second, an analysis of lightning by means of visual and electrical recording devices acting over time scales measured in microseconds. The first of the missing data was supplied beginning effectively in 1939–40, when Loeb and Meek in America (**1**)[2] and quite independently, Raether in Jena, Germany (**2**) on different approaches, developed the streamer mechanism for spark breakdown. The second of the missing data was supplied effectively beginning in 1934 when Schonland, Collens and their associates applied the technique of the Boys moving camera lens of high speed to a study of South African thunderstorms (**3**). Both advances were the culmination of years of effort by many workers leading up to the ultimate clarification. Among these, one must number the pioneering studies of Townsend on spark breakdown (**4**), the important time lag studies beginning in 1923 and the following years, the studies of photoelectric ionization of gases beginning with those of A. M. Cravath (**6**), and corona studies at the University of California (**7**). Likewise one must consider the pioneering studies of Walter (**8**) with moving camera, of C. V. Boys (**8**) and others who preceded Schonland, and the many engineering and other investigations of lightning strokes using electrical and magnetic pickups (**9**,**42**) whose value improved with the improvement of detecting, recording and oscillographic techniques.

Assume for simplicity the uniform field between parallel condenser plates in air at atmospheric pressure. The gap length is δ, the field $X = \frac{V}{\delta}$ with V the applied potential. The upper plate is the anode. In sparking phenomena the characteristic variable for the ionization phenomena is the ratio X/p of field strength X in volts per centimeter to pressure p in millimeters of Hg. For a gap with $\delta = 1$ cm., the sparking potential $V_s = 31.6$ kv. and $X_s/p = 41.6$ at atmospheric pressure. Assume one initiating electron to start from the cathode. As it crosses the gap of length δ, it creates an avalanche of $e^{\alpha\delta}$ new electrons and positive ions by cumulative ionization by collision in the gas (4). Here α is the first Townsend (4) coefficient representing the average number of ionizing acts of an electron per centimeter path along X. With $\delta = 1$ under the conditions cited for air, $\alpha\delta = 17$, and $e^{\alpha\delta} = 2.4 \times 10^7$ ions. Fifty per cent of these are produced within 0.041 cm. of the anode. A cloud track picture of such avalanches by Raether (2) is shown as Fig. 1 for three different impulsive time intervals below sparking thus yielding velocities of avalanche advance. The radius ρ of the avalanche can be estimated by the electron diffusion in the avalanche in the 5×10^{-8} sec. of crossing the gap (10). It is of the order of 0.013 cm. The electrons in this avalanche with velocities about 100 times that of the positive ions enter the anode and leave behind a positive ion space charge whose density creates a field X^1 at a distance ρ from the center of about 0.013 cm. of 6000 v. per centimeter. If the potential had been 5 per cent higher the value of X^1 would have been 140,000 v. per centimeter, for α is very field sensitive. In the avalanche head accompanying the ionization there were produced some $fe^{\alpha\delta}$ high energy photons (light quanta), capable of ionizing the gas molecules present (11). The value of f is not as yet known, though estimates are possible (40,41). These photons are mostly emitted over an interval of 10^{-7} sec. In the vector sensitive field $X^1 + X$ extending out a distance Δx^1 beyond ρ as shown in Fig. 2 the value of α is very much increased (12,13). If now with the high absorption coefficient μ for such photons in air (of the order of one half of these are absorbed in 10^{-1} cm. (14,40)) the photoelectrons produced in the sensitive volume can ionize in the distance

---

[1] Professor of Physics, University of California, Berkeley, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

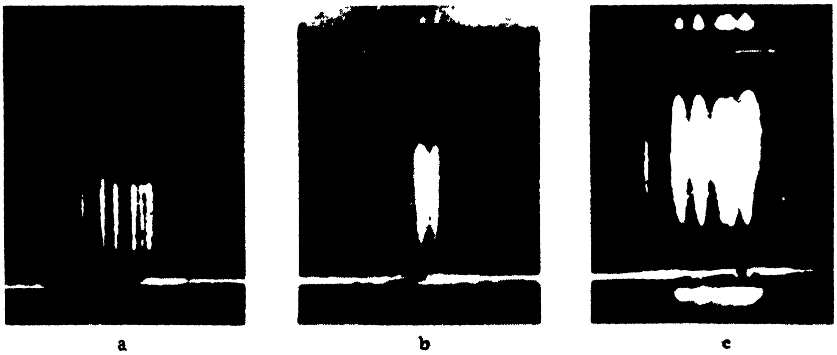


FIG. 1. Showing Raether's cloud track photographs of electron avalanches using pulsed fields of different duration below breakdown. From the time interval which increases progressively from a through c, the velocity of advance of avalanche tips can be determined.

Δx^1 to produce the same density of ions existing in the original space charge, the space charge will be extended into the gap by Δx^1 (11). The process will then repeat itself and the positive space charge will advance back across the gap from anode to cathode. As it does so, the electrons created by the advance stream up its length to the anode giving a conducting plasma which acts to distort the field in the gap and still more enhance $X + X^1$. The velocity of advance from anode to cathode of this process is high ($\sim 10^8$ cm. per second), and becomes higher as it approaches the cathode because of increased field distortion (2,7,12,15,16).

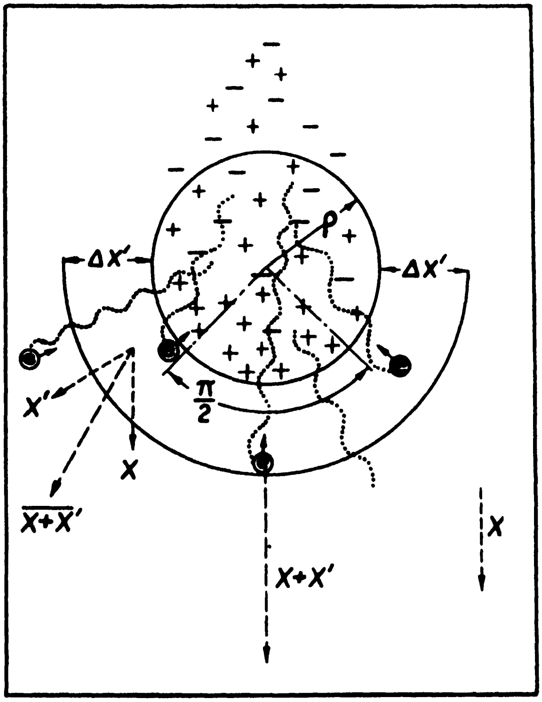


FIG. 2. Schematic diagram showing the positive space charge left behind by an electron avalanche. Its radius is ρ and the sensitive zone ΔX^1. Fields are represented by dashed arrows. Dotted curves indicate photons liberating electrons in different regions about the zone. The zone is of width ΔX^1.

The mechanism is called a *streamer*. It was discovered by Raether in cloud track pictures (2) and was first observed by Loeb and by Kip in positive corona (7). Its properties are now well established. The mechanism of streamer formation is illustrated in the schematic diagrams of Figs. 3 and 4, and cloud track pictures of such streamers by Raether (2) are shown in Figs. 5 and 6. A great deal is known about such streamers today (7,17): they have an ion density of the order of 10^{14} ions per cubic centimeter and 10^{10} ions are produced per centimeter of length; their velocity ranges from 2×10^7 to 2×10^8 cm. per second

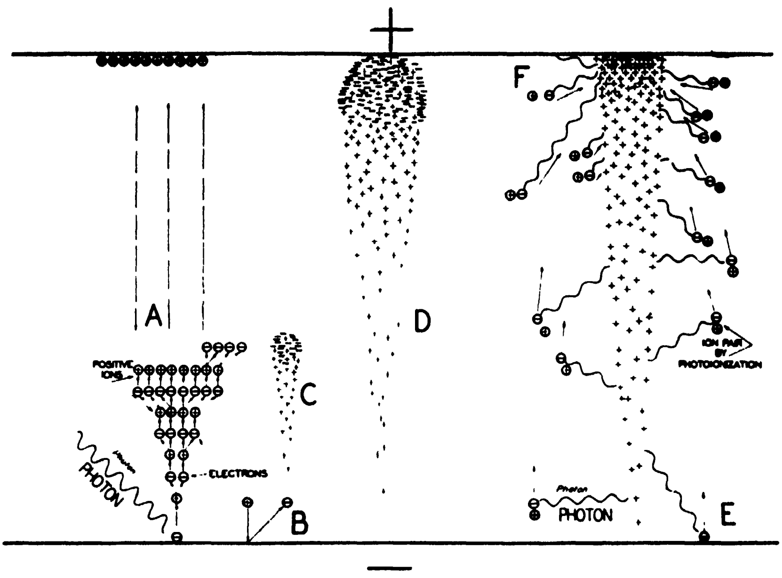


FIG. 3. Schematic diagram showing the creation of an avalanche between two parallel plate electrodes beginning with a photoelectron liberating an electron for the negative plate A, C, D, and F, representing the development of the avalanche. At F the avalanche has reached the positive plate and gives up its cloud of electrons leaving a positive space charge.

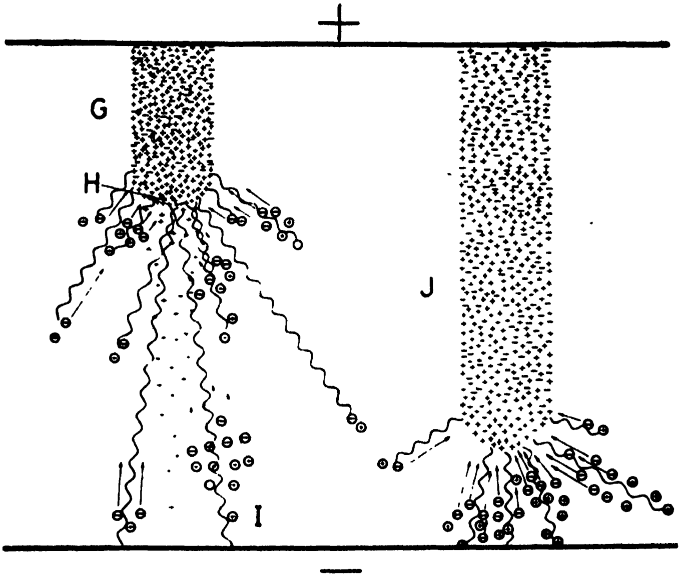


FIG. 4. A schematic diagram, a continuation of Fig. 3, showing the mechanism of positive streamer advance. At G the streamer has advanced part-way across the gap and at J it has nearly bridged the gap. Field distortion at this point is high. Note also that the diameter of the channel is very much exaggerated for clarity in reproduction.

depending on the imposed field and the field distortion produced; they represent a displacement current of the order of 0.1 to 1 amp. in the gap; and their initial rate of development $\frac{di}{dt} \sim 10^8$ amp. per second is capable of shock exciting electrical systems and produces much elec-

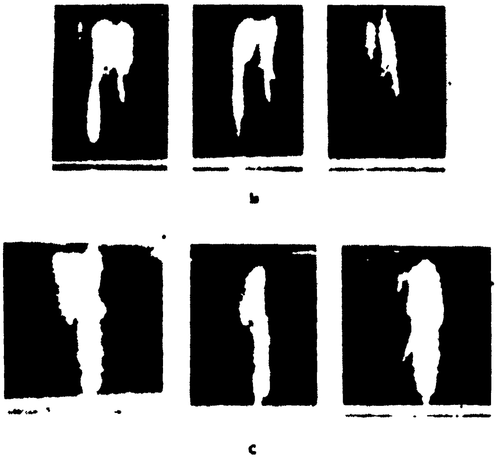


FIG. 5. Raether's photographs of streamers developing from avalanches. The negative electrode is at the bottom and the positive electrode is at the top. The avalanche can be seen in each case proceeding upward with an enlargement at its upper end representing the streamer advance before the streamer could cross the electrodes and give the spark. The time scale here is of the order of 10^{-7} sec., representing the time of crossing of the avalanche and the return of the streamer at higher velocity. By proper choice of timing Raether could calculate the approximate speed of the motion of the streamer toward the anode. Notice branching and on two of the photographs, pictures of avalanches that did not lead to streamers because they started their careers later during the pulse.[4]

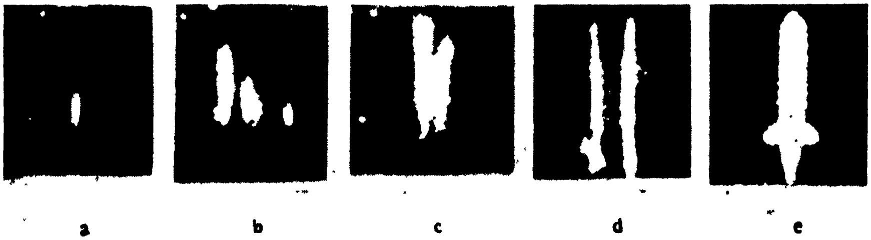


FIG. 6. Further pictures of Raether's showing avalanches producing streamers. These pictures are taken with different times of application after the sparking voltage had been applied. Beginning at the left, one sees avalanches crossing at three consecutively longer time intervals. The last two pictures indicate returning streamers. The one at the extreme right indicates a streamer with two equal branches. Note that the width of these photographs is highly exaggerated because of differences between the time of application of the voltage and the condensation of the cloud.

trical noise. Representing a field distortion the streamers can propagate themselves as long as the total field strength they are moving in is high enough, and the gaseous density is such as to insure photoionization in the gas. It has been estimated from corona studies that the minimum field under standard conditions in air is of the order of 4400 v. per centimeter (18).

When the streamer head approaches the cathode the intense local photoelectric ionization of the metal and the high field existing cause the release of such a flood of electrons from the cathode that a steep potential gradient suddenly develops and sweeps up the pre-ionized channel with a very high speed. This speed has not been measured on sparks (2) but is greater than 10^9 cm. per second for long sparks according to Meek and Allibone (19). In lightning it ranges from 10^9 to 10^{10} cm. per second (3). Similar speeds have been observed for such ionizing potential waves by Snoddy, Dieterich, and Beams at lower pressures (20). The velocity of such ionizing potential waves in plasma according to the latter authors, depends on the electron or ion density in the channel plasma and on the steepness of the potential gradient of the front of the wave. The intense ionization produced by the poten-

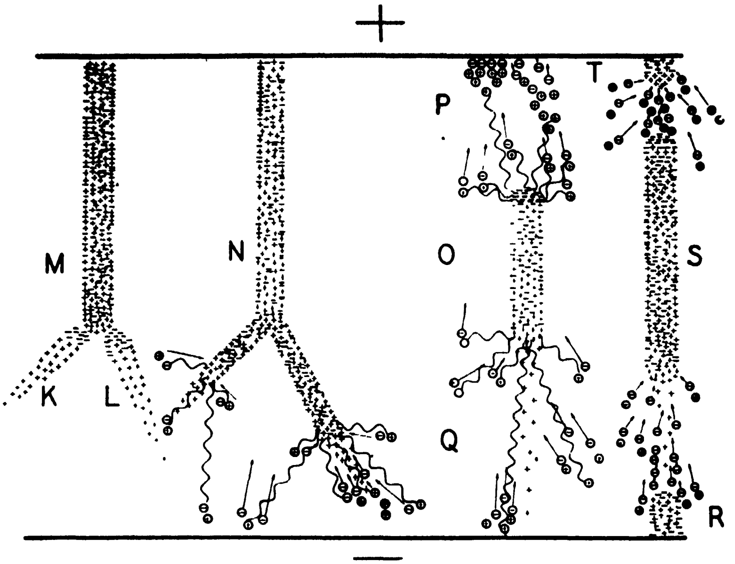


FIG. 7. Schematic diagram indicating the process of branching in streamer advance M and N, and the development of midgap streamers in over-volted gaps. Note that at O the avalanche proceeded part-way across the gap before starting a streamer toward the negative electrode. S is a later stage in which the streamer has advanced well across the gap causing serious distortion and producing an intense avalanche R at the negative electrode and an anode streamer T at the positive electrode. Such breakdown had been observed by Dunnington and White well before the streamer theory had been developed.

tial wave ionizing from possibly 1 to 30 per cent of the molecules in the channel leads to the brilliant return stroke. This intense adiabatic energy input is responsible for the light, heat, and noise of the spark. Following the return stroke, channel temperatures of the order of 10,000 to 18,000° are reached (21). The heated channel expands at the rate of about 10^5 cm. per second until the expansion cools the channel (22).

The quantitative condition for streamer advance has recently been indicated as roughly of the form $\epsilon f e^{\alpha\delta} = 1$ (11). Here ϵ is the chance that a photoelectron is created in the volume between ρ and Δx^1 (Fig. 2) such that it can create an avalanche sufficient to extend the streamer tip. ϵ is complex and depends on the imposed field X, the space charge tip field X^1 and thus on the size of Δx^1 as well as on μ the absorption coefficient for photoionization in the gas. Through ρ and μ the quantity ϵ is critically pressure dependent (23). For long gaps or divergent fields, or for slight over-voltages, $\frac{V - V_s}{V_s}$, the length of an avalanche x to produce a streamer can be less than the gap length δ. This leads to what are known as midgap and anode breakdown streamers first observed by Dunnington and later by White and by Allibone and Meek (24). The geometrical position of the single photoelectron, or the creation of two simultaneously appearing well located photoelectrons, in Δx^1 can lead either to crooked paths (usually in weak fields X) or to forked paths branching in the direction of progress, occurring chiefly in strong and in strongly divergent fields (25). The schematic diagrams illustrating the origin of midgap streamers and branches paths is shown in Fig. 7 as appearing in overvolted uniform fields. Figure 8 shows a crooked spark photographed by Terada and Nakaya (26). In Fig. 9 is shown a branched lightning stroke advancing from cloud to

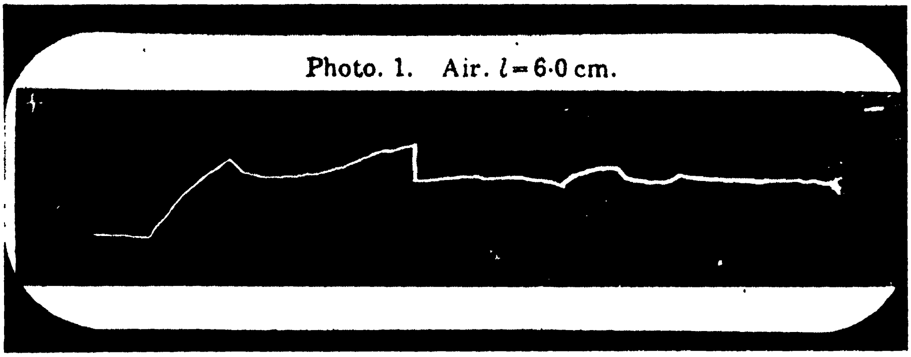


FIG. 8. A beautiful photograph of a spark by Terada and Nakaya, showing the crooked nature of a spark for a long spark with a weak guiding field.

ground, and Fig. 10 shows a branched streamer taken within 1.25 × 10^{-8} sec. in a highly divergent field produced by an overvolted point as photographed by Gorrill with a cloud chamber (15). Figure 11 is a photograph of a midgap breakdown observed by White in CO_2 in intervals of the order of 10^{-9} sec. with Kerr cell shutter, cathode at left (24).

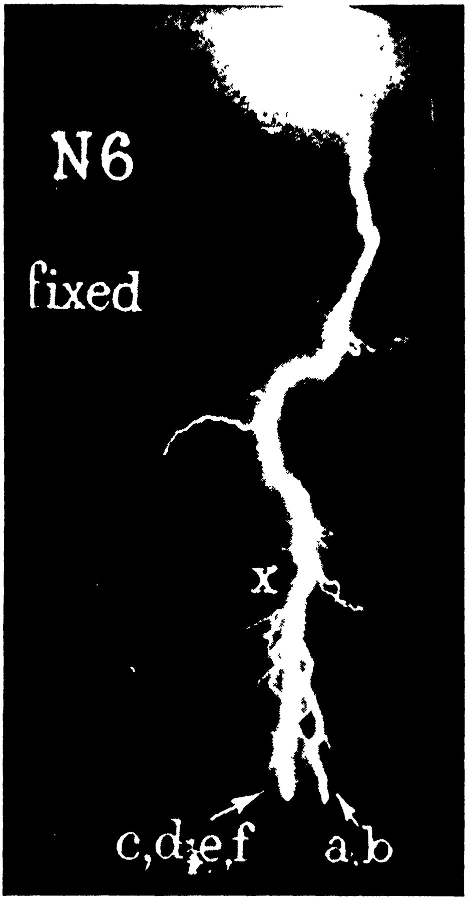


FIG. 9. A detail of one of Schonland's fixed camera photographs of a lightning stroke showing a branching in the direction of progress and some decrease in intensity through narrowing of the channel and less halation, as the return stroke moves upward. All luminosity here is caused by the return stroke.

When one turns from the discussion of the simple uniform field spark to long sparks in air and to lightning, conditions are somewhat altered. In such events the fields under consideration are often localized and very high with steep local gradients near points or clouds. Again most of the fields are impulsive;[3] that is they are suddenly created by surge generators in sparks or by the turbulent and violently convective gas mass motions about charged drops in thunder showers (27,28,29,42). All data on lightning discharge indicate the sudden and probably localized appearance of high field gradients very shortly before discharge. There are the rare cases where apparently high fields of longer duration appear causing St. Elmo's fire or corona from tall pointed conductors on the ground. Actually where such manifestations occur the cloud-to-ground field on the average is not high, ~2700 v. per centimeter, but

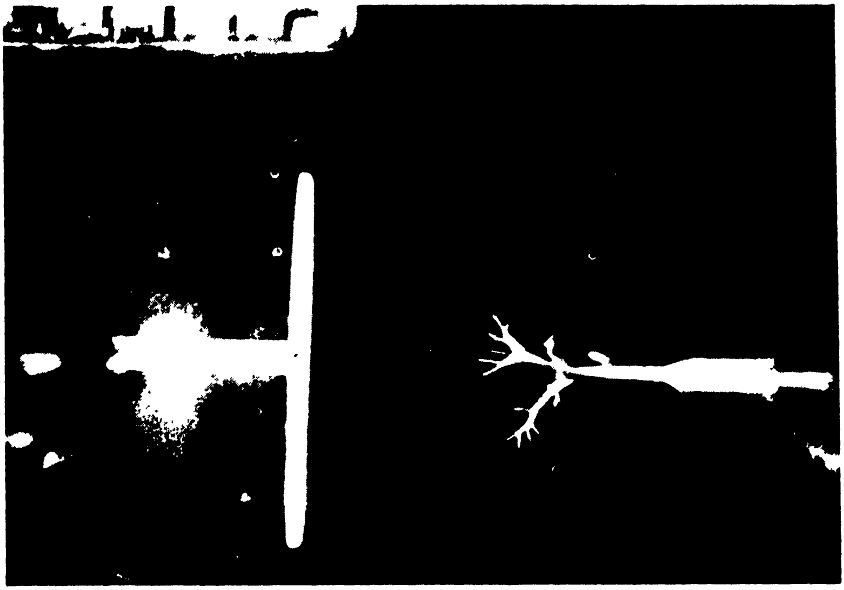


FIG. 10. Cloud photograph of a positive corona streamer from a positive point to negative plane taken by Gorrill with impulse voltage applied for 1.25×10^{-8} sec. Note that the streamer did not travel very far. Longer duration of the impulse field causes the branches to progress further across to the plate. Note the heavy branching in the direction of propagation.

long conductors projecting into the field distort the field and focus on themselves the high local gradient leading to St. Elmo's fire or even to invited lightning strokes. These are *not* the rule in the common lightning strokes observed and in which we are interested. Sounding balloon and airplane measurements indicate that there are in storms usually gradients less than 300 v. per centimeter or else sudden fields leading to discharge (27,28). There is some divergence of opinion as to fields measured near the ground or on airplanes in flight just before strokes, but they fix these on the average as lying between 2700 and 4000 v. per centimeter near the point struck. Probe measurements before breakdown of a corona gap by positive point streamers indicate minimum fields of about 4400 v. per centimeter to be required. It is on the basis of mean gradients of such values that the potentials leading to lightning discharge are estimated to be of the order of 10^8 to 10^9 v. depending on length of stroke. It is, therefore, to be anticipated that in all such sparks, despite field distortion by streamer formation and highly localized fields of high order at electrodes or in portions of the cloud that initiate discharge, there must be minimum field gradients of the order of 2700 v. per centimeter over the whole gap to be traversed before sparks occur. One must, therefore, consider briefly what occurs when sparks start from suddenly positively or negatively charged points or localized high field regions with adequate gradients of lower order between. Studies of long sparks by Allibone and Meek (27) and corona studies at atmospheric pressure indicate that for a ***positive metal point*** fields must exist very near the metal point surface giving X of about 70,000 v. per centimeter in air to insure deionization of existing negative

---

[3] It is not profitable to discuss breakdown under constant or alternating high fields for long sparks or lightning. In long sparks the mechanism is complicated by building up of corona and space charge accumulation. Lightning rarely gives high fields of long duration. Most strokes appear to be of the impulsive type.

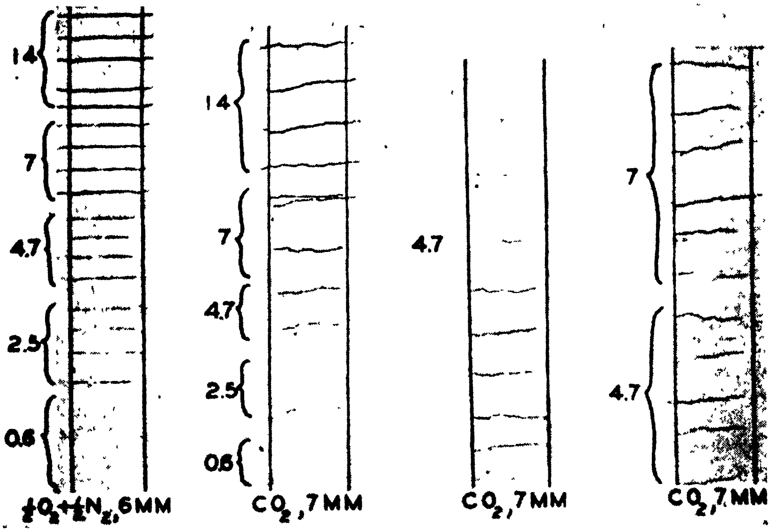


FIG. 11. Actual photographs of midgap streamer breakdown using Kerr cell shutter taken by White in CO_2 and an oxygen and nitrogen mixture at relatively lower pressures. The time scale is indicated by the figures on the left. Notice particularly the shortest time scale photographs in CO_2 at atmospheric pressure where the midgap streamer is just beginning to show. Longer intervals show the sparks crossing the gap. The negative electrode is to the left, the positive is to the right. The extension of the spark behind the electrodes is a reflection on the electrode surfaces. The exact time scales with Kerr cell shutter cannot be fixed. While drawings of visual observations show midgap streamers more clearly at the shorter time intervals, it is believed that actual photographs of these are more suitable for demonstration.

O_2 ions to start the discharge (7). As we saw, uniform fields of 1 cm. length require 30,000 v. per centimeter. The high fields initiating lightning could well be less. In more extended regions of uniform fields

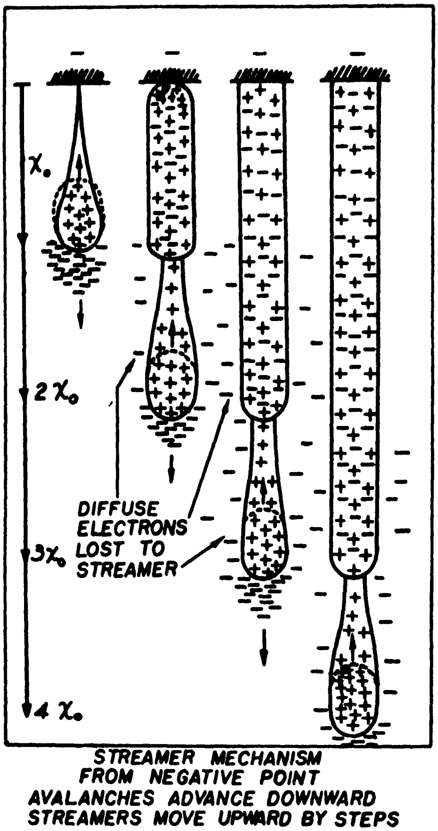

X_0
$2X_0$
$3X_0$
$4X_0$
DIFFUSE ELECTRONS LOST TO STREAMER
STREAMER MECHANISM
FROM NEGATIVE POINT
AVALANCHES ADVANCE DOWNWARD
STREAMERS MOVE UPWARD BY STEPS

FIG. 12. Schematic diagram illustrating a streamer mechanism for a negative point. The electron avalanche at the extreme right has developed to a distance x_0, where it starts to send a streamer back to the cathode. As this streamer advances, the electron cloud is able to advance $2x_0$ and start a new streamer by which time the first streamer has reached the cathode. Successive steps are indicated by the diagrams to the right extending the streamer mechanism a distance of $4x_0$ into the gap. It is probable that the successive steps x_0 after the first should be shorter because of the number of electrons participating, and it is indicated that both the streamer advance upward and the electron avalanche advance downward could produce crookedness and branching not indicated in the figure. The tendency will be for them to move straight forward owing to the number of electrons available for avalanche advance. Note also that with the electrons moving ahead, a certain number of them are lost to the channel by diffusion. These require a small current down the streamer channel in order to maintain the avalanche advance. It is this current which plays an important role in the increase of field gradients causing stepping.

of the order of 10,000 v. per centimeter in air, water drops will be drawn out in fine points to yield positive streamers, thus initiating spark discharge as Macky and earlier studies by Zeleny have shown (30). In any case, when adequate localized sudden fields of appropriate size occur, a discharge streamer mechanism starts. In detail this process will differ both as regards initiating field and mechanism depending on whether the local charge is positive or negative.

When such local fields about a positive point or cloud region occur, the streamer begins from an avalanche directed towards the cloud or positive point, and the streamer forges ahead outward across the gap pouring electrons from its advancing tip into the positive cloud or electrode. The problem is the same as for the uniform field except that streamer advance may not be as fast and the path in a weak field will be more tortuous. The width of the streamer channel will increase by diffusion in proportion to its time of travel. In long sparks and lightning with 1 to 10 m. of travel, the streamer channel width could be 0.5 to 1 cm. It would be visible to the naked eye close by in the dark were it not masked by brighter phenomena following. It could not be photographed at any distance (7). Breakdown would then follow when it crossed the gap to the cathode or ground although it could be met by a negative streamer process as it neared the cathode with the high field distortion produced. This has been observed in occasional long sparks (19,31).

In the case of the negative point the picture is more complicated (32). When an adequate field exists, which must now be higher than for the positive point, an electron liberated near the cathode or from it proceeds to form an avalanche. This avalanche will proceed out into the gap a distance x_0, until it has created a space charge density sufficient to produce a streamer in midgap. This streamer will now move *backward* towards the cathode as seen in the sequence shown in the schematic diagram of Fig. 12. As the streamer advances the electrons feeding into it allow the electrons ahead of the positive space charge again to move forward another x_0, or perhaps a shorter distance, at the end of which a new streamer proceeds back to join the first streamer which by that time has reached the cathode. There is thus a streamer channel $2x_0$ long from cathode out into the gap with the electrons moving further beyond $2x_0$ to make a streamer $3x_0$ long and so on. Depending on the nature of the imposed field, the successive steps x_0 can either be shorter or longer depending on how the field distortion is proceeding. When this process approaches the anode sufficiently closely seriously to distort the field, it may be met by a *positive anode streamer* at which point a potential wave will sweep up the channel from anode to cathode reionizing the plasma and giving a spark. The composite of streamer advance towards the negative point and electron avalanche towards the positive ground can have crookedness in a weak guiding field come both through the streamer advance and the vagaries of the advancing avalanches. However, with the number of these avalanches the forward advance should tend towards straighter paths than with streamers alone. What evidence we have from long sparks makes this appear to be true as seen in the photograph of the long spark by Allibone and Meek (19,31), where the negative charge is at the top

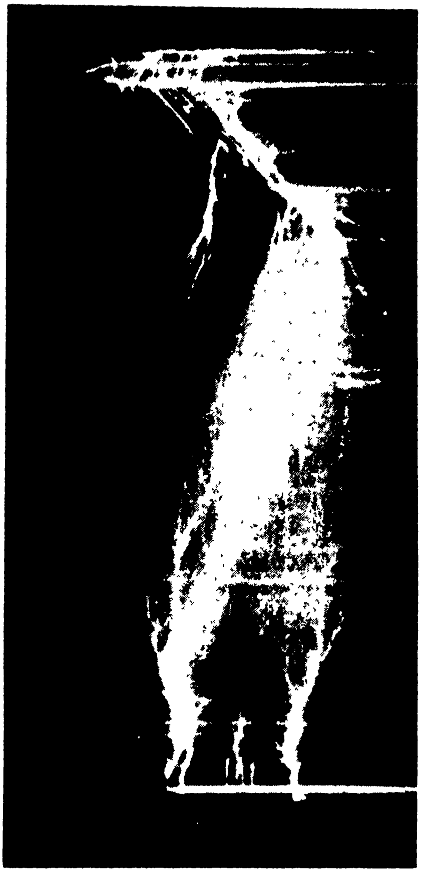


FIG. 13. A very illuminating photograph of a long spark in air from a negative point (top) to a grounded plane (bottom) with high resistance in series with the point. The moving film camera is moving from left to right as can be seen by the continuation of streaks. Note that the negative point streamer mechanism progresses along relatively straight lines, and that it continues about one fourth of the way from the negative points across the gap. Note the step-like procedures in its development which Allibone and Meek however, do not call proper stepping. In sparks of a meter length stepping should not occur naturally and the effect shown is produced by the use of a high resistance which lowers the potential of the point after a partial discharge. Note also that the negative point streamer is met by an upward branching positive streamer from the positive plate beginning at the extreme left which advances three fourths of the way across the gap meeting the negative streamer at that point. The return stroke can be seen for the positive point immediately to the right of the upward moving positive streamer and at the right of the downward moving negative streamer which completes the junction. Notice the intense illumination over the section of the channel where the two streamers meet resulting in much halation. Notice a subsequent positive streamer occurring at a later time to the right at the bottom of the picture.

of the Fig. 13. It is clear that for such cathode streamer processes the fields must be higher than for positive streamer processes. Experiments with long sparks bear this out in that the *average* potential gradient from positive point to negative plane is 4400 v. per centimeter while with negative point to plane it is of the order of 9000 v. per centimeter (32). This circumstance is very important for long sparks and especially for lightning since over 90 per cent of the observed cloud-to-ground strokes in all but mountain thunderstorms (42) are from negative clouds. The fact that a positive point starts a breakdown streamer at lower fields can influence the statistics and interpretation of lightning discharges if care is not taken to remember this fact.

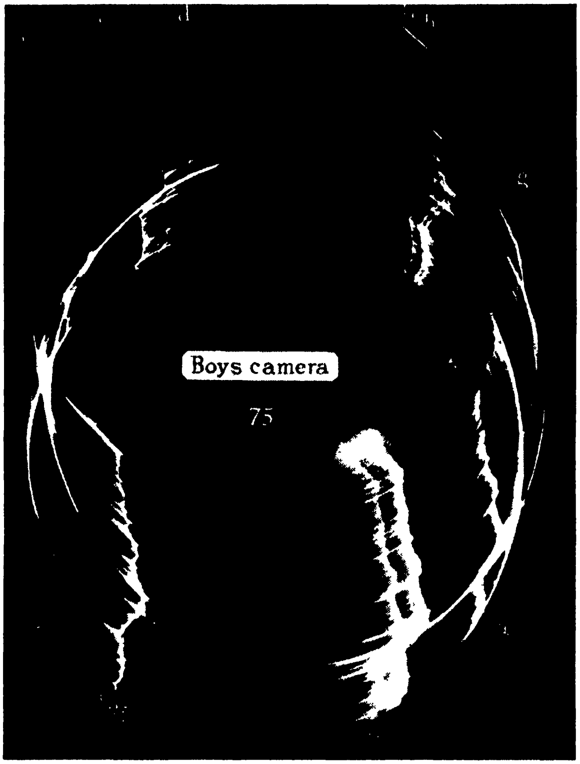


FIG. 14. A revolving lens Boys camera photograph taken by Schonland and Collens indicating the intricate nature of the pictures. It is to be noted that two lenses were used. The sequence of events is indicated by a, b, c, d, e, and f. a, b, c, and d, are given by successive strokes down the same channel. The motion of the lens in the lower half of the picture is such that the time progresses from right to left as indicated by the bright streaks. Note the length of the bright streaks coming from points of particularly intense ionization.

The first step in a real understanding of the lightning discharge mechanism came when following C. V. Boys, B. F. J. Schonland and H. Collens and their associates adapted a camera with a rapidly revolving lens to the study (3). This technique made it possible to study the lightning stroke on a time scale of microseconds. The revolving lens photographs are not easy to interpret as can be seen from the example shown in Fig. 14. However, the results were startling. Later Allibone and Meek (19,31) studied long impulsive sparks with a camera with rapidly moving film with, however, less temporal resolving power. Considering that most discharges observed are from negative cloud to ground, the sequence of events delineated by Schonland are as follows (3):

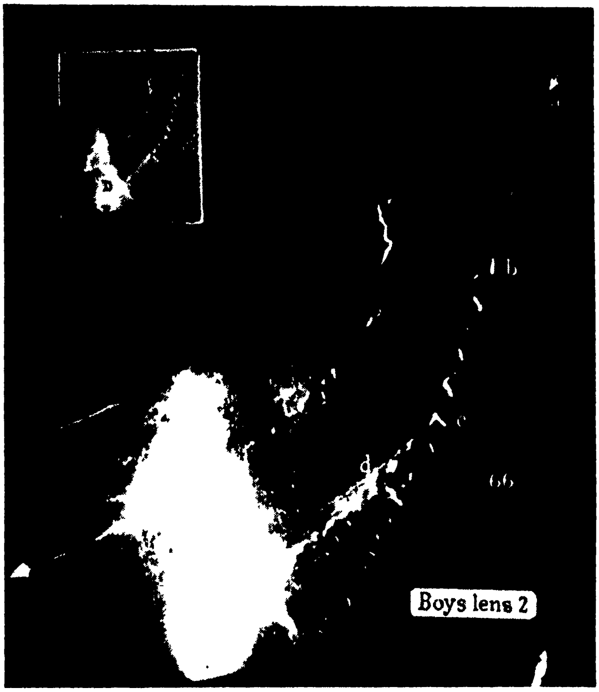


FIG. 15. This is a particularly beautiful photograph of the stepped leader stroke by Schonland and Collens, the large photograph being a detailed picture of the smaller one shown in the insert. The time extends from right to left. Note the separate steps of the leader and also the stepping in the progress of the main branches somewhat above.

1. An invisible process called a *pilot streamer* suddenly starts from a cloud towards the ground into virgin air at a velocity which is generally around 2 × 10^7 cm. per second but in unusual cases has been observed up to 10^8 cm. per second. Its presence must be inferred since while it itself cannot be seen, its tip is periodically illuminated by later occurrences, and it must be there to carve the path later made visible. This process forging into virgin air yields the tortuous path and the branching characteristic of lightning.

2. After an average of some 50 microseconds (the time varies from

30 to 90 microseconds and is remarkably regular in any stroke), its tip is illuminated by a *stepped leader stroke.* This proceeds from the cloud to the ground at a speed which varies somewhat in different strokes but is of the order of 10^9 cm. per second. Stepped leaders are beautifully illustrated in Figs. 15 and 16. Note the steps for advancing branches as well. Whether the pilot streamer has stopped or slowed just before the arrival of the stepped leader is not known. At any rate, after the arrival it surges forward with possibly increased branching at this time. Branching is invariably in the direction of advance as seen in Figs. 9, 14, 15, and 16. The movements of stepped leaders can be picked up by

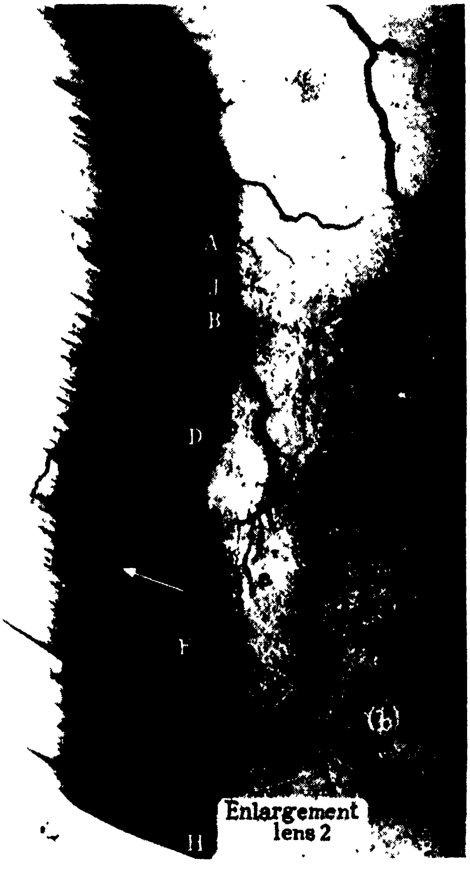


FIG. 16. Another beautiful picture of stepping from the work of Schonland and Collens, time extending from right to left. Note the stepping in the main pilot streamer, ef, and also the stepping of the branch xy. The return stroke is seen at the left and is much blackened by overexposure. The duration of illumination in the return stroke appears to be quite long and is probably caused by successive return strokes up the channel produced by the branching. Overexposure of the main stroke results when the sensitivity of the process is increased to observe the leader.

oscillograph through field disturbances as seen in diagrams of α and β strokes analyzed by Schonland in Figs. 17 and 18. Thus every 5 to 10 m. or 50 microseconds of advance the pilot streamer tip and channel are made visible by a *stepped leader.*

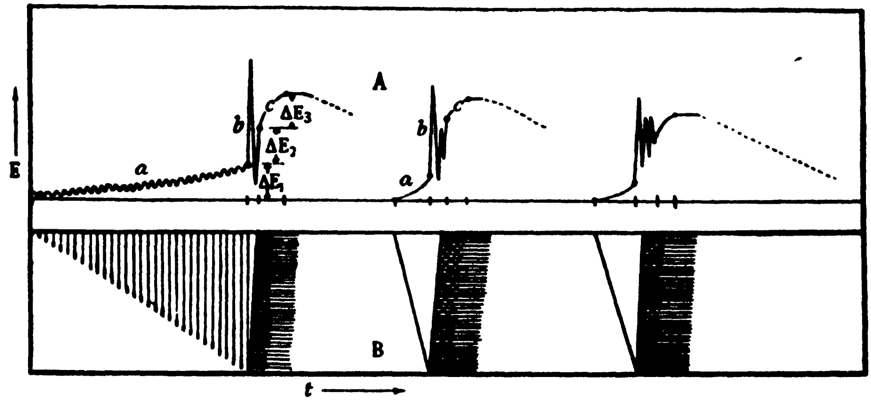


FIG. 17. Diagram from Schonland, Hodges, and Collens, of oscillograms of stepped leader strokes of the α type with schematic indication of the events correlated therewith underneath. Potential in the A part of the figure is upward and t is plotted along the axis of abscissae. Note the oscillations produced by each step and the intense oscillation B caused by the return stroke. The occurrences to the right indicate successive strokes with dart leader with no stepping.

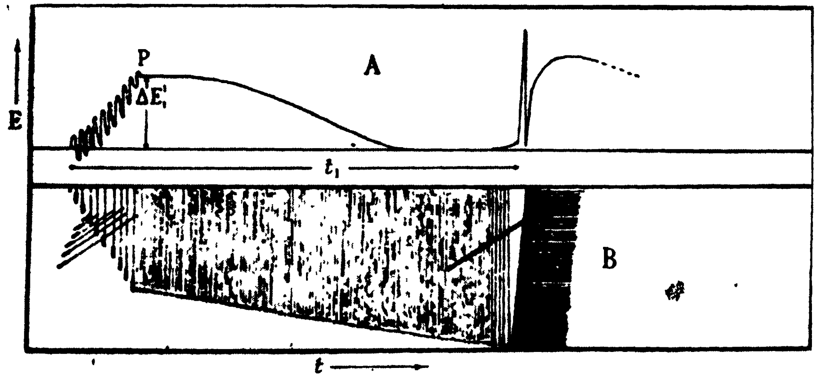


FIG. 18. Drawing of electrical events produced in β type strokes according to Schonland, Hodges, and Collens. The interpretation is similar to that for the α stroke of Fig. 17. Note here the very heavily stepped leaders calling forth intense branching and the slow leader advance of the steps over a period of about 5 milliseconds from the point P to the return stroke at the end of the time interval t_1.

3. Two principal types of strokes are observed. The α type, comprising 65 per cent of observed strokes, is one which advances with relatively even steps all the way to ground as seen in Fig. 17. As it approaches the ground it may be met with a positive streamer from a good

conducting element on the ground and the bright or return stroke appears. In the remaining 35 per cent of the strokes a β process occurs as illustrated in Fig. 18. Here initial stepping is very bright and strong and there is much downward branching. In this event the discharge seems to have spent itself before the streamer reaches ground. Thus the stepped leaders diminish in intensity and continue on with barely discernible intensity until the breakdown is complete. For intervals of time of the order of 5 milliseconds, this distorted field may drift along near the ground before the return stroke materializes. The average time taken for the pilot streamer to reach the ground in α strokes depends on the length of the stroke and on the average amounts to 5 milliseconds for 1-km. strokes. Thus the time for an α stroke to ground is 5 milliseconds per kilometer length while for the β type it would be about 10 milliseconds or more. For long strokes two stepped leaders can be seen at once, one in midair, the other near the streamer tip. Stepped leaders in general grow brighter as they approach the tip.

4. When the stepped leader process comes close to the ground, whether or not it is met by a positive streamer of notable length (38), the conducting path of ions is closed and a potential wave of high intensity accompanied by great luminosity passes *up* the channel from ground to cloud. Where the positive streamer meets the pilot above the ground, the junction point is exceptionally bright, as seen in the long spark of Fig. 13 (31,38). This "return stroke" travels with a velocity which runs *up* the channel at from 3×10^9 to 10^{10} cm. per second. The return stroke is indicated by a brightness which is that of the blinding part of the lightning flash. It diminishes in intensity as it passes branches and as it nears the cloud. It may appear broad because of several succeeding potential waves called forth largely by branches as noted by Schonland (3). The duration of luminosity following the stepped leaders or the return stroke is of the order of some microseconds although halation effects on the plate indicate longer times.

The duration of luminosity in sparks has been studied by J. D. Craggs and J. M. Meek (21,34) and indicates that while the time of light emission by excited atoms is of the order of 10^{-7} to 10^{-8} sec., both thermal ionization following the stroke and recombination of electrons and ions cause the longer emissions lasting some 3 microseconds. In argon metastable states carried this time out to 30 microseconds. The much longer duration of emission in certain *spots* in Figs. 13, 14, 15, and 16, cannot be associated with bends in the path in line of sight as at first assumed (39). They may be connected with certain peculiar conditions produced with very intense and dense ionization causing a persistence of excitation requiring wave mechanical interpretation (33,39).

5. The persistence of *ionization* in the channel beyond the period of luminosity is indicated by *repeated strokes* down the channel. According to Schonland, often associated with a Y-branching at the cloud

end successive strokes will follow down an old channel (3,42). As many as 20 repeated strokes have been observed with an average interval of 0.01 to 0.09 sec. between them. The average number per stroke is 3 (8). This is indicated in the moving camera photograph of Fig. 19 due to Walter (8), and in Fig. 14. Aside from the increased branching at the cloud end indicating that some other portion of the cloud has discharged through the old channel in such strokes these are signalized by the appearance of the *dart leader.* This leader differs in that it is a single leader stroke only stepped down very old channels. Thus it usually proceeds to the ground *without* stepping with an average velocity of 2×10^8 cm. per second. Its velocity ranges rather widely from 1×10^8 to 2×10^9 cm. per second depending on the age of the channel it follows and the steepness of the potential waves from the cloud end. It is in general slower than the stepped leader. On reaching the ground it is followed by the bright return stroke up the channel.

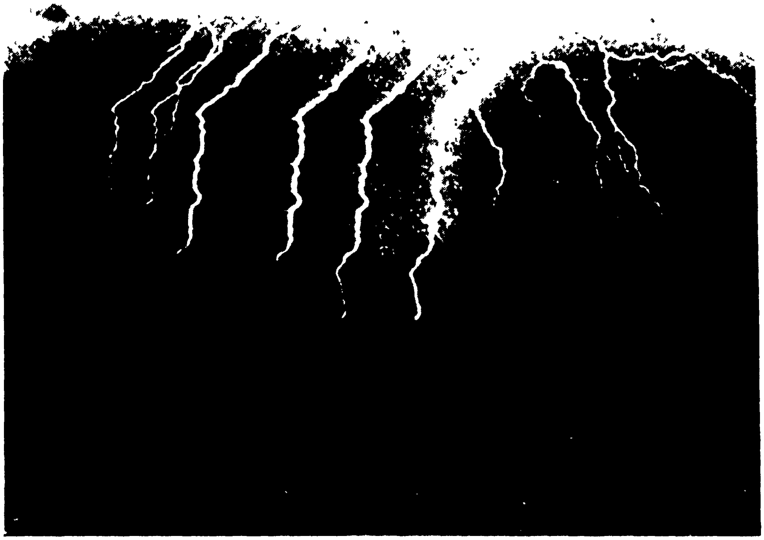


FIG. 19. Photograph of successive lightning strokes down the same channel taken with moving camera by Walters. These are to be compared with the photograph obtained on long sparks by Allibone and Meek shown in Fig. 20.

One may now interpret the occurrences as follows:

1. With adequate over-all field gradients to ground and local fields in a small section of the cloud ranging from 10 to 60 or more kv. per centimeter created slowly or suddenly by convective air currents, either a disrupted water drop induced discharge or a free electron causes ionization by collision and avalanche and streamer formation. This leads to

a positive streamer advance towards the cloud with further progress of the avalanche on the ground end of the streamer towards the ground in virtue of the field distortion produced. The negative point streamer mechanism already discussed is then initiated and the "pilot streamer" begins its career, drawing a small current of electrons down itself from the cloud to make up for diffusive loss of electrons from the avalanche's head. The streamer channel in 50 microseconds may reach a diameter of 2 to 10 mm. and will have about 10^{14} ions per cubic centimeter. The advance in weak guiding fields will be slow and tortuous and in stronger fields it will be fast and straighter, showing much branching.

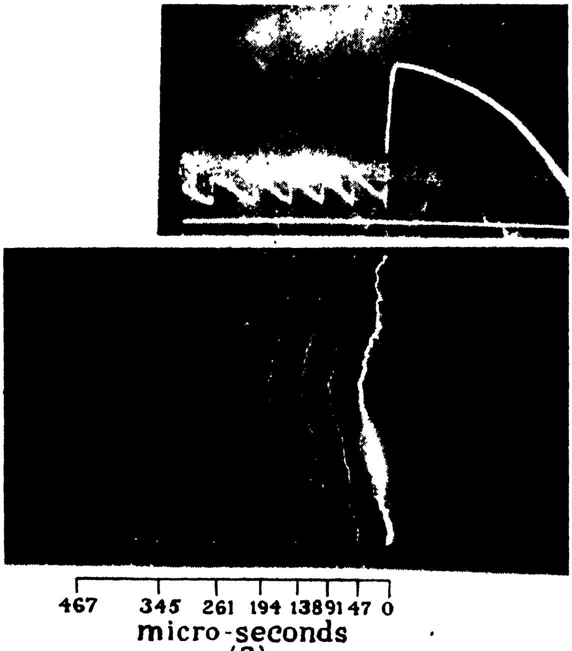
After some 50 microseconds of travel the upper end of the streamer channel has undergone some changes. Near the advancing tip the field gradients are high and the advancing electrons have much energy. They cannot attach to O_2 molecules to form ions. However, in the streamer channel the fields are much less and electrons rapidly lose energy so that loss of electrons to form negative ions is considerable. Positive and negative ions then begin to recombine and the conductivity in the older cloud end of the channel begins to decline. Following a statement by Schonland that "the unusual constancy of this phenomenon (time of stepping) cannot be ascribed to any process involved in the cloud but must be led back to some basic physical process," and a later statement that recombination of ions might be involved, J. M. Meek discovered that indeed *recombination* led to just such an expected constancy in the phenomenon (35).

The number of ions n out of an initial number n_0 remaining as a result of recombination after a time t is given by $n = \frac{n_0}{1 + n_0\beta t}$. Here β is the coefficient of recombination of ions which is of the order of 2×10^{-6} in air at 22° and 760-mm. pressure. It is to be noted that n will become inversely proportional to t and roughly independent of n_0 when $n_0\beta t$ becomes much greater than 1. This leads at once to the discovery that in 50 microseconds $n_0\beta t$ exceeds the value 1 if n_0 exceeds 10^{10}. Hence for ion concentrations in excess of 10^{10} ions per cubic centimeter, the decline of ionization becomes largely independent of initial concentration and reaches its value of 10^{10} in about 50 microseconds. Thus, irrespective of initial ion concentration which will be fairly constant in any given channel with minor variations, the ion density will decline to 10^{10} in about 50 microseconds. On this basis Meek evolved a theory (35) for the stepped leader stroke which was later modified by Loeb and Meek (36). This indicates that in any streamer channel, if f represents fraction of the electrons needed in the plasma to act as a current down the streamer channel to make up for diffusive electron loss at the avalanche head and if f_e is the fraction of free electrons left in the channel after a time t by attachment process to O_2 molecules the electrical field gradient at the cloud end of the streamer channel after 50 microseconds is given by $X = 2 \times 10^7 \left(\frac{f}{f_e}\right)^2$ v. per centimeter. With $f = 0.05$ and $f_e = 0.1$, which are not unreasonable values, X becomes 50250 v. per centimeter. Thus a high field gradient is created at the cloud end by the time the pilot streamer channel has advanced for 50 microseconds. With such a gradient the O_2^- ions in the channel can become deionized and with accompanying intensive electron ionization the potential distortion caused by this gradient sweeps down the channel at a speed that varies with the density of ionization left in the channel plasma. As this density increases down the newer portions of the pilot streamer channel, where there has been less recombination, the stepped leader speeds up and becomes brighter. All variations in the stepping time and velocity can be accounted for by variations in the initial density of ionization and the residual ionization existing at the time of the step. When the stepped leader arrives at the end of the pilot streamer, the latter forges ahead with renewed vigor and much branching and the process repeats.

In the case of β type strokes excessive branching so drains the energy of the cloud available that progressive and vigorous advance and stepping is impeded and the streamer forges ahead only slowly. When either α or β streamer tip advances close enough to the ground a positive streamer will move upward from the ground and at the instant of junction a very intense potential wave sweeps up the pre-ionized channel with very high velocity. This results from the fact that except for the iR drops down the streamer and leader channel the whole potential difference between cloud and ground centers at the junction point. This is clearly seen both in sparks and in lightning where positive streamers from the ground meet the leader well above the ground as in Fig. 13. In most *natural* strokes to ground good conducting points from which a streamer can start are few and far between so that the leader must in most cases advance pretty well towards the ground before the return stroke can materialize. Golde (38) recently estimates 5 m. to open ground and 16 m. to a lightning conductor. If, however, one places a good pointed conductor on top of a high building, the positive field about the good conductor is much increased and positive streamers will advance well up towards the cloud. Thus the strokes will appear to come from the ground as a positive streamer since these start at materially lower field gradients than from a negative cloud. In fact, such a system often *invites* strokes that would otherwise not occur. Hence many more strokes to the Empire State Building were recorded with good grounded conducting points projecting than normally occur. Thus the lightning strokes *usually observed* rarely show streamer processes (38) from ground while long sparks between metal electrodes show streamers passing well toward the center of the gap as seen in Fig. 13.

Allibone and Meek, however, observed that the longer the gap and the lower the resistance in series with the negative point, the more the junction point of positive ground streamer and negative leader approaches the ground (31).

Once junction has been established one observes, as in sparks, the intensely brilliant and noisy return stroke. This as indicated, depending on the ion density in the channel and the steepness of the potential gradient, travels up the channel giving what the eye perceives as the lightning stroke. As shown by the Boys and moving film cameras, the stepped leader and streamer when long enough occur earlier in the process, but the time scale is such that the human eye perceives only the return stroke. If there is much branching, the return stroke illumi-



(a)

FIG. 20. Lower photograph taken with moving film camera by Allibone and Meek of a meter long spark with aperiodic circuit and high resistance containing capacity giving successive sparks down the pre-established channel. Note that in this case time goes from right to left and that the spark proceeds from the positive point downward. The leader stroke being seen on the right and the return stroke on the left of the first stroke. In this case the positive streamer has been met by a *negative* streamer from the earthed plane. Note the downward branching of the positive streamer and the upward branching of the negative streamer mechanism. Also note the intense brilliance produced at the junction points of the two streamers. Subsequent strokes show less detail of structure. Above is a voltage potential oscillogram indicating the fall of potential with the advent of the return stroke and the subsequent build-up of potential to a lower breakdown value on each successive stroke.

nating them as well as the main channel loses in intensity and presumably in speed as it advances towards the cloud as seen in Fig. 9. The decay in ionization at the upper end of the channel since the last stepped leader makes the luminosity at the cloud end in the return stroke distinctly less (3). Ion densities produced by return strokes in sparks have been estimated by Stark effect broadening as of the order of 10^{19} per cubic centimeter by Dunnington (24) and 3.5×10^{18} per cubic centimeter by Margenau and Watson (37). Careful study by Craggs and Meek (21), and by Craggs and Hopwood (34), in sparks in H_2 and A from several different considerations yield of the order of 10^{17} ions per cubic centimeter. In lightning somewhat higher densities could be expected. The exact width of the channel observed in lightning strokes is a matter of controversy. The pilot streamer and stepped leader may be about a centimeter in diameter. With the high energy in the return stroke the radial adiabatic expansion occurs at about 10^5 cm. per second according to Meek from spark studies (22). In the 3 to 20 microseconds

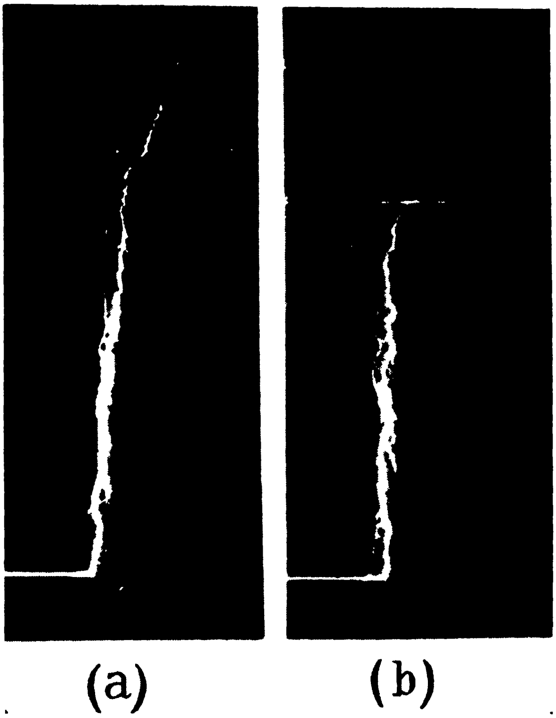


FIG. 21. Showing two sparks from positive points to grounded plane. The time extending from right to left. Note the leader stroke on the right and the return stroke on the left. The reproduction of these negatives is too poor to indicate this clearly. The stepping is produced on a meter long spark by introducing high resistance in connection with the point. It is, therefore, *not natural* stepping.

of brilliant luminosity, the channel could broaden by 4 to 5 cm. in both directions. Hence the width observed can range from 2 to 10 cm., which is within the range claimed by estimators. The width of the brightest phase is probably less. The breadth of channels shown by moving lens is partly due to multiple return strokes caused by branching.

Since *ionization* persists for a considerable time beyond the few microseconds of intense luminosity, it is natural to expect that subsequent discharges of a heavily charged cloud portion within the initiating cloud will take place to the recently discharged portion. Thus successive strokes after 10 or so milliseconds will proceed down the old channel possibly showing new branches at the cloud end when not obscured by cloud and rain. These will be initiated by a *dart* leader whose velocity and luminosity will depend on the residual ionization in the old channel. Its progress will generally be continuous as with the *last* step of the leader process and will be followed by a return stroke. Allibone and Meek (19) succeeded in obtaining successive strokes down an old channel with long sparks by putting a capacity across the gap in a non-oscillating circuit with high resistance in series with a negative point as shown in Fig. 20.

Allibone and Meek further reproduced certain of the other phenomena observed in lightning with their long sparks. It was observed that sparks with positive point and grounded plane proceeded more readily than with negative point. The streamer crossed directly as would be expected with velocities of the anticipated magnitude as shown in Fig. 21. The gaps of about a meter in length naturally precluded stepped leaders. *Artificial* stepping could be induced by placing a high resistance in series with the positive point as seen in Fig. 21(*b*). The stepping is, however, caused by a temporary reduction of the positive point potential by the neutralizing current of electrons to it and the recovery of potential at the point through the high resistance. Discharge from a negative point with grounded conducting plane showed the reluctance of the streamer mechanism to proceed smoothly. There was no stepping in the negative process (19,31). Instead there was a series of separate predischarges with final breakdown presumably following along one of the earlier channels as seen in Fig. 13. A tendency to breakdown with midgap streamers was also evident with a negative point indicating the difficulty of the negative streamer mechanism. Positive streamers from the ground always appear in these long sparks since they are favored by the high conductivity of the metal plane.

It is seen in this necessarily rapid survey that since 1935 we have come a long way in the interpretation of one of the most terrifying and conspicuous phenomena observed by mankind from times immemorable. A great deal of further study is needed. It is hoped that by thus bringing together the elements of our accumulated knowledge from widely different sources enough interest may be stimulated to bring other workers into this field to carry the studies further.

REFERENCES.

(1) L. B. LOEB, "Fundamental Processes of Electrical Discharge in Gases," New York, John Wiley & Sons, Inc., p. 426 (1939); J. M. MEEK, *Phys. Rev.*, Vol. 57, p. 722 (1940); L. B. LOEB AND J. M. MEEK, *J. Applied Phys.*, Vol. 11, pp. 438, 459 (1940); L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," Stanford Press (1941).

(2) H. RAETHER, *Zeits. f. Phys.*, Vol. 110, p. 611 (1938); Vol. 117, p. 386 (1941); Vol. 117, p. 524 (1941); *Archiv. für Elektrotech.*, Vol. 34, p. 49 (1940).

(3) B. F. J. SCHONLAND AND H. COLLENS, *Proc. Roy. Soc.*, Vol. A143, p. 654 (1934); Vol. A152, p. 595 (1935); Vol. A162, p. 175 (1937); Vol. A164, p. 132 (1938); Vol. A166, p. 356 (1938); D. J. MALAN AND B. F. J. SCHONLAND, *Proc. Roy. Soc.*, Vol. A191, p. 483 (1947).

(4) J. S. TOWNSEND, "Electricity in Gases," Oxford Press (1915).

(5) For the numerous references on this item see L. B. LOEB, "Fundamental Processes," *loc. cit.*, p. 449, References 41 through 50. Also LOEB AND MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 32, Reference 19.

(6) A. M. CRAVATH, *Phys. Rev.*, Vol. 47, p. 254(A) (1935); DECHENE, *Jour. de Phys. e Radium*, Vol. 7, p. 533 (1936); E. GREINER, *Zeits f. Phys.*, Vol. 81, p. 543 (1933); H. RAETHER, *Zeits. f. Phys.*, Vol. 110, p. 611 (1938); H. COSTA, *ibid.*, Vol. 113, p. 531 (1939); Vol. 116, p. 508 (1940); W. SCHWIECKER, *ibid.*, Vol. 116, p. 562 (1940).

(7) L. B. LOEB AND A. F. KIP, *J. Applied Phys.*, Vol. 10, p. 142 (1939). Also L. B. LOEB, "Fundamental Processes," *loc. cit.*, p. 514 ff.

(8) B. WALTER, *Jahrb. d. Hamburger. Wiss. Anst.*, Vol. 20 (1903); C. V. BOYS, *Nature*, Vol. 118, p. 749 (1926); Vol. 122, p. 310 (1928).

(9) F. W. PEEK, JR., *Trans. Am. Inst. Electrical Engrs.*, Vol. 48, p. 436 (1939); R. R. PITMAN AND J. J. TOROK, *ibid.*, Vol. 50, p. 568 (1931); E. J. WORKMAN AND R. E. HOLZER, Researches, Albuquerque, N. M., 1938–40. Not yet published; H. NORINDER, JOURNAL FRANKLIN INST., Vol. 225, p. 69 (1936); Vol. 244, pp. 109, 167 (1947); K. B. MCEACHRON, *ibid.*, Vol. 227, p. 149 (1939).

(10) L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 37.

(11) L. B. LOEB, *Reviews of Modern Physics*, Vol. 20, p. 151 (1948).

(12) H. RAETHER, *Archiv. f. Elektrotech.*, Vol. 34, p. 49 (1940).

(13) L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 40 ff.

(14) E. G. SCHNEIDER, *J. Optical Soc. Am.*, Vol. 30, p. 128 (1940).

(15) S. GORRILL, Thesis, University of California (1939).

(16) S. ZAYENTZ AND V. HEY, *Jour. Phys. USSR.*, Vol. 11, p. 413 (1946).

(17) Additional data come from as yet unpublished work of Meek and associates at Liverpool.

(18) V. B. WAITHMAN AND W. R. BAKER, *Phys. Rev.*, Vol. 57, p. 252A (1940).

(19) T. E. ALLIBONE AND J. M. MEEK, *Proc. Roy. Soc.*, Vol. A166, p. 97 (1938).

(20) L. B. SNODDY, DIETERICH, and J. W. BEAMS, *Phys. Rev.*, Vol. 50, p. 469 (1936); Vol. 52, p. 739 (1937); F. H. MITCHELL AND L. B. SNODDY, *Phys. Rev.*, Vol. 72, p. 1202 (1947).

(21) J. D. CRAGGS AND J. M. MEEK, *Proc. Roy. Soc.*, Vol. A186, p. 241 (1946).

(22) Paper delivered by J. M. Meek before Phys. Soc. London, April 23, 1947. Not yet published.

(23) C. G. MILLER AND L. B. LOEB, *Phys. Rev.*, Vol. 73, p. 84 (1948).

(24) E. O. LAWRENCE AND F. G. DUNNINGTON, *Phys. Rev.*, Vol. 35, p. 396 (1930); F. G. DUNNINGTON, *ibid.*, Vol. 38, p. 1535 (1931); H. J. WHITE, *ibid.*, Vol. 46, p. 99 (1934); T. E. ALLIBONE AND J. M. MEEK, *Proc. Roy. Soc.*, Vol. A169, p. 246 (1938).

(25) L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 58 ff.

(26) TERADA AND NAKAYA, *Sci. Papers Inst. Phys. Chem. Research*, Tokyo, Vol. 4, p. 129 (1923); Vol. 8, pp. 1, 63, 103, 197 (1925); Vol. 10, p. 43 (1929).

(27) SIR G. C. SIMPSON AND F. J. SCRASE, *Proc. Roy. Soc.*, Vol. A161, p. 309 (1937). Similar conclusions come from preliminary work of Seville Chapman in 1947.

(28) R. GUNN, *Phys. Rev.*, Vol. 71, p. 181 (1947); also R. GUNN, HALL AND KINZER, *Proc. Inst. Rad. Eng.*, Vol. 34, p. 156 P, April (1946).

(29) Analogous views were also expressed by R. E. HOLZER in a Seminar, University of California, Oct. 28, 1947, on basis of their studies.

(30) W. A. MACKY, *Proc. Roy. Soc.*, Vol. A133, p. 565 (1931); J. ZELENY, *Proc. Camb. Phil. Soc.*, Vol. 71, p. 18 (1914); *Phys. Rev.*, Vol. 16, p. 102 (1920); W. N. ENGLISH, *Phys. Rev.*, Vol. 74, July, 1948.

(31) T. E. ALLIBONE AND J. M. MEEK, *Proc. Roy. Soc.*, Vol. A169, p. 246 (1938).

(32) L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 71 ff.; also 89 ff.

(33) TH. NEUGEBAUER, *Zeits. f. Phys.*, Vol. 106, p. 474 (1937).

(34) J. D. CRAGGS AND W. HOPWOOD, *Proc. Phys. Soc. London*, Vol. 59, pp. 755, 771 (1947).

(35) J. M. MEEK, *Phys. Rev.*, Vol. 55, p. 972 (1939).

(36) L. B. LOEB AND J. M. MEEK, "Mechanism of the Electric Spark," *loc. cit.*, p. 101 ff.

(37) H. MARGENAU AND W. W. WATSON, *Phys. Rev.*, Vol. 8, p. 22 (1936).

(38) R. H. GOLDE, Letter to the Editor, *Nature*, Vol. 160, p. 395 (1947).

(39) J. D. CRAGGS, W. HOPWOOD, AND J. M. MEEK, *J. Applied Phys.*, Vol. 18, p. 919 (1947).

(40) L. B. LOEB, *Phys. Rev.*, Vol. 73, p. 798 (1948).

(41) L. B. LOEB, *Phys. Rev.*, Vol. 74, Aug., 1948; also L. B. LOEB and R. A. WIJSMANN, *Jour. App. Phys.*, Aug., 1948.

(42) EDMUND A. EVANS, "Study of Mechanism Lightning Discharge," Thesis No. 3781 S78E, Stanford University (1932).

CERTAIN MATTERS CONCERNING SCALING IN THE MAGNETRON WITH SPECIAL REFERENCE TO THE RELATIVE EFFICIENCY OF MAGNETRONS OF DIFFERENT SIZES.*

BY

W. F. G. SWANN,[1] D.Sc.

ABSTRACT.

The paper concerns the comparison of the behavior of two magnetron systems of different sizes, all apparatus from the input terminals of the magnetron to the final radiation antenna being contemplated in the picture. The mathematical comparison of the two systems assumes a specially simple form when the electrical conductivities of the corresponding parts of the systems in the two cases are adjusted to values in the inverse ratio of the scaling in linear dimensions. A scaling of this kind is therefore studied. The conclusions reached are as follows:

For an assigned potential applied to the two models, the power input is the same, the power radiated from the antenna is the same, the power dissipated in Ohm's law heating in all parts is the same, and the power dissipated in back-bombardment or at the anode is the same.

As a corollary to the foregoing, it is inferred that if the corresponding parts of the two models have the same electrical conductivity, then in so far as Ohm's law heating is an important factor at all, it is more important in the small model than in the large one.

INTRODUCTION.

Previous considerations of the problem of scaling have been given by A. M. Clogston [2] and by the writer.[3] The present paper is of more general application and is designed primarily for the discussion of power considerations. It is unlikely that the results here developed are not known, wholly or in part. However, I have found no statement of them, and discussions of the matter with specialists in this field have encouraged me to publish the presentation. The compactness of its nature may make it of interest even to those who are familiar with some or all of its consequences.

PRELIMINARY STATEMENT REGARDING ASSIGNED CONDITIONS.

In order to start the magnetron in operation it is necessary to assign certain magnitudes—a magnetic field, as produced by the magnet, and a "potential difference" between anode and cathode. Regarding the latter assignment, a little more care is necessary in statement. In general, in a problem involving non-statical considerations, there is no such thing as a potential difference between two points in the elementary sense of the phrase.[4] Even if we should have a simple case such as is represented in Fig. 1, there is no sense to speaking of the potential difference between *A* and *B* for a case where magnetic flux is changing through *ADBC*, for example. In such a case the line integral from *A* to *B* depends upon the path. In the case of the magnetron, there is no sense to speaking, without reservation, of the potential difference between the anode and cathode. The changing magnetic fields resulting from the oscillating currents render meaningless the elementary concept of potential difference between different parts of anode and cathode The electric field is everywhere given by

---

* A supplementary report on Contract No. OEMsr-358.

[1] Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Penna.

[2] Voltage and Wave Length Scaling of Magnetrons, Report 52—1/21/43, Radiation Laboratory.

[3] A Theorem on the Correspondence of Conditions in the Steady State Solutions for Magnetrons of Different Sizes. Appendix to Second Report on Contract OEMsr-358, Supplement 1, January 1, 1943.

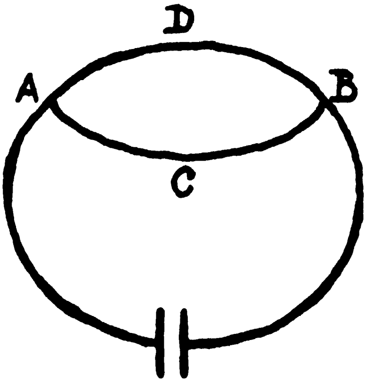


FIG. 1.

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} - \text{grad } \varphi,$$

where φ is the scalar and **U** the vector potential, but no less complete a specification of the field has any justification.

In spite of the foregoing remarks, there is sense to speaking of the line integral of **E** between two points for a definite path. In the case of the magnetron we shall consider this line integral as the thing specified when the two end points are the input terminals and the path is one which joins these two terminals outside the magnetron, or at any rate in such fashion that the magnetron contributes no changing magnetic flux through the circuit composed of this path and the remainder of the circuit leading to the pulser. In other words, if the magnetron be operated by attachment of its terminals to the poles of a hypothetical battery of infinitely low resistance, the aforesaid line integral would in this case be equal to the electromotive force of the battery.

---

[4] There are, of course, definite values for the vector and scalar potential differences between any two points, these quantities being understood in the sense implied in electrodynamics.

Having put this matter upon a firm basis, we shall follow the custom of speaking of the potential difference applied between the magnetron terminals, the term "potential difference" now being understood in the sense of the foregoing line integral.

THE CONTROLLING CONDITIONS FOR OPERATION.

We shall consider the system starting with the input terminals and comprising the magnetron proper, its associated coupling loop, an output antenna and any intermediate tuning apparatus. The operation of this compound device is governed by laws and assigned conditions *a*, *b*, *c*, *d*, as follows:

(*a*) For the present problem, where the permeability is unity in all relevant places.

$$\frac{1}{c}\left(\mathbf{i} + k\frac{\partial \mathbf{E}}{\partial t}\right) = \operatorname{curl} \mathbf{H}, \tag{1}$$

$$\rho = \operatorname{div} \mathbf{E} \tag{2}$$

$$-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = \operatorname{curl} \mathbf{E}, \tag{3}$$

$$0 = \operatorname{div} \mathbf{H}, \tag{4}$$

where **i** is the current density, and the significance of the other symbols is obvious. These equations apply within the conductors and within all cavities or other spaces in the conductors, also in the free space surrounding the antenna.

(*b*) In the material of the conductors, we shall suppose that the current density is controlled by Ohm's law[5]

$$\mathbf{i} = \sigma \mathbf{E}. \tag{5}$$

(*c*) At a point in the magnetron cavities, the motion of an electron is controlled by

$$\frac{d}{dt}(m\mathbf{u}) = e\left(\mathbf{E} + \frac{[\mathbf{uH}]}{c}\right), \tag{6}$$

where **u** is the velocity of an electron and *m* is its mass. In this connection, there is no need to limit ourselves to a non-relativistic mass, although the velocities involved are too small to render relativistic considerations important.

---

[5] If we should use the more complicated equation involving $[\boldsymbol{\omega}\mathbf{H}]/c$, where $\boldsymbol{\omega}$ is the mean velocity of the electrons in the material, that would simply mean including the Hall Effect in our calculations. We shall neglect this refinement, not because it is necessary to do so, but in order not to complicate the main derivation by the inclusion of things of subsidiary interest. However, in an Appendix to this paper, we shall point out that even this Hall Effect conforms to the scaling theorem exactly, and indeed it will appear that the fundamental equations (1) to (4) and (5) may be used in a more general form without affecting the scaling characteristics.

(*d*) Any mathematical problem requires, in addition to a specification of the fundamental laws which give the course of events, a specification of the necessary assignments of magnitude which must be made to provide for a unique solution of the problem. A formal procedure would call for a careful analysis of this matter. However, relying upon the generally accepted belief, as substantiated, moreover, by the experimental facts, we shall suppose that the conditions and magnitudes which must be assigned to give a unique solution for the steady state are:

1. The assignment of the magnetic field $\mathbf{H}_m$ of the magnetron magnet.
2. The assignment, in the sense already defined, of the potential difference V between the input terminals of the magnetron.
3. The assignment, for the cathode, of the temperature conditions which will insure operation under conditions of space change limitation, so that the normal component of $\mathbf{E}$ at the cathode is always zero.
4. Assignment of the usual boundary conditions at the junction of two media, viz., equality of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ and equality of the normal components of $D\ (= k\mathbf{E})$ and $B\ (= \mathbf{H}$ in this case, in all relevant places).

Now suppose that quantities in a certain magnetron system are denoted by a subscript unity attached to them, and that we consider another system, which we shall designate by the subscript 2, and in which all linear dimensions are scaled down on the ratio ϵ, where $\epsilon < 1$.

We shall define "corresponding" points and times in the two systems by the relation

$$(x_2, y_2, z_2, t_2) = \epsilon(x_1, y_1, z_1, t_1) \tag{7}$$

in which it will be observed that we are comparing events in system (1) with those at scaled-down times in system (2).

Recognizing that quantities with subscript unity hang together as a solution of (1) to (6), satisfying the boundary conditions, and making $V = V_1$, and $\mathbf{H}_m = \mathbf{H}_{m1}$, we shall consider as candidates for the solution in system (2) the following quantities:

$$\mathbf{E}_2 = \mathbf{E}_1/\epsilon; \qquad \mathbf{H}_2 = \mathbf{H}_1/\epsilon; \tag{8}$$

$$\rho_2 = \rho_1/\epsilon^2; \qquad \mathbf{i}_2 = \mathbf{i}_1/\epsilon^2; \qquad \mathbf{u}_2 = \mathbf{u}_1 \tag{9}$$

so that, of course,

$$\mathbf{H}_{m2} = \mathbf{H}_{m1}/\epsilon.$$

The quantities are to be applied at points x_2, y_2, z_2, and at times t_2 in system (2) related to the corresponding points x_1, y_1, z_1, and times t_1

for system (1) in the manner defined by (7) as already implied. However, we shall regard our system (2) as not only scaled down but as provided with different electrical conductivities as regards its various parts in such fashion that, at corresponding points,[6]

$$\sigma_2 = \sigma_1/\epsilon. \tag{10}$$

However, we shall not scale k. In other words, $k_1 = k_2$.

Now it is immediately obvious that, at corresponding points and times,

$$\frac{\partial \mathbf{E}_2}{\partial t_2} = \frac{1}{\epsilon}\frac{\partial}{\partial t_1}\left(\frac{\mathbf{E}_1}{\epsilon}\right); \quad \frac{\partial \mathbf{H}_2}{\partial t_2} = \frac{1}{\epsilon}\frac{\partial}{\partial t_1}\left(\frac{\mathbf{H}_1}{\epsilon}\right), \tag{11}$$

$$\left(\frac{\partial \mathbf{E}_2}{\partial x_2}, \frac{\partial \mathbf{E}_2}{\partial y_2}, \frac{\partial \mathbf{E}_2}{\partial z_2}\right) = \frac{1}{\epsilon}\left[\frac{\partial}{\partial x_1}\left(\frac{\mathbf{E}_1}{\epsilon}\right), \frac{\partial}{\partial y_1}\left(\frac{\mathbf{E}_1}{\epsilon}\right), \frac{\partial}{\partial z_1}\left(\frac{\mathbf{E}_1}{\epsilon}\right)\right], \tag{12}$$

$$\left(\frac{\partial \mathbf{H}_2}{\partial x_2}, \frac{\partial \mathbf{H}_2}{\partial y_2}, \frac{\partial \mathbf{H}_2}{\partial z_2}\right) = \frac{1}{\epsilon}\left[\frac{\partial}{\partial x_1}\left(\frac{\mathbf{H}_1}{\epsilon}\right), \frac{\partial}{\partial y_1}\left(\frac{\mathbf{H}_1}{\epsilon}\right), \frac{\partial}{\partial z_1}\left(\frac{\mathbf{H}_1}{\epsilon}\right)\right]. \tag{13}$$

Moreover, if ds is an element of the path of an orbit of an electron in the magnetron cavity, and $\mathbf{u}$ is the electron velocity,

$$\frac{d}{dt}(m\mathbf{u}) = \mathbf{u}\frac{d}{ds}(m\mathbf{u}).$$

Thus since at corresponding points $\mathbf{u}_2 = \mathbf{u}_1$, so that the masses at corresponding points are the same, we have, at corresponding points,

$$\frac{d}{dt_2}(m\mathbf{u}_2) = \frac{1}{\epsilon}\frac{d}{dt_1}(m\mathbf{u}_1). \tag{14}$$

It will now be immediately obvious that if $\mathbf{E}_1$, $\mathbf{H}_1$, ρ_1, $\mathbf{u}_1$, $\mathbf{i}_1$ are solutions of (1) to (6) for system (1) with $\sigma = \sigma_1$, then the corresponding values of these quantities as defined by (8) and (9) are solutions of (1) to (6) at corresponding points and times, provided that at corresponding points σ_2 is given by (10). If they are solutions at all *corresponding* points and times, they are solutions at *all* points and times.

As regards the supplementary conditions, we can regard $\mathbf{H}_{m1}$ as a part of the field of the problem and constituting the whole field at such a point as P for example. We thus see that we must have, as already stated,

$$\mathbf{H}_{m2} = \mathbf{H}_{m1}/\epsilon. \tag{15}$$

Since at corresponding points and times $\mathbf{E}_2 = \mathbf{E}_1/\epsilon$, and since corresponding elements of path in the second and first systems are in the ratio $\epsilon/1$, it is obvious that line integrals of $\mathbf{E}$ between corresponding points are the same in the two systems. Thus we must have

---

[6] The reason for this is simply one of mathematical convenience. There is, of course, *meaning* to a problem in which the conductivities are equal at corresponding points in the two systems. However, in such a problem the solutions are not related by such a simple transformation as is here employed.

$$V_2 = V_1. \tag{16}$$

Provided that the temperature conditions are properly chosen in system (2), the space charge limitation will be provided for in that system if it is provided for in the first system. Thus the normal component of **E** at the cathode will be zero in the second system if it is zero in the first.

It will be immediately obvious that the boundary conditions at the surface of two media are satisfied in the second system provided that they are satisfied in the first.

POWER COMPARISONS.

It will be convenient to define n as

$$n \equiv 1/\epsilon.$$

Power Input. The power input is VI. Now $V_1 = V_2$. Again, since at corresponding points $\mathbf{i}_2 = \mathbf{i}_1 n^2$, and since corresponding elements of area ds are related by $ds_2 = ds_1/n^2$, it is obvious that as regards the input current, $I_2 = I_1$. *Hence the input power is the same in the two systems.*

Power Radiated. The power radiated from the antenna is P_{r1}, where

$$P_r = AM^2\nu^4. \tag{17}$$

Here M is the maximum moment of the antenna, ν is the frequency, and A is the same for both systems.

Now considering corresponding elements of volume dv of the antennae in the two systems, we have $dv_2 = dv_1/n^3$. Since $\rho_2 = \rho_1 n^2$ if L is the distance of a point on the antenna from the neutral plane, the distance being measured parallel to the axis of the antenna, $L_2 = L_1/n$.

Now

$$M = \int \rho L dv,$$

where the integration extends all over the antenna. Hence

$$M_2 = \int \rho_2 L_2 dv_2 = \frac{n^2}{nn^3}\int \rho_1 L_1 dv_1 = \frac{M_1}{n^2}.$$

Hence, since $\nu_2 = n\nu_1$, we have, from (17),

$$(P_r)_2 = AM_2^2\nu_2^4 = \frac{AM_1^2}{n^4}\nu_1^4 n^4 = (P_r)_1.$$

Hence the power radiated is the same for the two systems.

Power Absorbed in Ohm's Law Heating. The power absorbed in an element of volume dv is $\mathbf{i}^2 dv/\sigma$. The total power absorbed in Ohm's law heating is P_o, where

$$P_o = \int \frac{\mathbf{i}^2}{\sigma} dv_1,$$

the integral being extended over all the conductors of the apparatus. Since $\mathbf{i}_2 = n^2\mathbf{i}_1$, and $dv_2 = dv_1/n^3$, and at corresponding places $\sigma_2 = n\sigma_1$

$$(P_o)_2 = \int \frac{\mathbf{i}_1{}^2 dv_2}{\sigma_2} = \frac{n^4}{n^3 n} \int \frac{\mathbf{i}_1{}^2 dv_1}{\sigma_1} = (P_o)_1.$$

Hence the power absorbed in Ohm's law heating is the same for the two systems.

Power Absorbed in Back-Bombardment or at the Anode. If dS is an element of the cathode surface, the power P_b absorbed in back-bombardment is given by

$$P_b = \int N u_n W dS,$$

where N is the number of electrons per cubic centimeter at a point near the cathode, W is the energy of an electron, u_n is the velocity of the electrons normal to the surface and where the integration is extended all over the cathode. Now

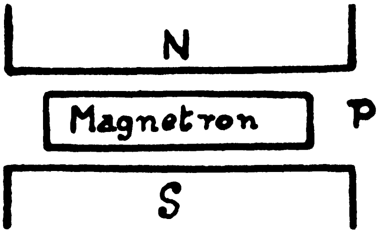


FIG. 2.

$$Neu_n = i_n;$$

thus

$$P_b = \frac{1}{e} \int i_n W dS.$$

Now since W depends only upon $\mathbf{u}$, it is the same for both systems. Hence, since $dS_2 = dS_1/n^2$, and $i_{n2} = n^2 i_{n1}$,

$$(P_b)_2 = \frac{1}{e} \int i_{n2} W_2 dS_2 = \frac{n^2}{n^2} \int i_{n1} W_1 dS_1 = (P_b)_1.$$

Hence the power absorbed in back-bombardment is the same in the two systems. A similar result holds for anode power absorption.

Comment Regarding the Foregoing. In spite of the fact that the input power, the power radiated, the power lost in Ohm's law heating, the power lost in back-bombardment, and the power lost at the anode are the same for the two systems above studied, we have to observe that the means of getting rid of the power lost in Ohm's law heating, in back-bombardment, and at the anode present a more serious problem in the case of the small magnetron than in that of the large one.

Inferences Regarding the Case Where the Electrical Conductivities are the Same in the Two Systems. In the case of the smaller system of the above problems the conductivity is larger than that for the larger system at corresponding points. If it is now made the same, all other *assigned* conditions being maintained, it is obvious that all currents will be reduced and the radiation will be reduced. If we now raise V so as to bring the power radiated from the antenna back to what it was before, we shall increase all the currents in the conductors and so the Ohm's law heat dissipated in them. The Ohm's law dissipation will therefore be no longer equal to the power radiated but will be greater. We thus conclude that for magnetron assemblies of the same material, in so far as Ohm's law losses are important at all, they are more important in relation to the energy radiated for small magnetrons than for large ones.

APPENDIX.

Generalization of the Foregoing Scaling Theorem. Reverting to the remark in footnote (5), it will be seen that had we replaced Eq. 5 by the more general form $\mathbf{i} = \sigma(\mathbf{E} + [\omega\mathbf{H}]/c)$, and observing that $\mathbf{i} = \omega\rho$ in the wire, we could write the equation as $\mathbf{i} = \sigma(\mathbf{E} + [\mathbf{iH}]/\rho c)$, and in the light of (8), (9), and (10), it will be observed that this more general equation scales properly, just as (5) does. Of course, the scaling of ρ in the wire is bound up with the scaling of σ and of the dimensions of the conductor.

We may proceed to even greater generality along the following lines. We may replace (1) to (4) and (5) by the relativistically invariant macroscopic equations [7]

$$\frac{1}{c}\left(\mathbf{i} + \rho\mathbf{w} + \frac{\partial \mathbf{D}}{\partial t}\right) = \operatorname{curl} \mathbf{H}, \tag{1.1}$$

$$\rho = \operatorname{div} \mathbf{D}, \tag{2.1}$$

$$-\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = \operatorname{curl} \mathbf{E}, \tag{3.1}$$

$$0 = \operatorname{div} \mathbf{B}, \tag{4.1}$$

$$\mathbf{i} = \frac{\sigma}{(1 - \mathbf{w}^2/c^2)^{\frac{1}{2}}}\left\{\left(\mathbf{E} + \frac{[\mathbf{wB}]}{c} - \frac{\mathbf{w}(\mathbf{Ew})}{c^2}\right)\right\}, \tag{5.1}$$

---

[7] See, for example, E. CUNNINGHAM, "The Principle of Relativity," Cambridge University Press (1914), p. 121.

together with the relativistically invariant relations

$$\mathbf{B} - \frac{[\mathbf{wE}]}{c} = \mu\left(\mathbf{H} - \frac{[\mathbf{wD}]}{c}\right)$$

$$\mathbf{D} + \frac{[\mathbf{wH}]}{c} = k\left(\mathbf{E} + \frac{[\mathbf{wB}]}{c}\right)$$

where $\mathbf{w}$ is the velocity of the medium, μ and k are constants, or at any rate quantities invariant under the transformation (7) and the restricted relativity transformation.

We also replace (6) by

$$\frac{d}{dt}(m\mathbf{u}) = e\left(\mathbf{E} + \frac{[\mathbf{uB}]}{c}\right) \tag{6.1}$$

where $\mathbf{u}$ is the velocity of the electron.

It then becomes immediately obvious along the lines of the main paper that (14) holds and the equations (1.1) to (6.1) assume the same form in $\mathbf{E}_2$, $\mathbf{H}_2$, $\mathbf{D}_2$, $\mathbf{B}_2$, $\mathbf{u}_2$, ρ_2, $\mathbf{i}_2$, as in $\mathbf{E}_1$, $\mathbf{H}_1$, $\mathbf{D}_1$, $\mathbf{B}_1$, $\mathbf{u}_1$, ρ_1, $\mathbf{i}_1$, provided that

$$\begin{array}{lll}
\mathbf{E}_2 = \mathbf{E}_1/\epsilon; & \mathbf{H}_2 = \mathbf{H}_1/\epsilon; & \\
\mathbf{D}_2 = \mathbf{D}_1/\epsilon; & \mathbf{B}_2 = \mathbf{B}_1/\epsilon; & \\
\rho_2 = \rho_1/\epsilon^2; & \mathbf{i}_2 = \mathbf{i}_1/\epsilon^2; & \\
\mathbf{w}_2 = \mathbf{w}_1; & \mathbf{u}_2 = \mathbf{u}_1; & \sigma_2 = \sigma_1/\epsilon; \\
V_2 = V_1; & \mathbf{H}_{m2} = \mathbf{H}_{m1}/\epsilon. &
\end{array}$$

It immediately follows that the results cited under "Power Comparisons" also hold. In this connection, it is perhaps desirable to treat the power absorbed in Ohm's law heating rather differently, as follows: The power absorbed in an element of volume dv is $(\mathbf{Ei})dv$. This includes power absorbed in Ohm's law heating in the element and power ultimately radiated. In view of the scaling relations, this power is the same in the scaled and unscaled systems. Now the Poynting flux from the element dv is c div $[\mathbf{EH}]dv$. In view of the scaling relations, this is the same in the scaled as in the unscaled system. Hence the difference between this quantity and $(\mathbf{Ei})dv$ cited above, which difference represents the energy dissipated as heat within dv, is the same in the scaled as in the unscaled system.

Valuable Oil From Weed Seed.—The possibility of making a valuable oil from the seed of pennycress, a European weed common in middle western and western States, has been suggested.

This weed, also called fan-weed, French-weed, dish mustard, treacle-wort, and bastard cress, favors grasslands, gardens, and waste places, and is a serious nuisance in grain fields. It is a major constituent of grain screenings in some areas. Its seeds yield about a third of their weight in oil; the residual oil-free meal has a protein content higher than that of undecorticated cottonseed meal obtained by mechanical expression which contains about 28 per cent protein.

The potential value of pennycress oil naturally lies in its close resemblance to oils of rape and mustard, which are valued ingredients of certain lubricants, and are employed in machinery operating under heavy loads at high temperatures such as marine and Diesel engines. Rape oil is also used in producing factice, a rubber substitute. Domestic supplies of such oils are insufficient, and the U. S. imports as much as 15,000 tons of rapeseed oil annually. Under irrigation, as high as 1,500 lb. of seed per acre have been obtained, and without irrigation, up to 500 lb. per acre.

Both rape and mustard oils are used in some food products, and it is probable that with proper refinement, pennycress oil is edible. The oil is semi-drying and would be excellent as a machinery lubricant and for bearing surfaces subject to the flushing action of water. The meal has a flavor of combined garlic and mustard. Thus this research suggests that a common nuisance may become a practical raw material for the production of a valuable vegetable oil, a feed material, and perhaps a condiment. Moreover, it is possible that investigation may reveal other weed seeds that also yield useful oils.

R. H. O.

Two and a Half Acres Per Stomach.—Productive land is "the master key to peace among peoples," says Dr. H. H. Bennett of the U. S. Department of Agriculture in a recent address in which he emphasized that misuse of land does not stop at any nation's boundaries, and that conservation is a world-wide task.

Developing this theme, he "took a look" at the situation in the world and in the United States. The survey was based on nutritional studies that show "we need at least 2½ acres per capita of good productive land to furnish each person with a minimum nutritious diet."

In the world, he finds, there are only about 4 billion acres of immediately arable land for a world population of more than 2 billion people, which is increasing at a rate of about 20 million persons a year. On the 2½ acre per capita basis there is already a world shortage, he finds.

In the United States the situation is somewhat better. The U. S. today has about 460 million acres of high grade cropland for a population of 144 million—about 3 acres per capita. But population is continuing to increase—by about 2.2 millions in 1946. The margin above the 2½ acres is getting narrower both as a result of increasing numbers and because of destruction of good land by misuse, particularly by erosion.

Many parts of the world are already beyond the borderline of this safe ratio of land to population. For the United States, says Bennett, "The conclusion is plain: Time and land are running out, and we dare not delay for a single unnecessary reason in safeguarding the productive land which we have left."

R. H. O.

METHOD OF SOLVING DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS.

BY

W. V. LYON.[1]

The following method of solving ordinary differential equations with constant coefficients employs only well-known classical principles and consequently should be of interest to those who have not taken time to become familiar with one of the operational methods of solving these equations.

There are three distinct parts to the classical solution:

1. Finding the particular integral. This is adequately treated in any good text on differential equations.
2. Finding the algebraic roots of the characteristic equation. This is a necessary part of any method of solution and is not considered here.
3. Finding the coefficients in the complementary function.

It is the purpose of this note to show how a general expression can be developed for these coefficients from which their numerical values are readily computed.

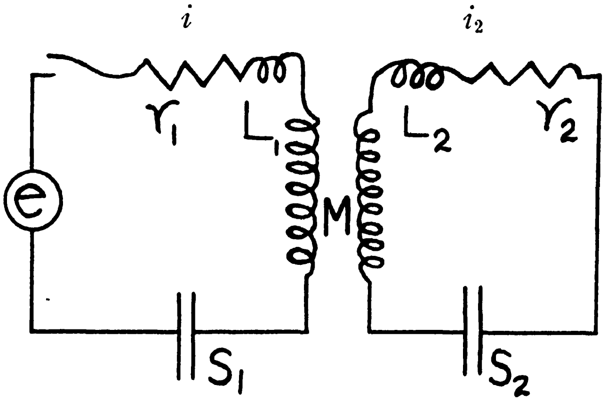


FIG. 1.

Consider the electrical circuit of Fig. 1 on which an emf. e is impressed at time $t = 0$. The differential equation which applies to this circuit is of the fourth order and the general solution for the current i is

$$i = u + C_1\epsilon^{p_1t} + C_2\epsilon^{p_2t} + C_3\epsilon^{p_3t} + C_4\epsilon^{p_4t}, \qquad (1)$$

where u is the particular integral and the roots $p_1, \cdots p_4$ of the characteristic equation are all different. At $t = 0$

---

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

$$C_1 + C_2 + C_3 + C_4 = i_0 - u_0, \tag{2}$$

where the zero subscript indicates the value is at $t = 0$. Differentiate Eq. 1 and obtain at $t = 0$

$$p_1C_1 + p_2C_2 + p_3C_3 + p_4C_4 = i_0' - u_0', \tag{3}$$

where the prime indicates the derivative.

Differentiate a second and third time and obtain at $t = 0$

$$p_1^2C_1 + p_2^2C_2 + p_3^2C_3 + p_4^2C_4 = i_0'' - u_0'', \tag{4}$$

$$p_1^3C_1 + p_2^3C_2 + p_3^3C_3 + p_4^3C_4 = i_0''' - u_0'''. \tag{5}$$

Eliminate C_4 from these equations in the following way:

Multiply Eq. 2 by p_4 and subtract from Eq. 3
Multiply Eq. 3 by p_4 and subtract from Eq. 4
Multiply Eq. 4 by p_4 and subtract from Eq. 5

Proceed in a similar manner and eliminate C_3 and then C_2. In the final equation for C_1 introduce the relations that exist among the roots of the characteristic equation and its coefficients. If the characteristic equation is

$$p^4 + ap^3 + bp^2 + cp + d = 0,$$

the sum of the roots equals $(-a)$, the sum of the products taken two at a time equals b, etc.

The expression for C_1 becomes

$$C_1 = \frac{\begin{array}{c}(i_0''' - u_0''') + (i_0'' - u_0'')(p_1 + a) + (i_0' - u_0') \\ \times (p_1^2 + ap_1 + b) + (i_0 - u_0)(p_1^3 + ap_1^2 + bp_1 + c)\end{array}}{(p_1 - p_2)(p_1 - p_3)(p_1 - p_4)}. \tag{6}$$

This expression for the complementary coefficient depends upon the initial values and the initial derivatives of the dependent variable, i, and the particular integral, u. When these values are given or are readily computed, Eq. 6 can be used for computing the coefficients of the complementary function. However, the numerator of Eq. 6 can be converted into other forms which are more convenient to use in most practical problems.

The fourth order equation of which Eq. 1 is the solution is derived from two or more simultaneous equations. For example, the Kirchhoff equations which apply to the relaxed circuit of Fig. 1 are

$$(r_1 + L_1D + S_1D^{-1})i + MDi_2 = e, \tag{7}$$

$$MDi + (r_2 + L_2D + S_2D^{-1})i_2 = 0, \tag{8}$$

where $D = \frac{d}{dt}$ and $D^{-1}i = \int_0^t i\,dt$. In general if a capacitor has an initial potential v_0, the subsequent value of the potential is $v_0 + SD^{-1}i$. Integrate both sides of the Kirchhoff equations, noting that, in general, $D^{-1}Di = \int_0^t \frac{di}{dt}dt = i - i_0$.

Since the circuit is relaxed the integrated equations are

$$(r_1D^{-1} + L_1 + S_1D^{-2})i + Mi_2 = D^{-1}e, \tag{9}$$

$$Mi + (r_2D^{-1} + L_2 + S_2D^{-2})i_2 = 0. \tag{10}$$

The current i_2 can be eliminated from these equations by treating the integrating operator D^{-1} as an algebraic quantity. Thus the integrated equation for i is

$$\begin{vmatrix} (r_1D^{-1} + L_1 + S_1D^{-2}), & M \\ M, & (r_2D^{-1} + L_2 + S_2D^{-2}) \end{vmatrix} i = \begin{vmatrix} D^{-1}e, & M \\ 0, & (r_2D^{-1} + L_2 + S_2D^{-2}) \end{vmatrix}$$

Expand both determinants and divide by the constant term in the coefficient of i, which in this case is $(L_1L_2 - M^2)$. The integrated equation can be written in the form

$$(1 + aD^{-1} + bD^{-2} + cD^{-3} + dD^{-4})i = f(D^{-1})e, \tag{11}$$

where $f(D^{-1})$ here equals $\frac{(r_2D^{-1} + L_2 + S_2D^{-2})}{L_1L_2 - M^2}D^{-1}$. The fourth derivative of Eq. 11 is the differential equation of the system

$$(D^4 + aD^3 + bD^2 + cD + d)i = f''''(D^{-1})e, \tag{12}$$

where $f''''(D^{-1})$ here equals $\frac{(r_2D^2 + L_2D^3 + S_2D)}{L_1L_2 - M^2}$. The characteristic equation is

$$p^4 + ap^3 + bp^2 + cp + d = 0. \tag{13}$$

In Eq. 6 first consider the part of C_1 which depends upon i and its derivatives at $t = 0$. Note that at $t = 0$, $D^{-1}i = (D^{-1}i)_0 = \int_0^0 i\,dt = 0$. Also as an abbreviation let $f(D^{-1})e = \phi$. At $t = 0$, Eq. 11 reduces to

$$(i)_0 = (\phi)_0. \tag{14}$$

Differentiate Eq. 11 and obtain at $t = 0$

$$(Di)_0 + a(i)_0 = (\phi')_0. \tag{15}$$

Differentiate a second and third time and obtain

$$(D^2i)_0 + a(Di)_0 + b(i)_0 = (\phi'')_0, \tag{16}$$

$$(D^3i)_0 + a(D^2i)_0 + b(Di)_0 + c(i)_0 = (\phi''')_0. \tag{17}$$

Multiply Eq. 14 by p^3, Eq. 15 by p^2, Eq. 16 by p and add their sum to Eq. 17. It is then seen that the part of Eq. 6 which depends upon i and its derivatives becomes

$$(\phi''')_0 + p_1(\phi'')_0 + p_1^2(\phi')_0 + p_1^3(\phi)_0.$$

Next consider the part of Eq. 6 which depends upon the particular integral u and its derivatives.

Subtract the characteristic Eq. 13 from the differential Eq. 12 and obtain

$$\{(D^4 - p^4) + a(D^3 - p^3) + b(D^2 - p^2) + c(D - p)\}u = \phi''''. \qquad (18)$$

$(D - p)$ is a factor of the left-hand side of Eq. 18 and hence the equation can be written:

$$(D - p)\left\{\begin{matrix} D^3 + pD^2 + p^2D + p^3 \\ + a(D^2 + pD + p^2) \\ + b(D + p) \\ + c \end{matrix}\right\} u = \phi''''$$

or

$$\{u''' + (p + a)u'' + (p^2 + ap + b)u' + (p^3 + ap^2 + bp + c)u\} = \frac{1}{D - p}\phi''''. \qquad (19)$$

Consequently the part of Eq. 6 which depends upon the particular integral and its derivatives at $t = 0$ is:

$$-\left[\frac{1}{D - p_1}\phi''''\right]_0$$

Therefore Eq. 6 can be written

$$C_1 = \frac{(\phi''')_0 + p_1(\phi'')_0 + p_1^2(\phi')_0 + p_1^3(\phi)_0 - \left[\dfrac{1}{D - p_1}\phi''''\right]_0}{(p_1 - p_2)(p_1 - p_3)(p_1 - p_4)}. \qquad (20)$$

In Eq. 20, ϕ is obtained from the integrated forms of the differential equations and in general will depend in part upon the initial values of the kinetic and potential energies. If the circuit is relaxed, as is assumed in Fig. 1, ϕ depends only upon the circuit constants and the applied emf. and consequently the fourth order integrated equation has the form

$$(1 + aD^{-1} + bD^{-2} + cD^{-3} + dD^{-4})i = (G_0 + G_1D^{-1} + G_2D^{-2} + G_3D^{-3} + G_4D^{-4})e. \qquad (21)$$

That is,

$$\phi = (G_0 + G_1D^{-1} + G_2D^{-2} + G_3D^{-3} + G_4D^{-4})e. \qquad (22)$$

At $t = 0$

$$\phi_0 = G_0(e)_0. \qquad (23)$$

Differentiate Eq. 22 and obtain at $t = 0$

$$(\phi')_0 = G_0(De)_0 + G_1(e)_0. \tag{24}$$

Differentiate a second and third time and obtain at $t = 0$

$$(\phi'')_0 = G_0(D^2e)_0 + G_1(De)_0 + G_2(e)_0, \tag{25}$$

$$(\phi''')_0 = G_0(D^3e)_0 + G_1(D^2e)_0 + G_2(De)_0 + G_3(e)_0. \tag{26}$$

Differentiate a fourth time and obtain

$$\phi'''' = G_0D^4e + G_1D^3e + G_2D^2e + G_3De + G_4e. \tag{27}$$

The value of $\frac{1}{D-p}\,\phi''''$ can be obtained by operating on the individual terms in Eq. 27.

Fifth term,

$$G_4\frac{1}{D-p}\,e = G_4\epsilon^{pt}\int \epsilon^{-pt}e\,dt. \tag{28}$$

Fourth term,

$$G_3\frac{1}{D-p}\,De = G_3\epsilon^{pt}\int \epsilon^{-pt}De\,dt.$$

Integrate by parts.

$$G_3\frac{1}{D-p}\,De = G_3\left(e + p\frac{1}{D-p}\,e\right). \tag{29}$$

Third term,

$$G_2\frac{1}{D-p}\,D^2e = G_2\left(De + pe + p^2\frac{1}{D-p}\,e\right). \tag{30}$$

Second term,

$$G_1\frac{1}{D-p}\,D^3e = G_1\left(D^2e + pDe + p^2e + p^3\frac{1}{D-p}\,e\right). \tag{31}$$

First term,

$$G_0\frac{1}{D-p}\,D^4e = G_0\left(D^3e + pD^2e + p^2De + p^3e + p^4\frac{1}{D-p}\,e\right). \tag{32}$$

If Eqs. 23, 24, 25, 26 and $\left[\frac{1}{D-p}\,\phi''''\right]_0$ are substituted in the numerator of Eq. 20

$$C_1 = \frac{(G_0p_1^4 + G_1p_1^3 + G_2p_1^2 + G_3p_1 + G_4)\left[-\frac{1}{D-p_1}\,e\right]_0}{(p_1 - p_2)(p_1 - p_3)(p_1 - p_4)}. \tag{33}$$

If Eq. 21 is differentiated four times the result is the differential equation of the system. The right-hand side of this differential equation is

$$(G_0D^4 + G_1D^3 + G_2D^2 + G_3D + G_4)e. \tag{34}$$

Therefore the coefficient of $\left[-\frac{1}{D - p_1} e \right]_0$ in Eq. 33 is found by replacing the differential operator D in the right-hand side of the differential equation by p_1.

In the following summary the values of the particular integral and $\left[-\frac{1}{D - p} e \right]_0$ are given for a few common values of the applied emf.

SUMMARY.

An emf. e is applied to a relaxed system the differential equation for which is

$$F(D)i = f(D)e.$$

The characteristic equation is

$$F(p) = 0,$$

and the roots, all different, are $p_1, \cdots p_n$.

The first term in the complementary function is

$$C_1\epsilon^{p_1t} = \frac{f(p_1)\left(-\frac{1}{D - p_1} e \right)_0}{(p_1 - p_2) \cdots (p_1 - p_n)} \epsilon^{p_1t}.$$

e	$\left[-\frac{1}{D - p} e \right]_0$	Particular integral
1	$\frac{1}{p}$	$\frac{f(0)}{F(0)}$
$\epsilon^{\alpha t}$	$\frac{1}{p - \alpha}$	$\frac{f(\alpha)}{F(\alpha)} \epsilon^{\alpha t}$
$\epsilon^{j\omega t}$	$\frac{1}{p - j\omega}$	$\frac{f(j\omega)}{F(j\omega)} \epsilon^{j\omega t}$
$\cos \omega t$	$\frac{p}{p^2 + \omega^2}$	$R_e\left(\frac{f(j\omega)}{F(j\omega)} \epsilon^{j\omega t} \right)$
$\sin \omega t$	$\frac{\omega}{p^2 + \omega^2}$	$R_e\left(-j \frac{f(j\omega)}{F(j\omega)} \epsilon^{j\omega t} \right)$
$t\epsilon^{\alpha t}$	$\frac{1}{(p - \alpha)^2}$	$\left(\frac{f(\alpha)}{F(\alpha)} \right) t\epsilon^{\alpha t} + \left(\frac{\partial}{\partial \alpha} \frac{f(\alpha)}{F(\alpha)} \right) \epsilon^{\alpha t}.$

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ELECTRON OPTICAL SCHLIEREN EFFECT.

Electron-microscope experiments conducted by Dr. L. L. Marton of the National Bureau of Standards have developed, in the electron optical analogy to the Schlieren effect, a valuable tool for the quantitative study of electrostatic or magnetic fields that are not susceptible to any other type of investigation. By means of the new technique, a dark-field image of magnetic or electric fields occurring between an electron source and a magnetic lens is formed beyond the focal plane of the lens. In this way it has been possible to obtain a visual representation of fringe fields from the small domains of spontaneous magnetization in ferromagnetic materials. The characteristics of such a pattern may then be used to compute the field distribution in the region under study. Extension of the principle provides a powerful means of broadening present knowledge concerning space-charge fields, fields produced by contact potentials, wave-guide problems, and the microstructure of metals.

The Schlieren method as applied to light rays involves the formation of an image of a light source on a convenient stop which intercepts all direct rays. If in the space between the source and lens there is a variation of the index of refraction, an image of that inhomogeneity will then be produced by means of the same lens in a conjugate plane beyond the stop. Thus a dark-field image of the variation in optical density of the refractive medium is obtained.

This method was adapted to electron-optical investigations at the Bureau in connection with the problem of measuring the magnetization of ferromagnetic wire used in the reproduction of sound and for recording data in electronic computing machinery. A magnetized wire was placed approximately 3 cm. from a magnetic lens having a free bore of about 16 mm. and was irradiated with an electron beam accelerated by a difference of potential of 40 kv. After a magnified image of the wire was properly focused on a fluorescent screen, the direct rays were intercepted by a center stop about 1 mm. in diameter. When the stop was placed partially across the visual field, a partly bright-field, partly dark-field image of the wire was obtained, in which the Schlieren image of the magnetic fields extending on both sides of the wire was readily recognizable. Thus, in addition to electron emission and scattering, a new category of "objects"—namely, field intensity patterns—was made available for study by means of electron optical systems.

---

* Communicated by the Director.

For a representation of ferromagnetic domains, experiments were carried out with thin laminar steel provided with a fine feather edge. After the samples were magnetized to saturation parallel to the edge, they were placed in an electron microscope, and the objective lens was slightly misaligned in order to bring in one edge of the objective aperture parallel to the observed edge. In this way the direct rays were intercepted by the objective aperture, and only the electrons scattered or deflected at the edge of the specimen reached the final image plane to form a dark-field image.[1] The image consisted of a bright line interrupted at irregular intervals averaging a few tenths of a micron. Whereever the line was interrupted, a faint pattern was visible at right angles to the edge. This pattern was assumed due to the fringe field of the ferromagnetic domains or of grain boundaries. Such a pattern is produced only if the average thickness of the specimen edge in the direction of the electron beam is no greater than one domain. This observation was repeated with the objective aperture at an angle to the observed edge of the sample. Part of the image was thus dark-field, and part was a bright-field image of the magnetized edge. By overexposing the bright-field portion, a fringe pattern at right angles to the edge corresponding to that of the dark-field image was observed in the bright-field image.

Perhaps the greatest value of the electron-optical Schlieren method lines in its utility for exploring complex electric and magnetic fields of extremely small dimensions or in which a probe of size greater than the electron would disturb the field under study. In the past, calculations of the field intensity at a point have been limited to those special cases in which the geometry of the field exhibits a high degree of symmetry. However, once images of the field have been obtained by the Schlieren method, the actual intensity in the neighborhood of a sample of any shape can be computed to a good approximation from the intensity distribution of the pattern or from the apparent displacement of a deflected image. The mathematical basis for these calculations has been worked out by S. H. Lachenbruch of the National Bureau of Standards.

For example, in a study of the field intensity about a magnetized wire the field may be assumed to be approximately that of an ideal magnetic dipole. If this assumption is made, an expected Schlieren intensity distribution based upon the theoretical trajectories of electrons approaching the wire in the midplane of a dipole can be calculated. Such a distribution may then be compared with the actual intensity variation of the Schlieren image in a line perpendicular to the image of the wire and bisecting the dipole pattern. In this way a better approximation to the field intensity in the midplane is obtained.

---

[1] The micrographs were made with the cooperation of Max Swerdlow of the Bureau's Electron Microscopy Laboratory.

The use of wave guides as conductors and circuit elements in ultra-high frequency radar and communication—and more recently in some experiments in nuclear physics—often leads to arrangements whose geometry is too complicated for expression in any known system of mathematical coordinates. The electronics engineer, having in many cases only an intuitive picture of the field distribution in such parts of the guide as elbows, must rely on empirical methods in designing wave-guide techniques and equipment. Experiments being undertaken at the Bureau by D. L. Reverdin are expected to provide a method utilizing the electron-optical Schlieren effect that may permit the calculation of field intensities in regions of a guide that are not at present susceptible to analytical treatment.

Ferromagnetism offers unique possibilities for investigation of the structure and elastic properties of metals, and it has been suggested that the electron-optical Schlieren effect may prove useful in metallurgical problems of this kind. Since the direction of magnetization in a single domain of a metallic specimen—and thus the magnetic properties of the specimen as a whole—are determined by the regularity and orientation of the atomic arrangement and the interaction between electrons of neighboring atoms, any factor tending to disturb these conditions must seriously affect the ferromagnetic properties. Magnetic data subject to reliable interpretation should thus provide valuable information on metallic purity, grain orientation, and strains caused by precipitation, grain boundaries, and unequal heating or cooling. Experiments now being planned at the National Bureau of Standards include a study of the behavior of the fringe field patterns as a function of temperature up to the Curie point (at which temperature ferromagnetic substances become paramagnetic) and repetition of these measurements on a single crystal material having very large domains. An extension to ferroelectric materials is also contemplated for the purpose of checking the domain theory of such materials.

CONTROL OF FADING LAMPS WITH LIGHT-SENSITIVE PAPER.

A practicable, inexpensive, and yet reliable means of controlling the performance of fading lamps has been developed in the Bureau's Textile Section as a solution to problems arising from the lack of reproducibility among such lamps. This is accomplished by the use of a special light-sensitive paper, which, when placed in the lamp with commercial materials being tested, will fade to an endpoint that can be visually matched with a standard. A very simple method of control independent of the unavoidable variation of radiant energy output of fading lamps is thus provided.

The use of arc lamps to test the light-stability of commercial products has become accepted procedure in many branches of industrial production and research. It is estimated that between one and two thousand irradiation units are in practically constant operation. The results are often used to assess the relative, and even absolute, light-stability of materials used in very large-scale applications. It is therefore important that such arc lamps operate reliably.

With increasing use of irradiating units, however, it has become apparent that these arcs do not function reproducibly from unit to unit or in a given unit from day to day. As a result of recent surveys it has been determined that the variations in performance of the same make of lamp are often quite large. Among textile fading lamps, which can be considered typical, some of the weakest lamps required from two to three times as long to produce the same fading as the strongest lamps. Over one-third of the lamps included in the group were found to deviate by more than 15 per cent from a mean value.

There are a variety of explanations for such behavior but the most generally applicable one is variation of line voltage. The radiant energy output in the 300 to 400 millimicron range, which is perhaps for most applications the most effective region of the glass enclosed arc, is decreased 11 per cent by a 4 per cent drop in line voltage. Voltage variations at most substations in the District of Columbia are about 4 per cent, which are probably smaller than in most communities because of the limited amount of heavy manufacturing in the District. To this must be added the variable voltage drop in the mains, transformers, and individual plant wiring which may be as high as 11 per cent. It is thus apparent that voltage difficulties can account for very large differences in lamp performance. Although the irradiating units are provided with transformer or rheostat taps for matching line voltages, the latter are difficult to measure without a recording voltmeter, and even then are subject to change after measurement. The taps, furthermore, may not adequately match the mean line voltage.

It is obvious that solving line voltage problems as a means of securing uniform arc lamp behavior would be difficult. An attempt has therefore been made to develop a simple, reliable method of gaging the performance of arc lamps by the use of a light-sensitive paper.

These papers are permanent in the dark, and can be produced in large uniform batches. In use, a piece of the paper is placed in the lamp along with the materials to be irradiated, and exposure is continued until the test paper matches a standard exposure of, for example, 20 "Standard Fading Hours," regardless of the actual time elapsed.

The "standard fading hour" now under trial was recommended by the committees on colorfastness to light of textiles of the American Association of Textile Chemists and Colorists, and of the American Society for Testing Materials. It is considered to be in agreement with the "Fade-Ometer hour" used by the dyestuff and textile industries for a number of years for rating colorfastness to light of dyed textiles.

The newer "Fade-Ometers" are somewhat faster in their fading action than those on which the concept of the "Fade-Ometer hour" was developed.

The standard exposures are produced from the same batch of paper by exposing it to the Bureau's standard fading lamp. This lamp has been developed as a means of obtaining accurately measured, definite dosages of radiant energy under reproducible conditions. The light-quantities are measured with "photon counters," light-integrating meters adapted for high-intensity sources. The standard exposures thus produced are permanent in the dark and change negligibly during a year's normal use. They are mounted in strip form in a protecting booklet along with plus and minus 10 per cent steps, to allow the user to estimate the degree of overexposure or to anticipate the end of the period.

The use of light-sensitive paper should make it possible to set up more rigid specifications of light-stability for all materials where this property is important. The work of the Bureau is still in the experimental stage. Neither the paper thus far developed nor the lamp used in standardization is considered to be the final form of this method for lamp control. In fact, there have been requests from diverse industries for papers useful for controlling exposures over much shorter and longer periods of time, and for use in lamps with water spray. The experience gained from this work has pointed the way toward more sensitive papers of wider scope. Papers useful over exposure periods lasting a few hours to a week are now under development and the radiometric technique is being improved.

ZEEMAN EFFECT OF NEUTRAL NITROGEN AND OXYGEN.

Excited atoms of the chemical elements emit radiations of various frequencies depending on the initial and final energy states between which the radiative process occurs. The total number of radiations that an atom is capable of emitting is its spectrum, each individual radiation being called a spectrum line. When the radiative process occurs in a magnetic field the energy states of the atom become complex so that previously single lines now are separated into an array of closely spaced lines of different intensities, symmetrically grouped about the undisturbed position of the line. This array, or magnetic pattern, known as the Zeeman effect, uniquely identifies the energy levels between which the radiative transition occurs. It furnishes for each energy level a numerical quantity called its *g*-value or splitting factor.

When the Zeeman patterns of such metals as molybdenum, tantalum, and manganese, are photographed in the red and infrared regions of the spectrum, there usually appear also on the spectrograms the patterns of atmospheric nitrogen and oxygen. The relative positions and estimated intensities in the magnetic patterns of these lines have now been determined for the first time from precise measurements of spectrograms by Dr. C. C. Kiess of the National Bureau of Standards and Dr. George Shortley of the Ohio State University.

Because the multiplet levels of these elements are very close to each other, the effect of the magnetic field is to displace the component levels from their positions of symmetry about the undisturbed position by amounts depending on numerical quantities, or quantum numbers, associated with the levels. As a consequence of this influence, known as the Paschen-Back interaction, the magnetic patterns of the lines are distorted out of their patterns of symmetry. In the case of nitrogen the distortions are slight; but in the case of oxygen they are so pronounced that the pattern bears no resemblance either to that of a weak field or to the triplet that results from complete Paschen-Back interaction.

The interpretation of the measurements of Kiess and Shortley has afforded an interesting application of the quantum theory to the elucidation of the Paschen-Back effect. The *g*-values derived for the nitrogen and oxygen energy levels are the first to be announced for neutral atoms of atomic number less than 10, or neon, and are found to conform with those required theoretically for the lighter chemical elements.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays, and Fridays from 9 A.M. until 5 P.M., Thursdays from 2 P.M. until 10 P.M. and Saturdays from 9 A.M. until 12 noon.

RECENT ADDITIONS.

AERONAUTICS.

GRAY, GEORGE W. Frontiers of Flight. 1948.
THEODORSEN, THEODORE. Theory of Propellers. 1948.

ARCHITECTURE AND BUILDING.

MOON, PARRY H. AND SPENCER, DOMINA E. Lighting Design. 1948.
SHOEMAKER, RICHARD W. Radiant Heating. 1948.

ASTRONOMY.

THOMPSON, ALLYN J. Making Your Own Telescope. 1947.

BACTERIOLOGY.

HAMMER, BERNARD WERNICK. Dairy Bacteriology. Third Edition. 1948.
SALLE, A. J. Fundamental Principles of Bacteriology. 1948.

BIBLIOGRAPHY.

UNITED STATES. NATIONAL ADVISORY COMMITTEE ON AERONAUTICS. Index of N.A.C.A. Technical Publications. 1948.

BIOGRAPHIES.

ARCHIBALD, RAYMOND CLARE. Mathematical Table Makers. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BAILEY, ALTON EDWARD. Cottonseed and Cottonseed Products. 1948.
BERGMANN, ERNST DAVID. The Chemistry of Acetylene and Related Compounds. 1948.
BOGUE, ROBERT HERMAN. Chemistry of Portland Cement. 1947.
CLARK, W. MANSFIELD. Topics in Physical Chemistry. 1948.
DE MENT, JACK AND DAKE, H. C. Handbook of Uranium Minerals. 1947.
DYSON, GEORGE MALCOLM. A New Notation and Enumeration System for Organic Compounds. 1947.
GARDNER, WILLIAM. Chemical Synonyms and Trade Names. Fifth Edition. 1948.
HICKINBOTTOM, W. J. Reactions of Organic Compounds. Second Edition. 1948.
HILDITCH, T. P. The Chemical Constitution of Natural Fats. Second Edition. 1947.
HOUWINK, R. Technology of Synthetic Polymers. 1947.
LONG, J. S. AND ANDERSON, H. V. Chemical Calculations. Fifth Edition. 1948.

Newer Methods of Preparative Organic Chemistry. 1948.
RALSTON, A. W. Fatty Acids and Their Derivatives. 1948.
SHEELY, M. L. Production of Synthetic Fatty Acids. No Date.
SNELL, F. D. AND SNELL, C. T. Colorimetric Methods of Analysis. 1948.
THEILHEIMER, W. Synthetic Methods of Organic Chemistry. Volume 1. 1948.
TYLER, CHAPLIN. Chemical Engineering Economics. Third Edition. 1948.
WILLIAMS, HERBERT E. Cyanogen Compounds. Second Edition. 1948.

DICTIONARIES.

ZIMMERMAN, O. T. AND LAVINE, IRVIN. Scientific and Technical Abbreviations, Signs and Symbols. 1948.

EDUCATION.

AMERICAN COUNCIL ON EDUCATION. American Universities and Colleges. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

DAS, M. M. Electricity. 1948.
FORSYTHE, WILLIAM E. AND ADAMS, ELLIOT Q. Fluorescent and Other Gaseous Discharge Lamps. 1948.
GLASOE, G. N. AND LEBACQZ, JEAN V. Pulse Generators. 1948.
KIMBARK, EDWARD WILSON. Power System Stability. Volume 1. 1948.
UNITED STATES. WAR DEPARTMENT. Electrical Communication Systems Engineering. 1945.
VEINOTT, CYRIL A. Fractional Horsepower Electric Motors. Second Edition. 1948.

ELECTRONICS.

GABOR, DIONYS. The Electron Microscope. 1948.
GREENWOOD, IVAN A., JR.; HOLDAM, J. VANCE, JR.; AND MACRAE, DUNCAN, JR. Electronic Instruments. 1948.
KIVER, MILTON S. Television Simplified. Second Edition. 1948.
LOVELL, BERNARD. Electronics and Their Application in Industry and Research. 1947.
WEINBACH, MENDELL P. Electric Power Distribution. 1948.
ZELUFF, VIN AND MARKUS, JOHN. What Electronics Does. 1948.

ENGINEERING.

BATLEY, R. L. AND BARBER, E. GORDON. Boiler Plant Technology. Second Edition. 1947.
CISSEL, JAMES H. Stress Analysis and Design of Elementary Structures. Second Edition. 1948.
DUDLEY, L. Light Metals in Structural Engineering. 1946.
ITERSON, F. K. T. VAN. Plasticity in Engineering. 1947.
TERZAGHI, KARL AND PECK, RALPH B. Soil Mechanics in Engineering Practice. 1948.

FOOD.

GORTNER, WILLIS A.; ERDMAN, F. S.; AND MASTERMAN, N. K. Principles of Food Freezing. 1948.

INDUSTRIAL MANAGEMENT.

CUSHMAN, FRANK AND CUSHMAN, ROBERT W. Improving Supervision. 1947.

LANGUAGES.

PERRY, JAMES W. Chemical Russian, Self Taught. 1948.

MANUFACTURE.

BARRON, HARRY. Modern Rubber Chemistry. 1948.
CADY, EDWIN LAIRD. Precision Investment Castings. 1948.
DE CARLE, DONALD. Practical Watch Repairing. 1946.
GRANT, JULIUS. Wood Pulp and Allied Products. 1947.

Rawlings, A. L. The Science of Clocks and Watches. 1948.
Scoville, Warren C. Revolution in Glassmaking. 1948.

MARINE ENGINEERING.

King, Reno C., Jr. Practical Marine Engineering. 1948.

MATHEMATICS.

Finney, D. J. Probit Analysis. 1947.
Fort, Tomlinson. Finite Differences and Difference Equations in the Real Domain. 1948
Kattsoff, Louis O. A Philosophy of Mathematics. 1948.
Uspensky, J. V. and Heaslet, M. A. Elementary Number Theory. 1939.

MECHANICAL ENGINEERING.

Hunter, Marcus C. Inman. Rotary Valve Engines. 1946.
Merritt, Henry E. Gear Trains. 1947.
Parkinson, A. C. and Dawney, W. H. Gears, Gear Production and Measurement. 1948.
Stepanoff, A. J. Centrifugal and Axial Flow Pumps. 1948.
Vincent, E. T. Supercharging the Internal Combustion Engine. 1948.
Woodcock, Frederic L. Design of Metal Cutting Tools. 1948.

METALLURGY.

American Society for Testing Materials. Symposium on Powder Metallurgy. 1943.
Brown, Hiram. Aluminum and Its Applications. 1948.
Bullens, Dennison K. Steel and Its Heat Treatment. Volume 2. 1948.
Ellis, Owen W. Copper and Copper Alloys. 1948.
Harrington, R. H. The Modern Metallurgy of Alloys. 1948.
Heat-Treater. Fundamental Principles and Applications of Induction Heating. 1947.
Palmer, Frank R. and Luerssen, George V. Tool Steel Simplified. 1948.
White, A. E. and Siebert, C. A. Literature Survey on Low Temperature Properties of Metals. 1947.
Woldman, Norman E. Metal Process Engineering. 1948.

PATENTS.

Meinhardt, Peter. Inventions, Patents and Monopoly. 1946.
Toulmin, Harry Aubrey. Patent Law for Executive and Engineer. Second Edition. 1948.

PETROLEUM.

Weil, Benjamin Henry and Lane, J. C. Synthetic Petroleum from the Synthine Process. 1948.

PHARMACY AND PUBLIC HEALTH.

Bamford, Frank. Poisons. 1948.

PHOTOGRAPHY.

Bond, Fred. Kodachrome and Ektachrome from all Angles. Third Edition. 1947.
Custis, Eleanor Parke. Composition and Pictures. 1947.
James, T. H. and Higgins, G. C. Fundamentals of Photographic Theory. 1948.
Smith, W. J.; Turner, E. L.; and Hallam, C. D. Photoengraving in Relief. Second Edition. 1947.

PHYSICS.

Bates, Leslie Fleetwood. Modern Magnetism. Second Edition. 1948.
Faires, Virgil Moring. Elementary Thermodynamics. 1948.
Geyer, E. W. and Bruger, E. A. Tables of Properties of Gases. 1948.
Jánossy, L. Cosmic Rays. 1948.
King, Horace W.; Wisler, C. O.; and Woodburn, J. G. Hydraulics. Fifth Edition. 1948.
Luhr, Overton. Physics Tells Why. 1943.

MAURAIN, CHARLES. La Foudre. 1948.
MORSE, PHILIP MCCORD. Vibration and Sound. Second Edition. 1948.
SUITS, C. G. AND OTHERS. Applied Physics. 1948.
TOLANSKY, S. Introduction to Atomic Physics. 1948.
VENABLE, WILLIAM MAYO. The Interpretation of Spectra. 1948.

PHYSIOLOGY.

FRANKLIN INSTITUTE. BIOCHEMICAL RESEARCH FOUNDATION. Neutron Effects on Animals. 1947.
LEA, D. E. Actions of Radiation on Living Cells. 1947.

PLASTICS.

HICKS, JOSEPH S. AND FRANCIS, R. J. Low Pressure Laminating of Plastics. 1947.
MARK, H. AND PROSKAUER, E. S. The Science of Plastics. Volume 1. 1948.

RADIO.

BOYCE, WILLIAM F. AND ROCHE, JOSEPH J. Radio Data Book. 1948.
GRAMMER, G. AND GOODMAN, B. The A.R.R.L. Antenna Book. 1948.
MENZEL, DONALD H. Elementary Manual of Radio Propagation. 1948.
RIDER, JOHN FRANCIS AND USLAN, SEYMOUR D. FM Transmission and Reception. 1948.
SPANGENBERG, KARL R. Vacuum Tubes. 1948.
VAN VOORHIS, S. N. Microwave Receivers. 1948.

RAILROADS.

ALEXANDER, EDWIN P. The Pennsylvania Railroad; a Pictorial History. 1947.
KIEFER, P. W. A Practical Evaluation of Railroad Motive Power. 1947.

SANITARY ENGINEERING.

HOPKINS, EDWARD S. Water Purification Control. Third Edition. 1948.

SCIENCE.

DAVIS, HARRY M. Energy Unlimited; the Electron and Atom in Everyday Life. 1947.
GIEDION, SIEGFRIED. Mechanization Takes Command. 1948.
STEWART, IRWIN. Organizing Scientific Research for War. 1948.

TEXTILES.

AMERICAN VISCOSE CORP. TEXTILE RESEARCH DEPT. Rayon Technology. 1948.
BERGEN, WERNER VON AND MAUERSBERGER, H. R. American Wool Handbook. Second Edition. 1948.
LINTON, GEORGE E. AND PIZZUTO, JOSEPH J. Applied Textiles. 1948.
OLNEY, LOUIS ATWELL. Textile Chemistry and Dyeing. Tenth Edition. 1947.

WOOD-WORKING.

HANSEN, HOWARD J., Editor. Timber Engineers' Handbook. 1948.

PUBLICATIONS RECEIVED.

Microwave Transmission Circuits, edited by George L. Ragan, 725 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., 1948. Price, $7.00.

Mechanics of Machinery, by C. W. Hamm and E. J. Crane. Third edition, 538 pages. 16 × 23 cm., tables and drawings. New York, McGraw-Hill Book Co., 1948. Price, $5.00,

Popular Gemology, by Richard M. Pearl. 316 pages, 14 × 21 cm., illustrations and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $4.00.

Principles of Servomechanisms, by Gordon S. Brown and Donald P. Campbell. 400 pages, 15 × 24 cm., tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

M.I.T. in World War II, by John Burchard. 354 pages, 15 × 23 cm., illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.50.

Rotary Valve Engines, by Marcus C. Inman Hunter. 216 pages, 14 × 21 cm., illustrations, tables and drawings. New York, John Wiley & Sons, Inc., no date. Price, $5.00.

The Chemical Constitution of Natural Fats, by T. P. Holditch. Second edition, 554 pages, 15 × 25 cm., tables and drawings. New York, John Wiley & Sons, Inc., 1947. Price, $9.00.

Steam, Air, and Gas Power, by William H. Severns and Howard E. Degler. Fourth edition, 509 pages, 15 × 23 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $4.75.

The Chemistry of the Polysaccharides by Robert J. McIlroy. 118 pages, 14 × 22 cm. New York, Longmans, Green & Co.; London, Edward Arnold & Co., 1948. Price, $2.50.

History of Factory and Mine Hygiene, by Ludwig Teleky. 342 pages, 14 × 22 cm., tables. New York, Columbia University Press, 1948. Price, $4.50.

Mineral Resources of the United States, by the Staffs of the Bureau of Mines and Geological Survey. 212 pages, 21 × 28 cm., maps, charts and tables. Washington, D.C., Public Affairs Press, 1948. Price, $5.00.

The Theory of Mathematical Machines, by Francis J. Murray. Revised edition, 139 pages, 21 × 28 cm., drawings. New York, King's Crown Press, 1947 and 1948. Price, $3.00 (Paper).

Television How it Works, by John F. Rider. 203 pages, 21 × 28 cm., drawings and illustrations. New York, John F. Rider, 1948. Price, $2.70 (Paper).

BOOK REVIEWS.

SUPERCHARGING THE INTERNAL COMBUSTION ENGINE, by E. T. Vincent. 315 pages, illustrations, plates, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.00.

The earliest experiments with supercharging were conducted almost fifty years ago. The increase in power output was then incidental, as supercharging was applied with the idea of controlling the maximum temperature of the cycle.

Practical application did not occur until about ten years later (1910), when the German companies of Junkers and Sulzer utilized the principle in conjunction with diesel engines.

By the end of the First World War experimentation was in an advanced stage, and supercharging was quickly applied to the aircraft engine.

The most rapid development took place between 1920 and 1930, when considerable advances were made in developing materials which would stand up under the high exhaust-gas temperatures.

Professor Vincent's book brings together the accumulated knowledge in this specialized field. In the first chapters the thermodynamics and the engine cycle are discussed, before the types of compressors and their integral parts are introduced. Chapters estimating the operating conditions and describing the testing of superchargers, turbosuperchargers, and axial-flow compressors follow. A consideration of supercharging the oil engine concludes the book.

Persons in the internal-combustion-engine field who are not directly connected with the theory or development of superchargers will find the material contained in this book to be a valuable and comprehensive reference.

H. N. MICHAEL.

AIRCRAFT ENGINES, by Rollen H. Drake. 348 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1948. Price, $5.90.

This book is written principally for the layman, the teacher, and the airplane mechanic. It deals with the fundamentals of structure and operation of aircraft engines. The value of this book is enhanced by the fact that it contains enough information to enable the student to qualify for a Civil Aeronautics Administration's aircraft engine mechanic certificate.

The book is outstanding in its simplicity of presentation. To secure this desirable trait, the author has avoided the use of unnecessary graphs, formulas, tables, etc., which so often discourage the non-technically trained, yet interested and capable student.

In its twenty-two chapters the book covers most of the fundamentals pertinent to aircraft power units. The principles of internal combustion, the metals and alloys used, and the types of engines, are taken up first. After this general treatment, the component parts are dealt with: the cylinders, piston assemblies, crankshafts, crankcases, valves, etc. The accessory systems are treated next and include the ignition, fueling, supercharging, lubricating, and cooling systems. There are also chapters on propeller fundamentals, engine instruments, and on the later developments in jet propulsion.

The book is very adequately illustrated with drawings and photographs.

It may be recommended as outstandingly practical and easy to understand.

H. N. MICHAEL.

JOURNAL OF THE SOCIETY OF MOTION PICTURE ENGINEERS. INDEX, JANUARY, 1936 to DECEMBER, 1945. 156 pages, 15 × 23 cm. New York, Society of Motion Picture Engineers, 1948. Price, $2.00.

This is the third general index to be issued by the Society of Motion Picture Engineers since the appearance of its initial publication in 1916. Two indexes are included in this volume, one to authors and one to subjects. The subject index is arranged alphabetically by rather broad headings, with subdivisions under the larger groupings. "Sound recording" with eight and a half pages of references and "sound reproduction" with nearly five pages might well have also been subdivided. Numerous cross references are provided, both from headings not used and to other headings bearing on related topics. Under the subject headings the entries are arranged with the most recent first.

As files of a periodical accumulate, general indexes become an invaluable aid in making their contents readily accessible. The Society of Motion Picture Engineers merits commendation for its efforts in making such indexes available for its Journal.

G. E. PETTENGILL.

LORAN, LONG RANGE NAVIGATION, edited by J. A. Pierce, A. A. McKenzie, R. H. Woodward. 476 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $6.00.

One of the important uses of radio introduced during the last war is indicated in the derivation of its coined name "Loran," from *lo*ng *ra*nge *n*avigation. Developed to meet the needs of the Navy in convoy operations and to provide all-weather navigation for aircraft by day and night, it promises to have great usefulness in peacetime activities as well. Figures revealing its development are impressive. At the end of the war there were 70 Loran transmitting stations providing night-time service over 60 million square miles or three tenths of the earth's surface; 75,000 receiver-indicators had been manufactured, and 2¼ million charts prepared and shipped.

Although dependent upon the transmission and reception of pulsed radio signals like radar, Loran differs in that it employs lower radio frequencies and does not involve reflection from a target. "The primary fact upon which Loran is based is that the transmission time taken by a radio pulse to travel over a distance measures the distance." As actually operated Loran depends upon the absolute delay between the emission of pulses alternately from two stations and the time difference between their reception by the navigator.

In its treatment of the subject this volume commences with a brief comparison of Loran with other methods of navigation by means of radio aids. A historical chapter gives details of its development from the first proposal in October, 1940, through the various experimental stages to its practical operation. The different stations set up in the various theaters of war are discussed as well as the history of charting and training.

The principles of Loran are thoroughly treated with considerable attention being paid to the geometry of Loran, indicating the service area of the pair, triplets, chains and quadrilaterals. Different types have been developed, namely, standard Loran, sky-wave synchronized Loran, and low frequency Loran. Procedures for its use in navigation are indicated and a chapter on future trends considers possible developments and uses.

Part II is devoted to Loran equipment with chapters on timers, switching equipment, transmitters, antenna systems, and receiver-indicators. This material is quite detailed on the special applications and includes many diagrams, both schematic and block. An appendix gives the Loran program in the Hydrographic Office, a list of Loran ground stations and other data. The bibliography is comprehensive and provides a helpful indication of where material may be secured, especially necessary because of the nature of much of the material cited.

This volume well serves its purpose of recording the development of Loran and until significant changes occur, will undoubtedly remain the standard work on the subject.

G. E. PETTENGILL.

PRACTICAL MARINE ENGINEERING, by Reno C. King, Jr. 470 pages, illustrations, diagrams. 14 × 20 cm. New York, Prentice-Hall, Inc., 1948. Price, $6.00.

Designed for the operator of a marine steam power plant, this volume is thoroughly practical in nature. The author has based it on many lectures which he has given to applicants for Merchant Marine Engineering-Officer licenses. In addition to considering the various aspects of propulsion machinery, boilers, steam engines and steam turbines, the book also includes material on other subjects with which the marine engineer must be familiar—electricity, refrigeration, pumps and unfired pressure vessels, respiratory apparatus and fire-fighting equipment. Adequate attention has been paid to both routine and emergency operation and maintenance.

The illustrations and many diagrams as well as the questions, which are typical of those given in the license tests, make the book well adapted for self study.

G. E. PETTENGILL.

HOUSEHOLD ELECTRIC REFRIGERATION, by John F. Wostrel and John G. Praetz. Second ed. 458 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

From the time of the first printing of this book in 1938, there were few developments in household refrigeration. Thus, the new, second edition follows the first rather closely, with the exception of amplifications on mechanical details and service instructions for refrigerators that were introduced to the market after 1938.

The book, written in non-technical language, is particularly well suited for those servicing household refrigerators. The various steps are taken up very thoroughly. The basic physical laws governing refrigeration are explained before the systems are described. For all practical purposes there are only two principles employed in the refrigerators of today—compression and absorption. These are explained before the variations of the different commercial products are taken up.

The emphasis of the book is on servicing and a major part of it is devoted to the control devices, to installation, testing and servicing.

Chapters on food preservation and insulation materials should also prove helpful to those engaged in the production and distribution branches of the field.

H. N. MICHAEL.

CENTRIFUGAL AND AXIAL FLOW PUMPS, by A. J. Stepanoff. 428 pages, illustrations, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $7.50.

During recent years the rapid advancement in the design and application of centrifugal pumps has caused radical adjustments to be made in the former limitations of pressure, capacity, temperature, speed of operation, etc. The problems arising out of such changes are numerous and may be grouped under several headings: mechanical, hydraulic, operating, and economic.

Dr. Stepanoff's book deals with the necessary theoretical and experimental studies which make the advance in pump design possible. These include cavitation, operation outside the normal head capacity and speed range, axial and radial thrust, effect of stuffing boxes on the critical speed of pump shaft, control of head capacity and brake-horsepower curves, and the pumping of viscous liquids.

Of particular interest is Dr. Stepanoff's theoretical treatment of the pump impeller. Based on a single pattern of flow, identical reasoning, and similar procedure of impeller layout for centrifugal, mixed flow, and axial flow pumps, it results in a simple diagram which combines the design elements and performance characteristics of all centrifugal pumps, thus creating a "Mollier diagram" for all centrifugal and axial flow pumps.

The book is well supplied with charts, diagrams, and illustrations.

H. N. MICHAEL.

NICOLAUS COPERNICUS. DE REVOLUTIONIBUS. PREFACE AND BOOK I, translated by J. F. Dobson, assisted by S. Brodetsky. 32 pages, plates, diagrams, 17 × 25 cm. London, Royal Astronomical Society, 1947. Price, 3s 6d.

JOHN COUCH ADAMS AND THE DISCOVERY OF NEPTUNE, by W. M. Smart. 56 pages, portraits, 17 × 25 cm. London, Royal Astronomical Society, 1947. Price, 5s.

These two pamphlets appeared originally as *Occasional Notes* 10 and 11, respectively, of the Royal Astronomical Society. The first comprises a translation into English of parts of Copernicus' *De Revolutionibus*, which is one of the foundation works of astronomical literature, with its outlining of the theory that the earth moves around the sun. The translation is readable and is accompanied by extensive notes.

The other paper is two-fold in scope—in it Dr. Smart recounts the major facts about the life of John Couch Adams and he surveys the controversy which arose over the discovery of Neptune. Adams and Le Verrier had both become interested in the possibility of a trans-Uranium planet and both men, nearly simultaneously, deduced from mathematical calculations the existence and position of such a planet. Its subsequent telescopic discovery on September 23, 1846, in Berlin, gave rise to controversy as to the relative claims of the two men for the honor of initial discovery. This balanced appraisal of the various aspects of the matter will prove valuable in placing their work in proper perspective.

Both pamphlets should appeal to those concerned with the history of astronomy.

G. E. PETTENGILL.

LA FOUDRE, by Charles Maurain. 215 pages, illustrations, 11 × 16½ cm. Paris, Librairie Armand Colin, 1948. Price, 120 fr. (paper).

Lightning, which has been the subject of investigation ever since the middle of the eighteenth century has become an object of even greater study in the twentieth century. The various manifestations of atmospheric electricity have assumed an increasing importance with the growth in the use of electricity and the development of aerial travel.

Mr. Maurain discusses the general electrical properties of the atmosphere and then proceeds to consider the structure and properties of the lightning flash, the thunder cloud, and the theories relative to the production of electricity in the thunder cloud. Considerable attention is also paid to the question of atmospherics. The final chapter offers some comments on lightning arresters.

A bibliography and an index of names are included. This book is a useful summary of the present knowledge of lightning.

G. E. PETTENGILL.

ENCYCLOPEDIA OF CHEMICAL REACTIONS, by C. A. Jacobson. Volume II, 917 pages, 15 × 24 cm. New York, Reinhold Publishing Corp., 1948. Price, $12.00.

The first volume of this work was reviewed in the JOURNAL for August, 1946. This second volume includes some 3331 reactions for the elements cadmium, calcium, carbon, cerium, cesium, chlorine and chromium. It has the same general arrangement as the previous volume, first as to the formulas of the reactants and next as to those of the reagents.

The list of "Journals used by abstractors" has been revised and some changes made in the abbreviations employed. Unfortunately *Chem. Anal.* now stands for two publications, *Chemische Analyse* and *Chemist Analyst.* One is also rather surprised at the form of *Bio Chem. J.* and *Bio Chem. Z.*

The amount of volunteer labor that has gone into the production of this set merits the thanks of chemists everywhere who will profit by the accessibility of the compiled data.

G. E. PETTENGILL.

DAIRY BACTERIOLOGY, by Bernard W. Hammer. Third edition, 593 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.00.

Originally written as a text-book for a beginning dairy bacteriology course, this book, through its comprehensive coverage, has also become an important reference to the technical personnel of the dairy industry. It presumes a knowledge of general bacteriology. In the third edition of this work, minor rearrangements of material included in the earlier editions were made and new important developments have been added. The increased production of milk powder during the war, for instance, has presented many problems, the solution of which provided much useful material.

There is an entirely new chapter on dairy-plant water supply, in which particular attention is paid to the spoilage that a deficient water supply may cause in milk products.

Only special dairy laboratory methods are discussed. The standardized procedures are to be sought in the latest edition of "Standard Methods for the Examination of Dairy Products."

H. N. MICHAEL.

THE GENIUS OF INDUSTRIAL RESEARCH, by D. H. Killeffer. 263 pages, diagrams, 15 × 23 cm. New York, Reinhold Publishing Corp., 1948. Price, $4.50.

Industrial research is playing an ever greater role in the world today. Consequently, knowledge of its techniques and methods is of vital importance. In the present volume the author has stated some of the principles which he feels are generally applicable. He has leaned heavily on the case method and a large part of the text consists of extracts from the writings of others describing research which they have done. The examples have been chosen from the field of industrial chemistry, although the principles will be germane to most industrial research.

After a consideration of the difference between pure and applied research, the author points out the necessity of ability to state the problem in such terms that its solution becomes practical. Then the two basic methods of research are discussed—the Baconian or Edisonian method of trying everything, and the Aristotelian method of formulating a theory and then searching for facts to bolster it. Examples of both types are cited from the work of Thomas Midgley, Jr. The "discovery of the effectiveness of tetraethyl lead in inhibiting fuel knock in internal combustion engines" represents the Edisonian method, while the Aristotelian method method is demonstrated in Midgley's research leading to the fluorinated hydrocarbons as safe refrigerants.

The necessity of thinking out the problem is reviewed and then the author turns his attention to different types of research. These are *process research* which involves developing a new process for the manufacture of a known compound or product; *product research* which contemplates the development of new chemical products to meet existing needs—for example, rayon, cellophane, Freon; and *equipment research* which investigates the changing of a design to adapt it for performing a new function.

The progressive development of an idea through various stages is traced using the synthesis of rubber as an example. Other topics considered are the value of imagination and ingenuity, the importance and operation of a pilot plant, the writing of reports, and the evaluation of research.

The treatment of the subject is logical and the presentation stimulating. The quoting of the actual words of the researchers adds greatly to the interest of the illustrative material. Students will surely profit from a reading of the volume and the experienced chemist, plant supervisor or development engineer will undoubtedly find suggestions of value.

G. E. PETTENGILL.

GROWTH OF PLANTS. TWENTY YEARS' RESEARCH AT THE BOYCE THOMPSON INSTITUTE, by William Crocker. 459 pages, illustrations, 16 × 23 cm. New York, Reinhold Publishing Corp., 1948. Price, $10.00.

This book reports on the major and minor projects that have been conducted by the staff of the Boyce Thompson Institute since its foundation in 1924. Some of these projects are still being continued. The twelve major projects are described in the first eleven chapters, and the concluding twelfth chapter is devoted to the miscellaneous minor problems of plant research.

Among the early problems approached by the Institute were the "yellows" and virus diseases of plants and the "duck food problem" which deals with winter feeding grounds of migratory wild fowl in the tide-water regions of Virginia and North Carolina. These undertakings are described in the first chapter.

Chapters II and III deal with the life span of seeds and dormancy in seeds, respectively.

The next two chapters are devoted to the study of the physiological effects of ethylene and other unsaturated carbon-containing gases and of lethal gases upon plant growth.

In Chapter VI, Dr. P. W. Zimmerman attacks the problem of plant hormones, a problem still under investigation at the Institute.

Researches leading to the discovery of chemicals that force buds out of their normal state of dormancy are described in Chapter VII, entitled "Dormancy in Buds."

Problems relating to the formation and structure of plant cell membranes have been approached through microscopic, chemical and X-ray diffraction analyses of cell membranes. Mrs. Wansen K. Farr reports these studies in Chapter VIII.

Next the growth of plants under controlled environmental conditions is described. The controlled conditions included the following: light of various intensities, quality and daily duration; artificial light with various day lengths, with controlled temperatures and humidities, and with increased concentrations of CO_2; combinations of sunlight and artificial light; growth under a full solar spectrum and under various portions of the solar spectrum. The plants produced under these conditions were studied as to rate and nature of growth, dry and wet weight increase, chemical composition, and pigment development. Content, size, form, and anatomy of the several organs, as well as vegetative or reproductive development were also considered.

The last two major research projects are described by Drs. Hartzell and McCallan. They deal with insecticides and fungicides, respectively.

Chapter XII presents fourteen shorter projects, most of which have been finished or discontinued. The subject matter here is diverse, dealing with such phenomena as "Diurnal and Autumn Changes in Leaves of Deciduous Plants," or "Effect of Nitrogenous and Carbohydrate Reserves on Growth of Seedlings," *et al.*

Dr. Crocker's book provides a convenient critical summary of articles which appeared in the past in the various pertinent publications, as well as those that have not been hitherto published. It should prove of considerable value to further research as well as to the fields of application.

H. N. MICHAEL.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Radiation Survey Meter.—T. Enns. A simple portable instrument with a wide range of sensitivity has been constructed. The apparatus consists of an aluminum ionization chamber, and an electrometer circuit using the Victoreen VX 41 electrometer tube.

The electrometer circuit is shown in Fig. 1. It is based on the circuits described by Penick[1] and is balanced so that small changes (approximately 5 per cent) in the potential of the power battery produce no appreciable change in the reading of the output meter. All power for the electrometer circuit is supplied by a 7½-v. Burgess 5540 "C" battery. Throughout the useful life of the battery (170 hr.) the potentiometer P_1 can be adjusted to completely compensate for the drop in battery potential. This adjustment is made so that the filament emission as measured by the space charge grid current remains unchanged. The space charge grid current is measured by disconnecting the plate and setting the plate resistor P_2 at maximum value. The space charge grid current is chosen at 193 microamperes, or a meter reading of 20 microamperes.

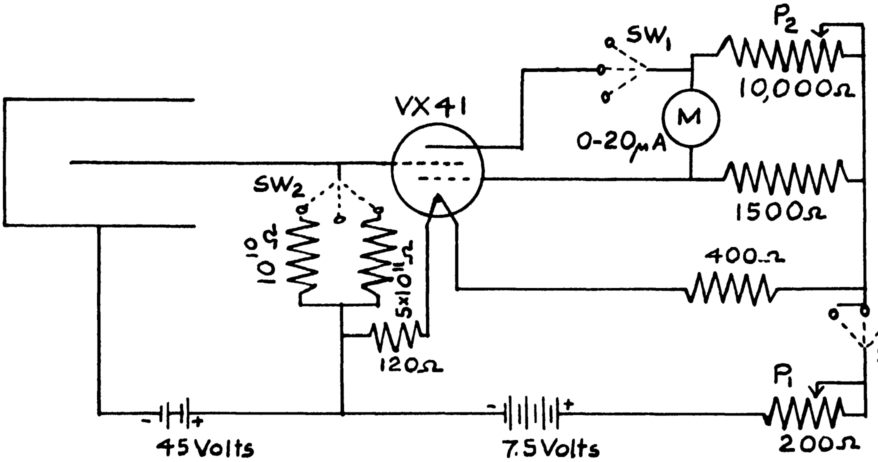


Fig. 1. Electrometer circuit.

The grid resistor switch SW_2 is a standard low capacity switch with silver contacts. The bakelite insulator has been replaced by a lucite insulator. It has three positions, two of them connect the grid to the resistors and the third gives a free grid for the rate of drift measurements.

---

[1] D. B. Penick, *Rev. Sci. Instruments*, 6: 115 (1935).

The circuit is housed in two standard metal cases each 4 by 4 by 2 in. combined to form a 4 by 4 by 4-in. case. The ion chamber is a 2-in.-diameter-by-2½-in.-long aluminum coil shield projecting from this case and insulated by a bakelite base.

Approximate sensitivities of the instrument to gamma radiation are:

with 10^{10} grid resistor—one microampere = 0.720 *r* per hour
with 5×10^{11} grid resistor—one microampere = 0.0144 *r* per hour
with rate of drift setting—one microampere per second = 0.108 *r* per hour.

Maximum range of the instrument is 14.4 *r* per hour. Minimum practical range is one microampere per minute or 0.0018 *r* per hour.

CURRENT TOPICS.

The Stroboflow, An Oscilloscope for Hydraulics.—The Stroboflow, for reproducing desired rates of flow in liquids, comprises a precision-made turbine, mounted on stainless steel pivots in Graphitar bearings between transparent panes, and allowed to rotate, practically floating in synchronism with the liquid.

The rotational speed of the turbine is used as a measure of the quantity of liquid flowing. Since there is a straight-line relationship between the speed of the rotor and the quantity of the liquid, and since the Stroboflow provides a very precise means of determining rotor speed, these two factors are combined in this instrument and constitute a highly accurate method of flow-rate indication.

The rotor's speed is determined from the stroboscopic effect of light viewed through an aperture in the turbine disc. The number of virtual images so obtained is a measure of the rotational speed, which in turn is directly proportional to rate of flow. The Stroboscope is calibrated in terms of stationary patterns of these images. Once a predetermined pattern is established, indicating a desired rate of flow, any deviation however minute from this rate of flow, will cause the pattern to precess.

One of the outstanding features of Stroboflow operation is the means by which the range is extended without sacrifice in the degree of accuracy. This is accomplished by a simple change of jets, interchangeable in any Stroboflow without change in calibration. Five standard jets, supplied with each instrument and for which each instrument is calibrated, provide forty different rates of flow in one instrument.

Since the rate of flow, the speed of the rotor, and the jet characteristic are all in direct, linear relationship over the entire range of the instrument, it is practicable to adapt several, separate Stroboflows to indicate different rates of flow while exhibiting identical patterns. This simplifies monitoring of flow rates in processes requiring coordinated multiple flows.

R. H. O.

Rocket Motor Test Station.—A test station which will static-test rocket propulsion motors up to 50,000-lb. thrust, equivalent to 500,000 hp., has been constructed near Schenectady.

This test station, the first of its kind, has been in operation for over a year, contributing invaluable data for basic rocket-motor design and fuel development.

The test station has three heavily-constructed steel-re-inforced concrete rocket-motor static-test pits. Each pit is set into the forward slope of a small hill, and earth is mounded around its sides.

Three covered stairways lead down into the interior of the test pit, which is divided into four rooms; control room, motor room, and two reactant rooms. Three-foot steel-re-inforced concrete walls separate the motor room from the rest of the structure, while all other walls and the roof are 2 ft. in thickness. Heavy steel doors seal off each room when tests are in progress.

The control room, lying parallel to the motor room, contains all the valves and controls used in regulating "runs" of rocket motors under test in the pit. Roughly, two thirds of the wall space is filled with observation ports and panels containing valves, switches, and gages. The rest of the area contains banks of photoelectric recorders which calibrate and record data continuously during the tests.

Fire is an ever-present danger in rocket-motor development and extensive precautions have been taken throughout the entire test station.

No smoking is permitted in or near the test pits. Power driven ventilation fans clear any fumes from the reactant rooms. Automatic alarm systems are located in the interior and outside of each test pit.

Within the pits a high-pressure water system, at 140-psi., can operate either automatically or manually by means of controls located in the control room. In the motor room, three different flooding systems can be used. In each of the reactant rooms, high-pressure nozzles are directed down upon the reactant tanks. Carbon dioxide extinguishers are located throughout the sites.

Sirens, starting five minutes before and continuing through each "run," as well as flashing red lights above the pits, warn personnel in the test area. Guards are also placed a safe distance from the pit under test to prevent persons entering the area. Sirens continue to sound some time after the "run" to allow a safety period after shutdown.

R. H. O.

New Type Manometer.—Unprecedented simplicity and high precision in making either absolute or differential measurements of pressure and vacuum are now obtainable through a new type manometer which reduces the arduous task of cleaning and filling a closed end manometer to the same relatively simple process of cleaning and filling an open end U-tube type. Only a single reading is required to obtain a precise measurement automatically corrected for ambient temperature and reduced to a height of mercury at 0° C.

The principle of operation of this new absolute and differential manometer is as follows: The closed end is effected by means of a U-tube which traps mercury to form the vacuum seal and an oblique bore stopcock above the trap to hold up the mercury seal. For differential readings the stopcock is kept open and connected to the lower source of pressure. The ratio of the diameters of the manometer tube and reservoir is adjusted exactly to counteract the increase in mercury height due to its lower density at room temperature than at 0° C.

Thus, a single reading of the mercury level in the manometer tube gives the true corrected pressure in mm. of Hg at 0° C. The coefficient of expansion of the vinylite scale is of such value that a precision of 0.1 mm. is maintained for room temperature variation between 20° and 30° C. A vernier is provided to simplify reading of the scale to a precision of 0.1 mm.

The glass part and scale of this new instrument are mounted on a functionally designed aluminum stand having a rigid three point support. The vinylite scales also have high dimensional stability and are designed to preserve the markings with a laminated cover of transparent vinylite over a milk white base upon which the markings are clearly printed in black. This new type manometer, Range: 0–200 mm., vernier reading to 0.1 mm., is made of pyrex brand glass, and is supplied mounted on aluminum stand, but without mercury.

Liquid Rheostats.—The liquid rheostat is particularly suited for control of larger motors, especially where other types of control cannot be used because of rating limitations.

Speed control systems using liquid rheostats are noteworthy for their stepless speed control, minimum maintenance and maximum dependability. Most applications in recent years have been on motors from 1,000 to 30,000 hp. in size.

Basically, the liquid rheostat consists of metal electrodes, half of which are movable, submerged in a tank of electrolyte that is usually a solution of sodium carbonate in water. The stationary electrodes are insulated from the steel in the tank by an insulating cell. Current flows between the electrodes and resistance is approximately proportional to the distance between them. Resistance properties of the electrolyte depend upon the concentration of sodium carbonate and the temperature of the solution.

There are three types of rheostat control systems: (1) The manual control system, where the electrodes are positioned by manual control through the pilot motor rheostat; (2) The speed regulating control system, offering a more refined scheme of speed adjustment, with the pilot motor being energized by an amplidyne; and (3) The kind of control that can be applied to maintain constant input to the motor or to prevent the load on the motor from exceeding a selected value.

The liquid rheostat, as a system element in conjunction with the proper control, can be applied in other ways to regulate temperature, flow, level, and pressure.

R. H. O.

Heat Conducting Glass Keeps Ice from Windshield.—Heat-conducting glass, put to use for de-icing windshields in one of the severest winters on record in the last quarter century, stands ready to give the American traveler new safety and additional comfort in cold weather.

Glass normally is used as an insulator, but it becomes a conductor of electricity by coating it with an invisible film.

The research staff of Libbey-Owens-Ford Glass Co., which holds patent rights on the new windshield, declares that the winter-long tests bear out successful results of earlier laboratory experiments with this novel glass, and satisfy every qualification for its commercial manufacture.

"Ice doesn't have a chance; it just can't form on the warm surface of the glass," according to R. A. Gaiser, one of the researchers who installed a windshield on his automobile and tested it in repeated freezing rains and sleet storms as well as in sub zero weather.

Higher voltage than is normally found on the average family automobile is required for efficient operation of the de-icer. The most feasible method of supplying current, Mr. Gaiser believes, is to replace the conventional generator with an a-c. generator. The alternating current then can be transformed to voltages sufficiently high to perform the necessary de-icing operation. Many police and fire cars and ambulances have a-c. equipment for radio transmitting purposes. No additional equipment would be needed in those cars.

Carbonyl Iron Powders.—Because of the widespread interest that has been aroused in Carbonyl Iron Powders as a result of an invention of a new magnetic clutch, in which these powders are used, by Jacob Rabinow, of the National Bureau of Standards, the following brief history and description of the powders has been prepared.

Iron Carbonyl Powders are manufactured exclusively by General Aniline & Film Corporation in a plant constructed expressly for the purpose at Grasselli, New Jersey, shortly after the outbreak of World War II. The manufacture of these powders expanded rapidly to meet the heavy war time demand for use as an important agent in various electrical devices, particularly in the field of communication, including radar, short wave transmitters, carrier telephony and in frequency modulation devices. One major application is in the form of cores in high frequency magnetic fields.

It is estimated that over 90 per cent of all automobile radio sets constructed since January 1947 are built with cores made from Carbonyl Iron Powders. They have been accepted by radio engineers as the "heart of inductance (push button) tuning." These cores have made push button tuning in auto radio sets possible.

Carbonyl Iron Powders are manufactured under rigid production control. They are made by the reaction of carbon monoxide gas with iron-containing ores to form liquid Iron Penta Carbonyl. This liquid is then heated to a temperature where it decomposes into powdered iron and the carbon monoxide gas is liberated. The results of this method are unique: Iron Penta Carbonyl is obtained which is chemically pure, and the iron powder particles formed by its decomposition are all minute spheres.

While the process of manufacture appears simple, the closest attention to detail is necessary to insure a product whose characteristics never vary. To illustrate: the powders are never exposed to air, but only to inert gases; ammonia is added to the packing containers to check any possible action of moisture and carbon dioxide during storage and shipment.

The new type of electromagnetic clutch, developed by Jacob Rabinow at the National Bureau of Standards, has the automobile, aviation, marine and farm implement people—in fact just about everyone in the motive power transmission field—enormously interested.

Rabinow's development, based on his discovery that frictional forces between solid surfaces and certain types of fluid media can be controlled by application of magnetic fields, is being studied by engineers, and Rabinow's visitors at his laboratory in Washington include representatives of the major automobile companies and clutch manufacturers.

The clutch is characterized by ease of control, high efficiency, smooth operation, long life and simplicity of construction.

Two clutch discs, one on the drive shaft and one on the shaft to be driven, are contained in a housing, with the discs immersed in oil in which millions of particles of iron powder are held in suspension. When this mixture of oil and iron is acted on by a magnetic field, the iron particles are mutually attracted and become "bound" together in the field. The magnetic field is, of course, produced by an electric current so that it is simple to control the binding force. The amount of power required is very small and the control is smooth from the minimum—characterized by a viscous 'drag'—to the maximum immediate rigidity following magnetic saturation of the iron.

Having no axially moving parts, the clutch is easy to build and the models show no sign of wear after long tests.

29 OCT 1948

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 246 SEPTEMBER, 1948 No. 3

STABILIZATION OF CARRIER-FREQUENCY SERVOMECHANISMS. III. METHODS OF OBTAINING REQUIRED CARRIER PHASE-SHIFT.*

BY

ANDREW SOBCZYK, Ph.D.[1]

ABSTRACT.

In a servomechanism using a two-phase alternating current control motor, a 90° difference is required in the phases of the carrier-frequency voltages applied to the fixed and control windings. This part describes and compares various methods of obtaining the phase difference.

The question of the possibility of a phase-shifting proportional-derivative parallel "T" is answered in the negative, by the result that in any parallel "T" transfer characteristic, if the quadratic factor in the numerator is of the proportional-derivative form at the correct resonant frequency, the amount of phase shift which may be obtained from the remaining portion of the transfer characteristic is less than arc tan $(2/n)$, where n is twice the carrier frequency divided by notch width. Thus for values of n high enough to have an appreciable stabilizing effect, the maximum obtainable intrinsic phase shift is negligible.

In order to obtain a large phase shift it is necessary to add either a series input or a load impedance to the parallel "T," or to use a phase-shifting network preceding or following the parallel "T." Formulae and design charts are given for determination of the values of the components of phase lag networks.

The method of calculation of tolerance requirements on the components, in terms of allowable deviation from the correct phase, is illustrated by an example of a phase lag network used in conjunction with a bridge "T" proportional-derivative network.

1. INTRODUCTION.

As mentioned in Part I, it is necessary that the phase of the carrier of the modulated error signal applied to the control winding of the two-phase motor differ by 90° from the phase of the voltage on the fixed winding. Actually in the servo controller a phase shift different from

---

* This paper is a continuation of Parts I and II, published in the JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 246, July, p. 21, and August, p. 95 (1948).

[1] Formerly with Radiation Laboratory, Massachusetts Institute of Technology; now a member of the Department of Mathematics, Boston University, Boston, Mass.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute

90° is usually required, in order to compensate for phase shift which occurs in the selsyns or elsewhere in the servo circuit.

All methods of obtaining phase shift of the carrier cause the frequency characteristic of the a-c. servo controller to become unsymmetrical about the carrier frequency, and also affect the phase characteristic. This distortion of the frequency-phase characteristics, and the question of its effect on servo performance, would not arise in a case where it was feasible to shift, instead, the phase of the voltage applied to the fixed winding. (Assuming the simplified motor, the effect of the distortion on stability and error in any case may be determined by the method of Part I.)

So far as obtaining the proportional-derivative effect on the modulation is concerned, the essential portion of the transfer characteristic of the parallel "T" is the factor $\left[1 + jT_d \frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]$ in the numerator. Accordingly, in the following first section we investigate the phase shift which can be obtained from the remaining portion of the parallel "T" transfer characteristic, preserving the proportional-derivative factor in the numerator.

2. PHASE SHIFT AT ω_0 IN TERMS OF INDEPENDENT CONSTANTS IN THE PARALLEL "T."

Let the numerator and denominator of the parallel "T" transfer characteristic

$$\frac{E_2}{E_1} = \frac{T_1T_2T_3p^3 + T_1(S_2 + T_3)p^2 + (T_1 + S_1)p + 1}{T_1T_2T_3p^3 + [T_1(S_2 + T_3) + T_2(T_1 + S_1 + T_3)]p^2 + (T_1 + S_1 + T_2 + T_3)p + 1}$$

be multiplied by $n\omega_0/2p$, and let $p = j\omega$. Then in order that the numerator consist of the proportional-derivative characteristic

$$\left[1 + jT_d \frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]$$

multiplied by a linear factor $(Uj\omega + 1)$, the following conditions must be satisfied by the fundamental time constants:

$$T_1T_2T_3 = \frac{u}{\omega_0^3}, \tag{1}$$

$$T_1(S_2 + T_3) = \frac{1}{\omega_0^2}\left(1 + \frac{2u}{n}\right) \tag{2}$$

$$T_1 + S_1 = \frac{1}{\omega_0}\left(\frac{2}{n} + u\right) \tag{3}$$

Suppose that these conditions are satisfied. Then we may consider T_1 and T_2 as independent variables, T_3, S_1, and S_2 as dependent variables: $T_3 = u/\omega_0{}^3T_1T_2$ from Eq. 1, and then S_2 and S_1 may be expressed in terms of T_1, T_2 from Eqs. 2 and 3. For positive S_1 and S_2, we see that T_1, T_2 must be restricted to lie in the region $T_2\omega_0 > nu/(n + 2u)$, $T_1\omega_0 < (2 + nu)/n$.

In terms of $T_1\omega_0$, $T_2\omega_0$, u, the phase shift ϕ at ω_0 is given by:

$$\tan\phi = \frac{u^2 - z}{(u^2 + 1)\left[a + \frac{n}{2}(1 + z)\right] + u(1 + z)}, \tag{4}$$

where $a = T_1\omega_0$, $z = T_1T_2\omega_0{}^2$, $n = T_d\omega_0$. Thus if $T_1T_2\omega_0{}^2 < u^2$, the phase shift is a lead; and if $T_1T_2\omega_0{}^2 > u^2$, it is a lag. If $T_1T_2\omega_0{}^2 = u^2$, then not only is there zero phase shift at ω_0, but also it may be verified that the factor $(Up + 1)$ of the numerator is also a factor of the denominator, so that the transfer characteristic is of the form

$$\frac{E_2}{E_1} = \frac{(Up + 1)\left(\frac{1}{\omega_0{}^2}p^2 + \frac{2}{n\omega_0}p + 1\right)}{(Up + 1)\left(\frac{1}{\omega_0{}^2}p^2 + \frac{2}{m\omega_0}p + 1\right)} = \frac{m}{n}\frac{\left[1 + jT_d\frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]}{\left[1 + j\frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]}.$$

The gain at ω_0 is given by:

$$g = \frac{2a(u^2 + 1)^{\frac{1}{2}}}{[(2a + nz + n + 2u)^2 + (2au + nu + 2z + znu)^2]^{\frac{1}{2}}}$$

$$= \frac{2a}{2(a + u) + n(1 + u^2)} \quad \text{in case } z = u^2.$$

Let us examine Eq. 4 for $\tan\phi$ to determine the maximum amount of phase shift which can be obtained. The expression:

$$\frac{u^2 - z}{K(u^2 + 1) + u(1 + z)}$$

is a monotonic function of u; its value increases from $-z/K$ at $u = 0$ to $1/K$ as $u \to \infty$. Therefore the tangent of the angle of phase lead cannot be greater than $\frac{1}{a + n(1 + z)/2} < \frac{2}{n}$. The tangent of the angle of phase lag cannot be greater than

$$\frac{z}{\left[a + \frac{n}{2}(1 + z)\right]} = \frac{1}{\left[\frac{a}{z} + \frac{n}{2}\left(\frac{1}{z} + 1\right)\right]} < \frac{2}{n}.$$

Thus for $T_d\omega_0 = n = 15$, the upper bound of possible phase shift is less than 7.6°. Actually, the bounds as found by calculation, for the case of equal condensers, were 3.8° lag, 0.5° lead.

It may be verified that Eqs. 1, 2, and 3 are invariant when the input and output of the parallel "T" are reversed, that is, when $R_2 \rightleftarrows R_3$ and $C_1 \rightleftarrows C_3$. They are also invariant when one of the individual "T"'s of the two "T"'s in parallel is reversed, that is, when either $R_2 \rightleftarrows R_3$, or $C_1 \rightleftarrows C_3$. Thus any proportional-derivative parallel "T" is reversible, without changing T_d, and the individual "T"'s likewise are reversible, to within the practically negligible amount of phase shift of the carrier.

3. LOAD IMPEDANCE AND SERIES INPUT IMPEDANCE.

Though the intrinsic phase shift obtainable in the parallel "T," together with the proper proportional-derivative characteristic for usably high values of T_d, is very small, an appreciable phase shift at the carrier frequency may be obtained (1) by loading the output of the parallel "T," or (2) by inserting a resistor or other impedance in series with the input to the parallel "T." A third method is to insert a one- or two-stage R-C or C-R phase-lag or phase-lead network either preceding or following the proportional-derivative parallel "T." (With any of the three methods, there is of course an accompanying effect on the gain-phase margin diagram, which may be calculated as in Part I.)

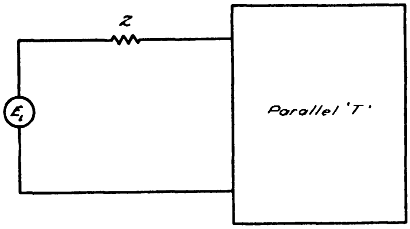


FIG. 1.

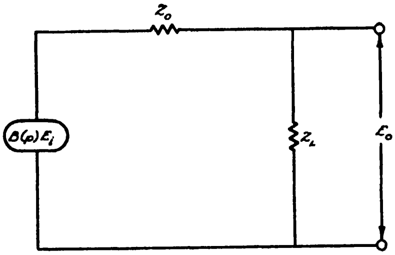


FIG. 2.

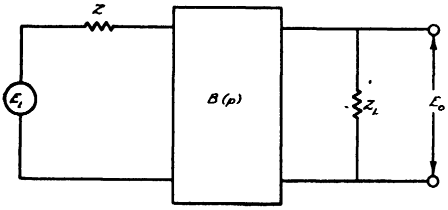


FIG. 3.

In this section we consider (1) and (2). If the input impedance to the parallel "T" is $A - jB$, and the source of E_i has series impedance Z, Fig. 1, the phase shift is $\angle\,[(A - jB)/(A + Z - jB)]$. Thus if Z is resistive, the phase shift approaches a lag of arc tan B/A as $Z \rightarrow \infty$; and if Z is inductive, a phase lag up to 90° + arc tan B/A may be obtained.

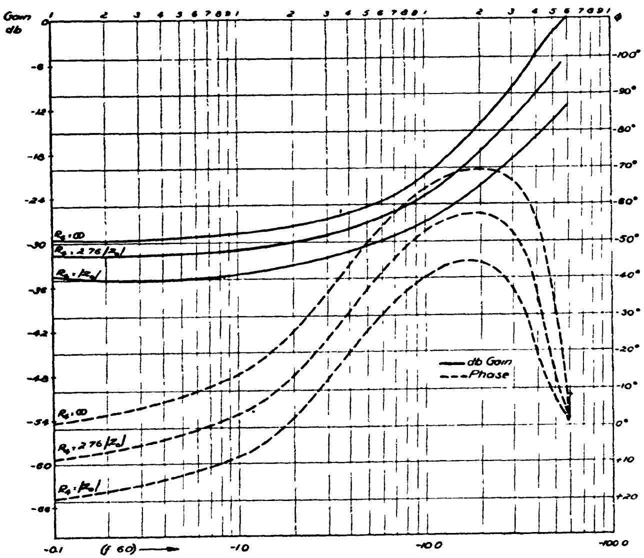


FIG. 4.

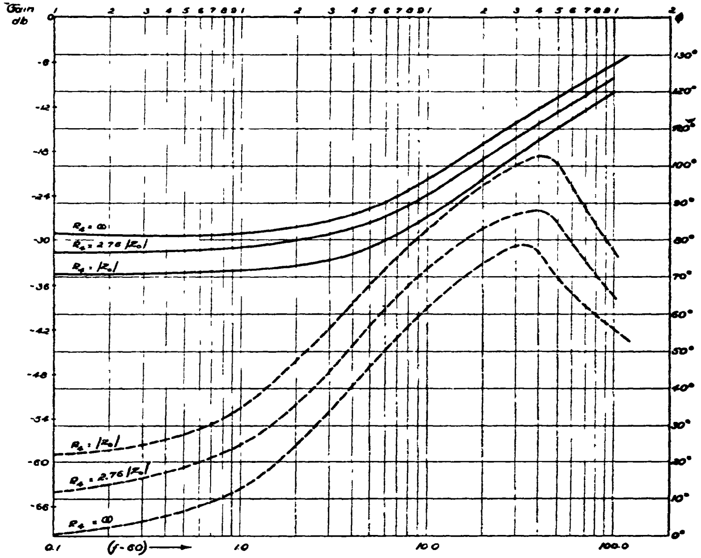


FIG. 5.

$$\text{Output Impedance } Z_o = \frac{6.9 + 32.2\left(\frac{\omega}{\omega_0}\right)^2 - 5.82\left(\frac{\omega}{\omega_0}\right)^4}{1 + 2.9\left(\frac{\omega}{\omega_0}\right)^2 + 56.4\left(\frac{\omega}{\omega_0}\right)^4 + 0.422\left(\frac{\omega}{\omega_0}\right)^6} \times 10^{-3}$$

$$+ j\,\frac{-\,31.6\left(\frac{\omega}{\omega_0}\right) - 11.1\left(\frac{\omega}{\omega_0}\right)^3 - 2.66\left(\frac{\omega}{\omega_0}\right)^5}{1 + 2.9\left(\frac{\omega}{\omega_0}\right)^2 + 56.4\left(\frac{\omega}{\omega_0}\right)^4 + 0.422\left(\frac{\omega}{\omega_0}\right)} \times 10^{-3}.$$

Notch width = 8 cps. ($T_d\omega_0 = 15$)
60 cps. Parallel "T"
$Z_o = 0.001285 + (-\,0.001282)$ at 60 cps.
$Z_o = 0.00182$ megohms
In case $C_1 = C_2 = C_3 = 1$ μfd., load resistor $R_4 = \infty$, 0.00182, 0.00514.

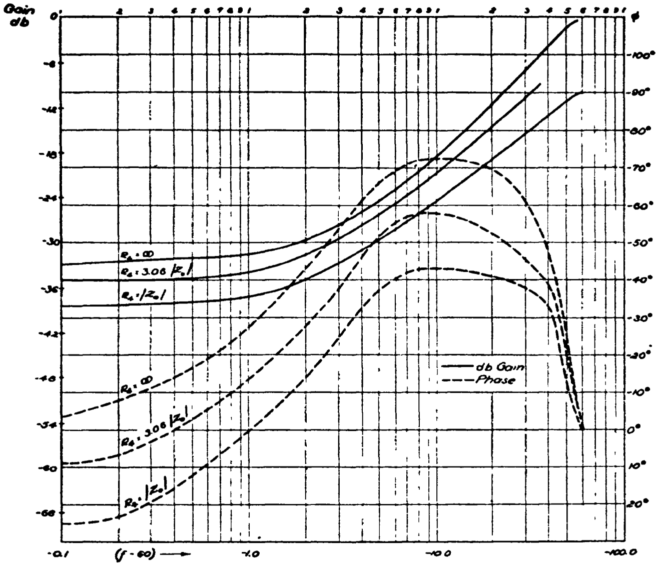


FIG. 6.

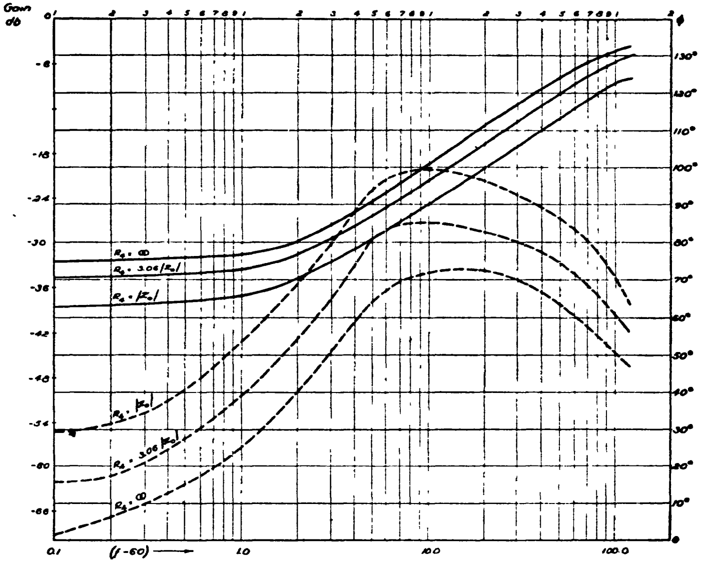


FIG. 7.

$$\text{Output Impedance } Z_o = \frac{7.19 + 25.4\left(\frac{\omega}{\omega_0}\right)^2 + 7.30\left(\frac{\omega}{\omega_0}\right)^4}{1 + 1.47\left(\frac{\omega}{\omega_0}\right)^2 + 84.7\left(\frac{\omega}{\omega_0}\right)^4 + 0.0486\left(\frac{\omega}{\omega_0}\right)^9} \times 10^{-3}$$

$$+ j\,\frac{-24.7\left(\frac{\omega}{\omega_0}\right) - 63.4\left(\frac{\omega}{\omega_0}\right)^3 - 266\left(\frac{\omega}{\omega_0}\right)^5}{1 + 1.47\left(\frac{\omega}{\omega_0}\right)^2 + 84.7\left(\frac{\omega}{\omega_0}\right)^4 + 0.0476\left(\frac{\omega}{\omega_0}\right)^6} \times 10^{-3}.$$

Notch width = 4 cps. ($T_d\omega_0 = 30$)
60 cps. Parallel "T"
$Z_o = 0.00128 + (-0.001502)$ at 60 cps.
$Z_o = 0.00197$ megohms
In case $C_1 = C_2 = C_3 = 1$ μfd., load resistor $R_4 = \infty, 0.00197, 0.00602$.

If a load impedance Z_L is connected to the output of the parallel "T," by Thevenin's theorem the circuit is equivalent to Fig. 2, where $B(p)$ is the parallel "T" transfer characteristic. If the output impedance is $Z_o = C - jD$, the phase shift is $Z_L/(C - jD + Z_L)$. Thus if Z_L is resistive, the phase shift approaches a lead of arc tan D/C as $Z_L \rightarrow 0$; and if $\angle Z_L = \theta > 0$, a phase lead up to $\theta +$ arc tan D/C may be obtained by taking Z_L to be sufficiently small.

Since the gain of the transfer characteristic is small, the effects of series input impedance and load impedance are nearly independent, and the total phase shift with both is approximately equal to the sum of the phase shifts which would be caused by each separately. Thus for example if Z is a large resistance, and Z_L a large capacity, a total phase lag of approximately $\angle Z_i + (90° - \angle Z_o)$ may be obtained.

If series input impedance Z and load impedance Z_L are connected to a transfer characteristic $B(p)$, Fig. 3, it may be shown that the resulting transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{B(p)}{\left(1 + \dfrac{Z}{Z_i}\right)\left(1 + \dfrac{z_{22}}{Z_L}\right) - \dfrac{Z_\imath}{Z_L}[B(p)]^2},$$

where $z_{22} = Z_o + Z_\imath[B(p)]^2$ is the output impedance with the input open-circuited.

Figures 4, 5, 6, and 7 show the effect of a load resistor on the parallel "T" transfer characteristic, for the case of equal condensers, with $T_d\omega_0 = 15$ and 30.

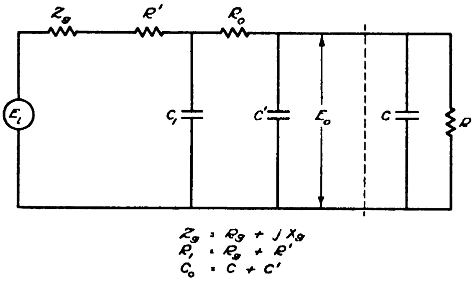


FIG. 8.

4. FORMULAE FOR PHASE-LAG NETWORKS.

Let the load on the phase-lag network at carrier frequency ω_0 be replaced by an equivalent resistor R and condenser C in parallel, as in Fig. 8. The capacity C is included in C_0; the source impedance Z_g, if purely resistive, is included in R_1. (A procedure for dealing with the case where Z_g is complex is indicated later.) The transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{R}{RT_1T_0p^2 + [R_0T_1 + R(T_0 + S_1 + T_1)]p + R + R_0 + R_1},$$

where $T_0 = R_0C_0$, $T_1 = R_1C_1$. Thus the angle of phase *lag* through the network is:

$$\phi = \text{arc tan}\,\frac{\omega_0[R(R_0 + R_1)C_0 + T_1(R_0 + R)]}{R + R_0 + R_1 - RT_0T_1\omega_0^2}.$$

Let $\tan\phi = -k$. Then the gain is:

$$g = \frac{R}{\pm\,(RT_0T_1\omega_0^2 - R - R_0 - R_1)(1 + k^2)^{\frac{1}{2}}},$$

where the plus sign corresponds to $\phi > 90°$, the minus sign to $\phi < 90°$.

Suppose that k, R, C_0, C_1 are given. Then the following formulae determine R_0, R_1 for maximum gain.

$$c(1 - kb)mR_0^2 + [b(m + 1) + c(n + mr - kbn) + k(1 + m)]R_0 + n\,(b + cR) + k(R + n) = 0,$$

$$R_1 = mR_0 + n,$$

where

$$b = \omega_0RC_0, \quad c = C_1\omega_0, \quad m = (b^2 + 1)/[(b^2 + 1) + bcR],$$
$$n = R/[(b^2 + 1) + bcR].$$

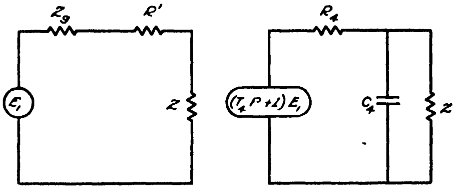


FIG. 9. Equivalent circuits.

If k, R, C_0, C_1 are given, the following formulae determine R_0, R_1 for a specified gain g:

$$c(1 - kb)R_0^2 - c(1 - kb)AR_0 - kR - (k + b)A = 0,$$
$$R_1 = A - R_0,$$

where $A = R[(kb - 1)g^{-1}(1 + k^2)^{-\frac{1}{2}} + kb - 1 - k]/(b + k + 1 - kb)$.

In case Z_g is complex, let $Z_g + R' = x + jy$. Then the voltage source E_1 with impedance $x + jy$, at frequency ω, is equivalent to a source $(T_4p + 1)E_1$ with resistance R_4, paralleled by a (positive or negative) capacity C_4, where $x = R_4/(1 + T_4^2\omega^2)$, $y = -R_4T_4\omega/(1 + T_4^2\omega^2)$, $T_4 = R_4C_4$, or $R_4 = x + y^2/x$, $T_4\omega = -y/x$. (See Fig. 9.) Thus to compute the phase shift, R_4 may be substituted for R_1, and C_1 modified by the addition of C_4. Then the phase lag with respect to E_1 is ϕ − arc tan $T_4\omega_0$, and the gain is $g(1 + T_4^2\omega^2)^{\frac{1}{2}}$.

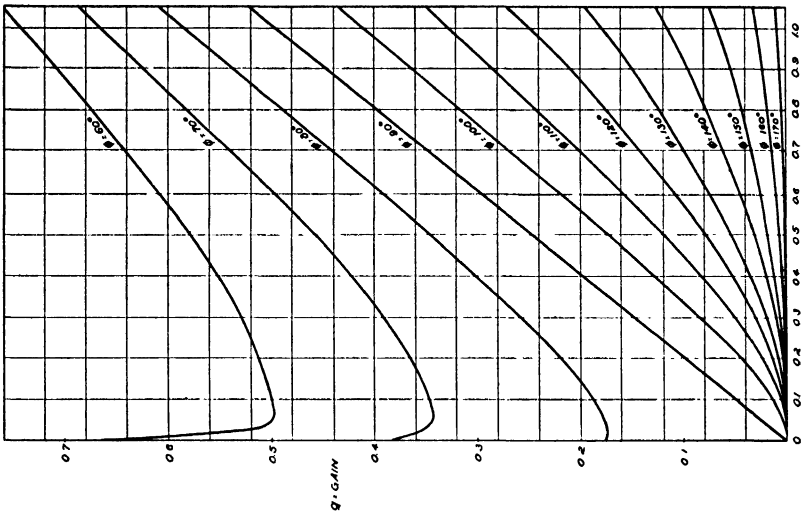

φ = 60°
φ = 70°
φ = 80°
φ = 90°
φ = 100°
φ = 110°
φ = 120°
φ = 130°
φ = 140°
φ = 150°
φ = 160°
φ = 170°
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
η = ζ⁻¹
0.1
0.2
0.3
0.4
0.5
0.6
0.7
q = GAIN

FIG. 11.

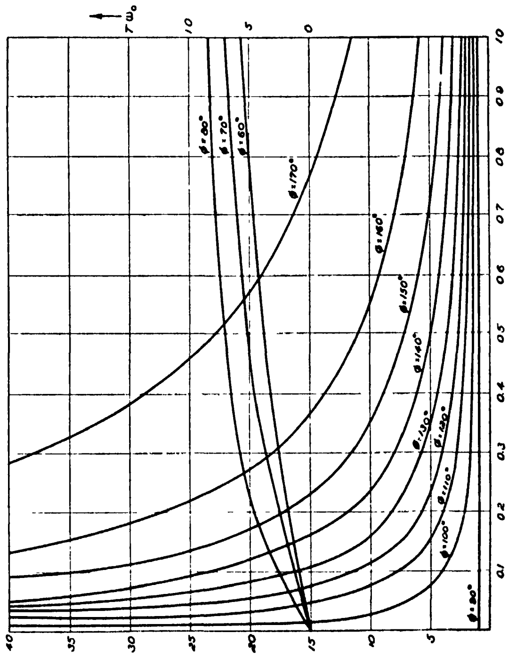

Tω₀
10
5
0
φ = 80°
φ = 70°
φ = 60°
φ = 170°
φ = 160°
φ = 150°
φ = 140°
φ = 130°
φ = 120°
φ = 110°
φ = 100°
φ = 90°
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
η = ζ⁻¹
40
35
30
25
20
15
10
5
Tθ₀

FIG. 10.

5. NUMERICAL VALUES FOR PHASE-LAG NETWORKS.

Consider the transfer characteristic

$$\frac{1}{T^2p^2 + 2\zeta Tp + 1}.$$

The phase lag ϕ of this transfer characteristic at frequency ω_0 is

$$\phi = \arc \tan \frac{2\zeta T\omega_0}{(1 - T^2\omega_0^2)}.$$

Figure 10 is a graph of the values of $\zeta \geqq 1$ and $T\omega_0$ for phase-lag angles ϕ from 60° to 170°, and Fig. 11 is a graph of corresponding values of gain at ω_0 as a function of $\zeta^{-1} = \eta$.

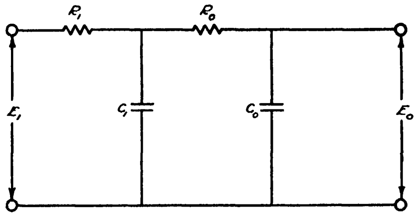


FIG. 12.

The transfer characteristic for the two-section phase-lag network with no load resistor, Fig. 12, is:

$$\frac{E_0}{E_1} = \frac{1}{T_1T_0p^2 + (T_0 + S_1 + T_1)p + 1},$$

where $T_1 = R_1C_1$, $T_0 = R_0C_0$, $S_1 = R_1C_0$. The input impedance is

$$Z_i = \frac{T_1T_0p^2 + (T_0 + S_1 + T_1)p + 1}{C_1T_0p^2 + (C_0 + C_1)p}.$$

The output impedance (with the input short-circuited) is

$$Z_o = \frac{R_0T_1p + R_0 + R_1}{T_1T_0p^2 + (T_0 + S_1 + T_1)p + 1}.$$

If the condensers C_0, C_1 are specified, the following formulae give the two solutions for R_0, R_1 to realize any desired ζ and T:

$$R_1 = \frac{\zeta T \pm T[\zeta^2 - 1 - (C_0/C_1)]^{\frac{1}{2}}}{(C_0 + C_1)},$$

$$R_0 = \frac{\zeta T \mp T[\zeta^2 - 1 - (C_0/C_1)]^{\frac{1}{2}}}{C_0}.$$

These solutions, for several sets of values for C_0, C_1, are the basis for the

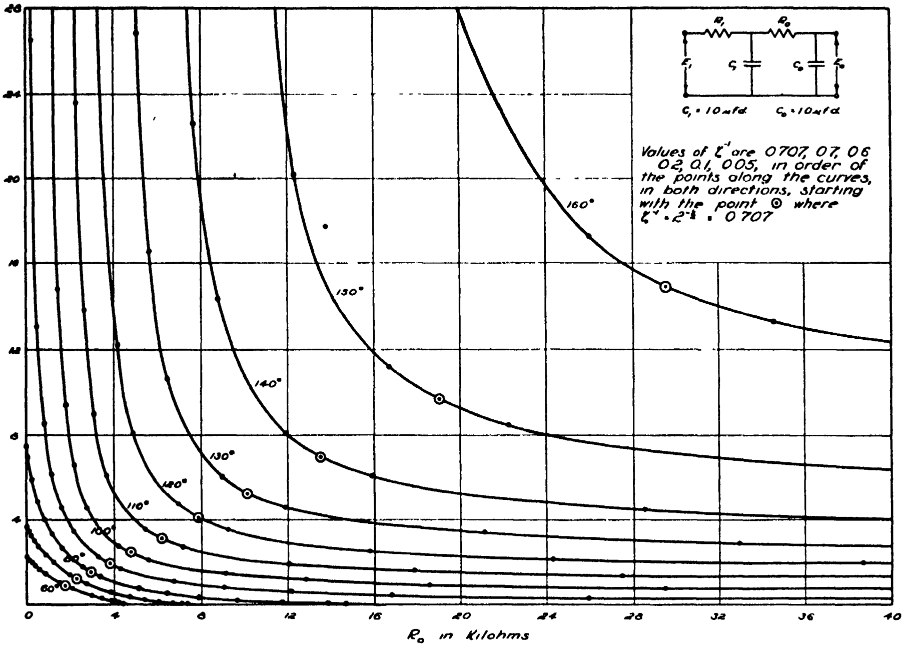


FIG. 13. Design chart for phase-lag networks.

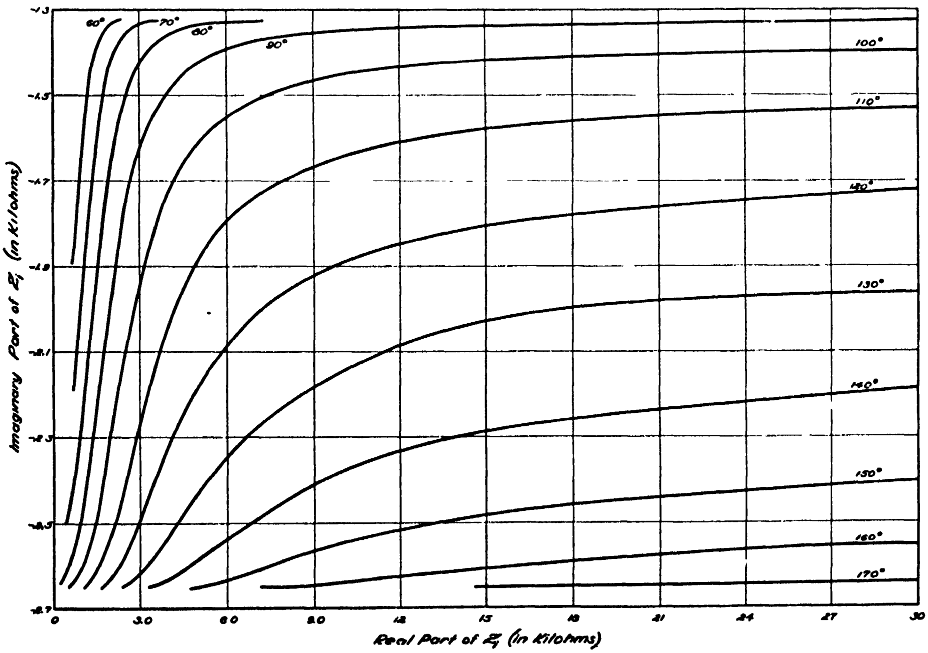


FIG. 14. Input impedances of phase-lag networks.
$C_1 = 1.0$ μfd., $C_0 = 1.0$ μfd., ζ, R_1, and R_0 as in Fig. 13.

design charts of Figs. 13, 16, 19, 22, and 25. Corresponding values of the impedances Z_i, Z_o may be obtained from Figs. 14, 15, 17, 18, 20, 21, 23, 24, 26, and 27.

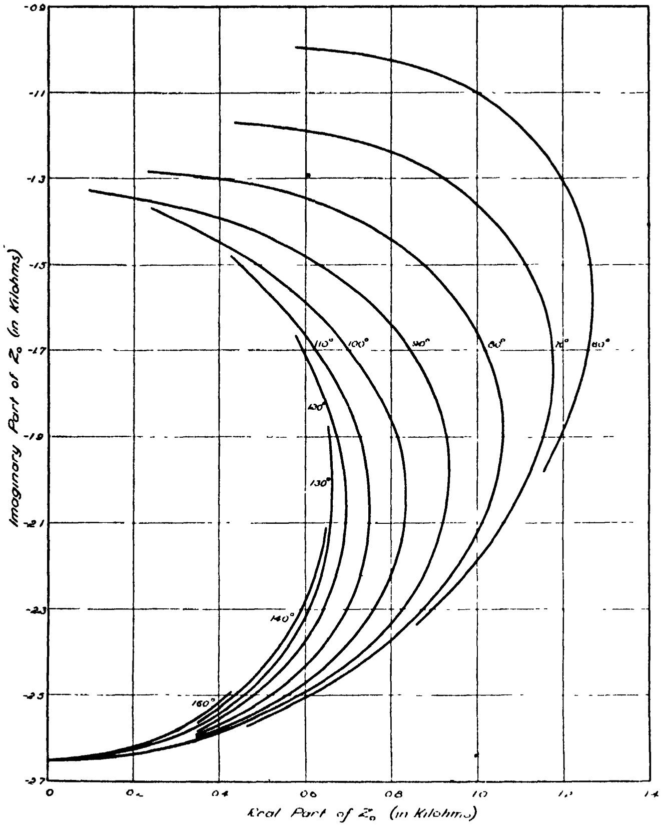


FIG. 15. Output impedances of phase-lag networks.
$C_1 = 1.0$ μfd., $C_0 = 1.0$ μfd., ζ, R_1, and R_0 as in Fig. 13.

If a load resistor R is connected across the output in Fig. 12, as in section 4, the transfer characteristic is:

$$\frac{E_0}{E_1} = \frac{R/(R + R_0 + R_1)}{T_1T_0p^2R/(R + R_0 + R_1) + [R_0T_1 + R(T_0 + S_1 + T_1)]p/(R + R_0 + R_1) + 1}.$$

Thus T and ζ become T' and ζ', where $T' = [R/(R + R_0 + R_1)]^{\frac{1}{2}}T$, $2\zeta'T' = [R_0T_1 + 2\zeta TR]/(R + R_0 + R_1)$. If T' and ζ' are calculated, the new phase shift may be determined from Fig. 10, and the new gain is $R/(R + R_0 + R_1)$ times the value corresponding to ζ' and T' in Fig. 11. Or, if the output impedance Z_o of the phase-lag network is at

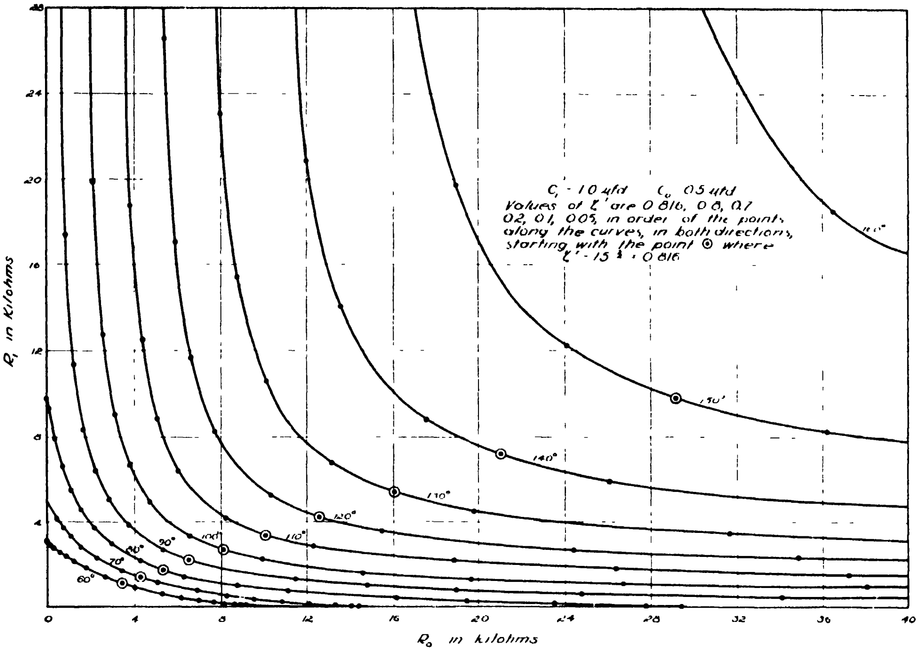


FIG. 16. Design chart for phase-lag networks.

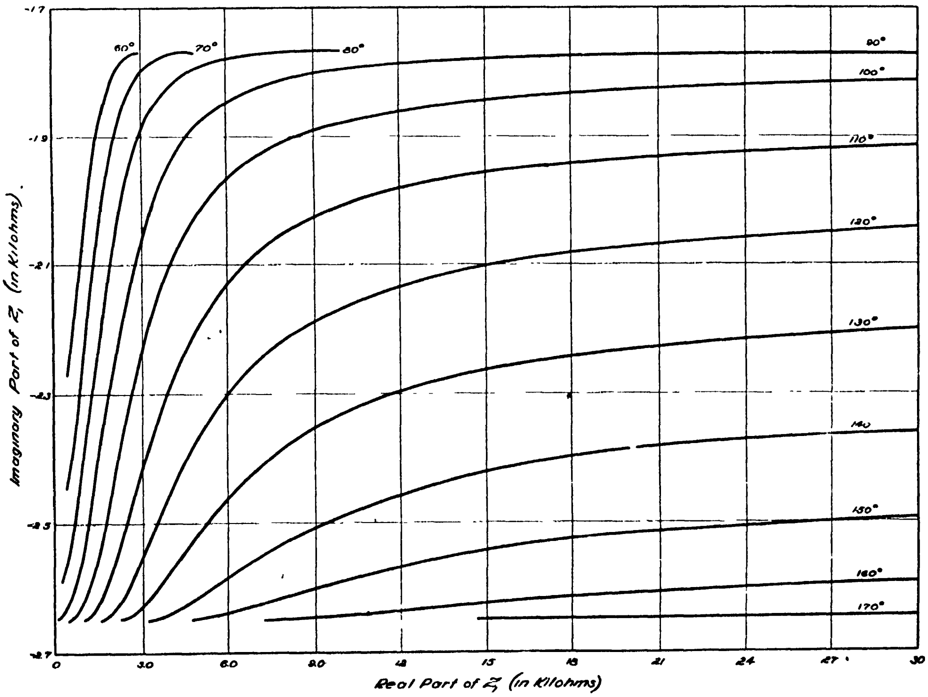


FIG. 17. Input impedances of phase-lag networks.
C_1 = 1.0 μfd., C_0 = 0.5 μfd., ζ, R_1, and R_0 as in Fig. 16.

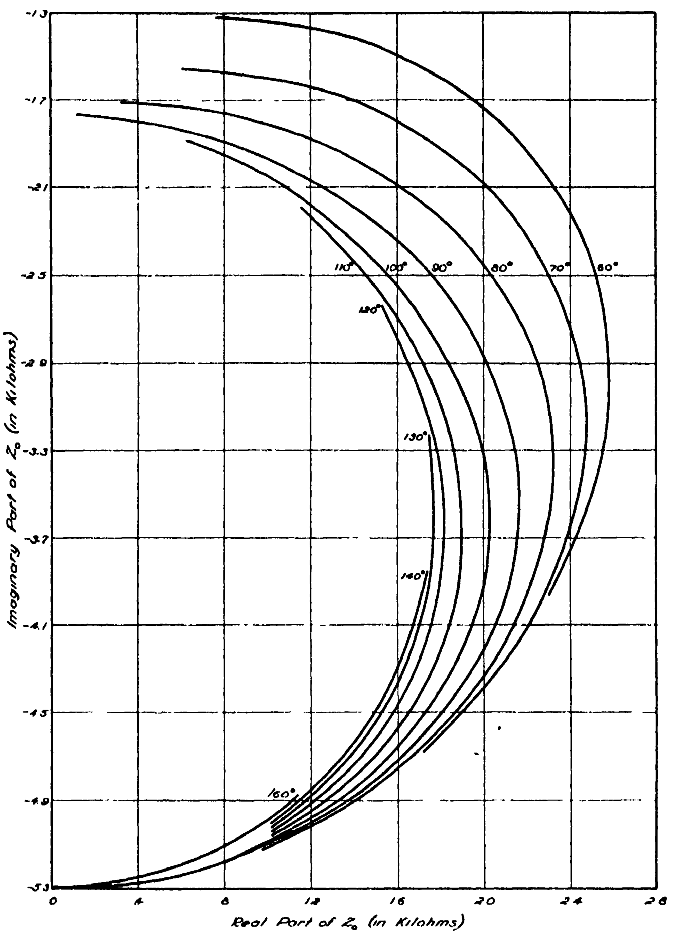


FIG. 18. Output impedances of phase-lag networks.
C_1 = 1.0 μfd., C_0 = 0.5 μfd., ζ, R_1, and R_0 as in Fig. 16.

hand, the change of phase due to any load impedance may easily be calculated.

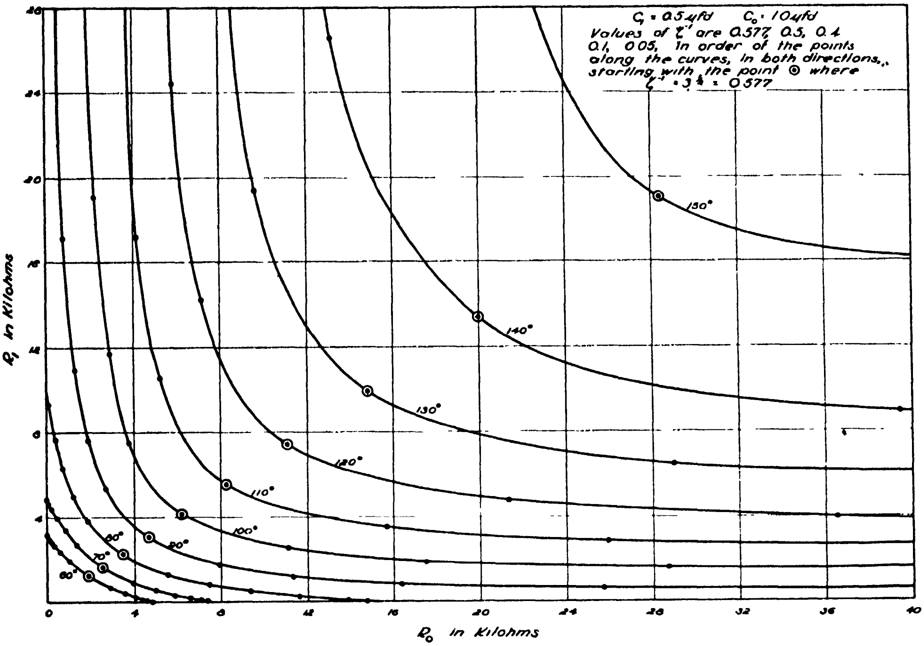


FIG. 19. Design chart for phase-lag networks.

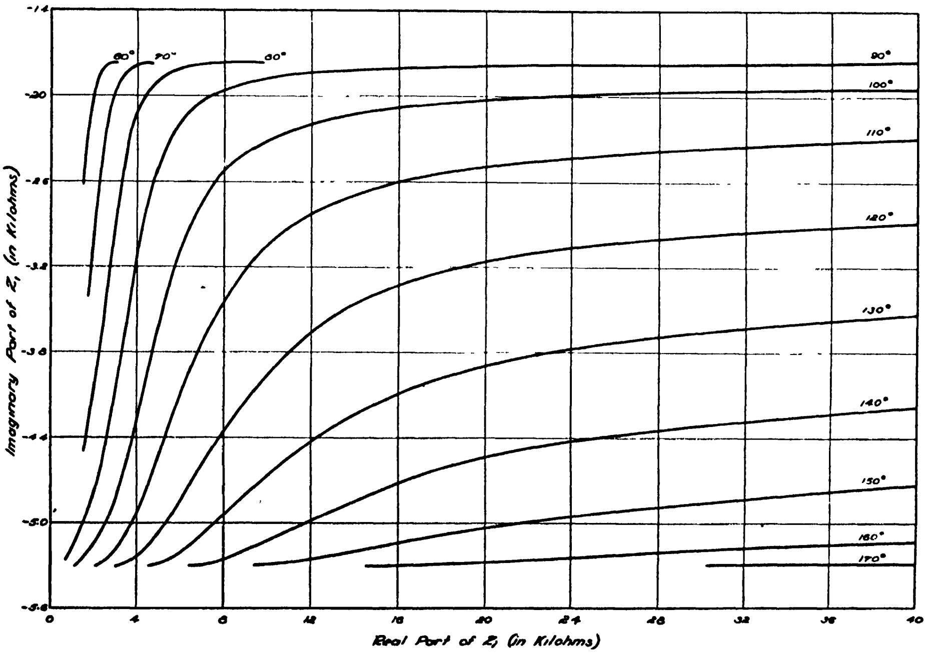


FIG. 20. Input impedances of phase-lag networks.
$C_1 = 0.5$ μfd., $C_0 = 1.0$ μfd., ζ, R_1, and R_0 as in Fig. 19.

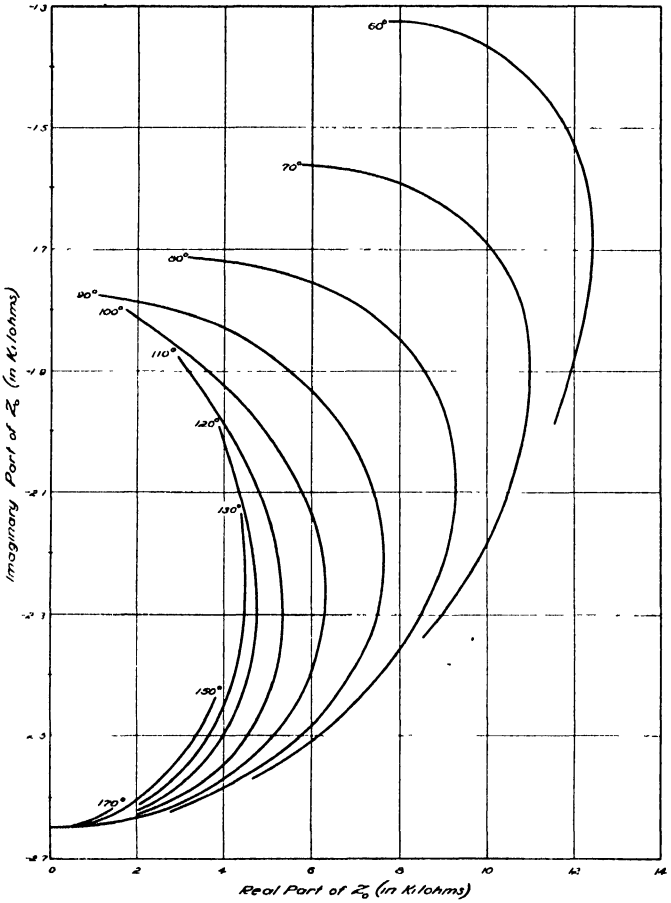


FIG. 21. Output impedances of phase-lag networks. $C_1 = 0.5$ μfd., $C_0 = 1.0$ μfd., ζ, R_1, and R_0 as in Fig. 19.

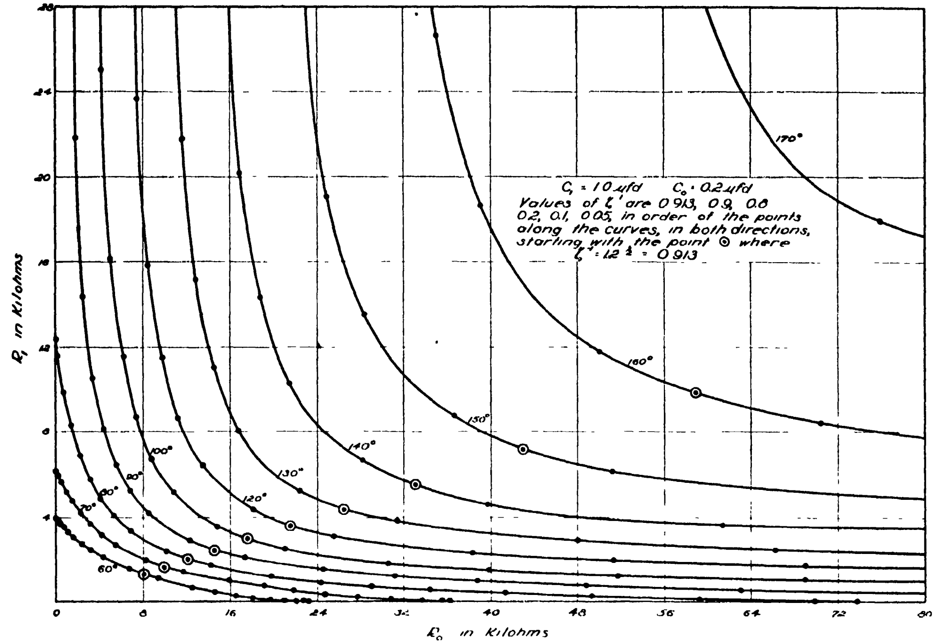


FIG. 22. Design chart for phase-lag networks.

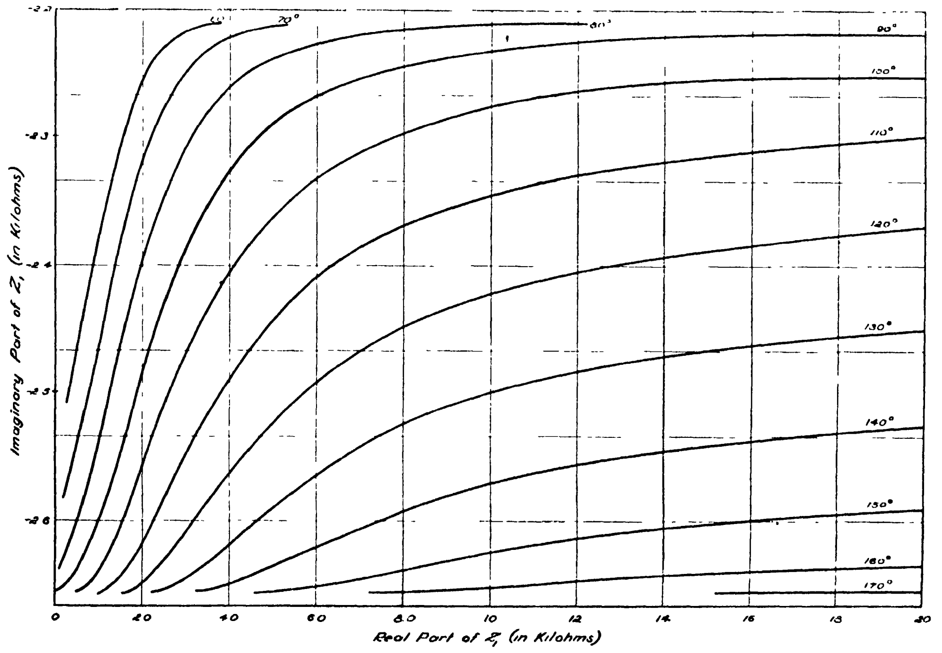


FIG. 23. Input impedances of phase-lag networks.
C_1 = 1.0 μfd., C_0 = 0.2 μfd., ζ, R_1, and R_0 as in Fig. 22.

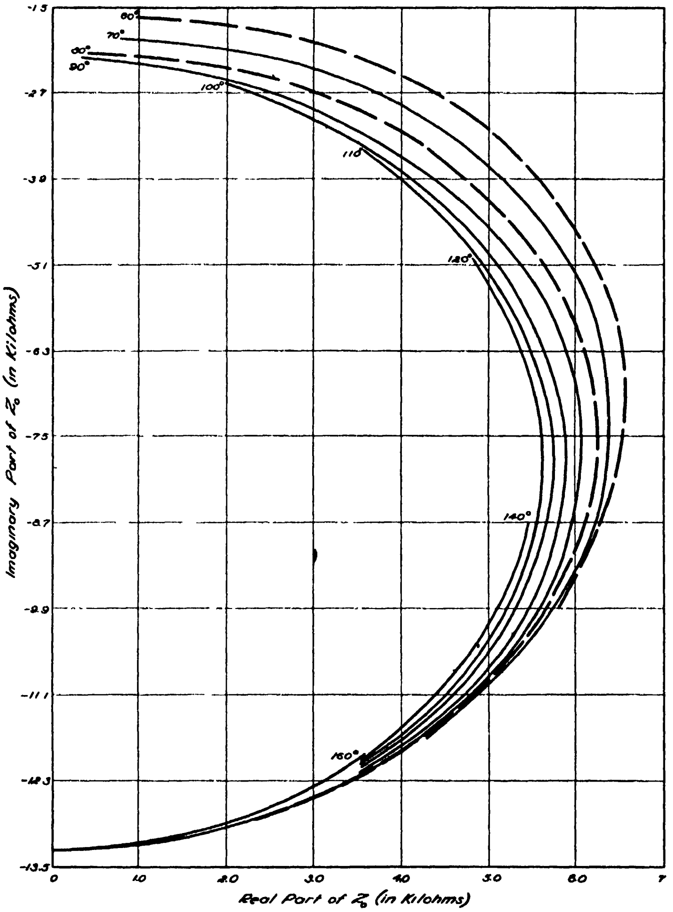


FIG. 24. Output impedances of phase-lag networks. C_1 = 1.0 μfd., C_0 = 0.2 μfd., ζ, R_1, and R_0 as in Fig. 22.

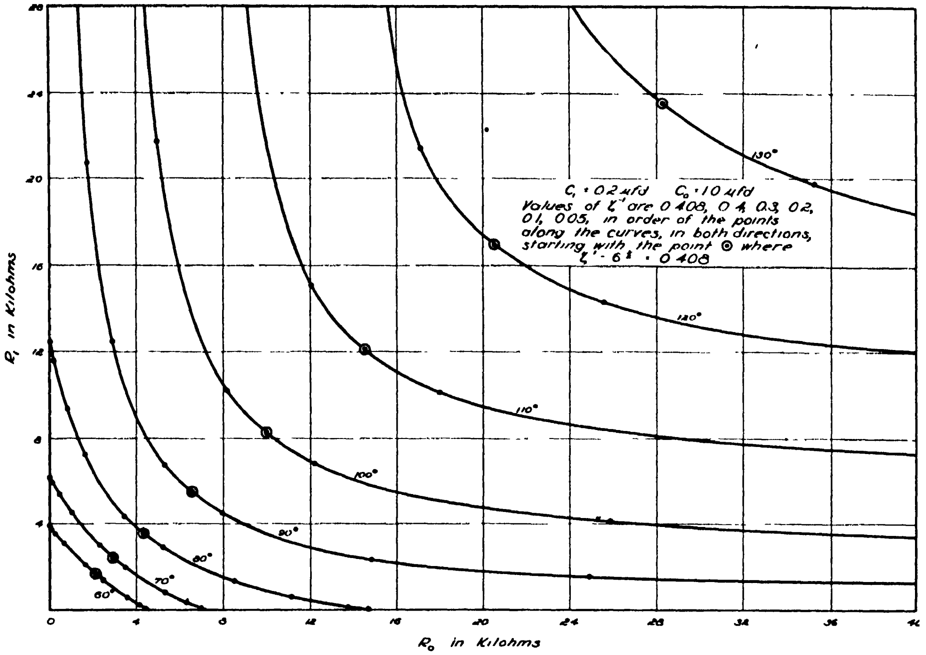


FIG. 25. Design chart for phase-lag networks.

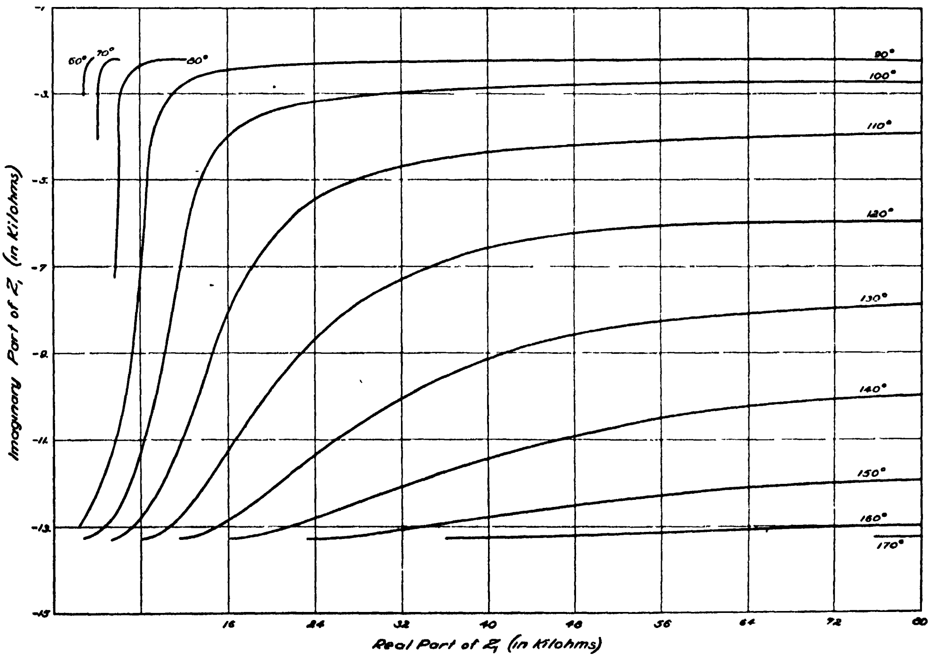


FIG. 26. Input impedances of phase-lag networks. $C_1 = 0.2\ \mu$fd., $C_0 = 1.0\ \mu$fd., ζ, R_1, and R_0 as in Fig. 25.

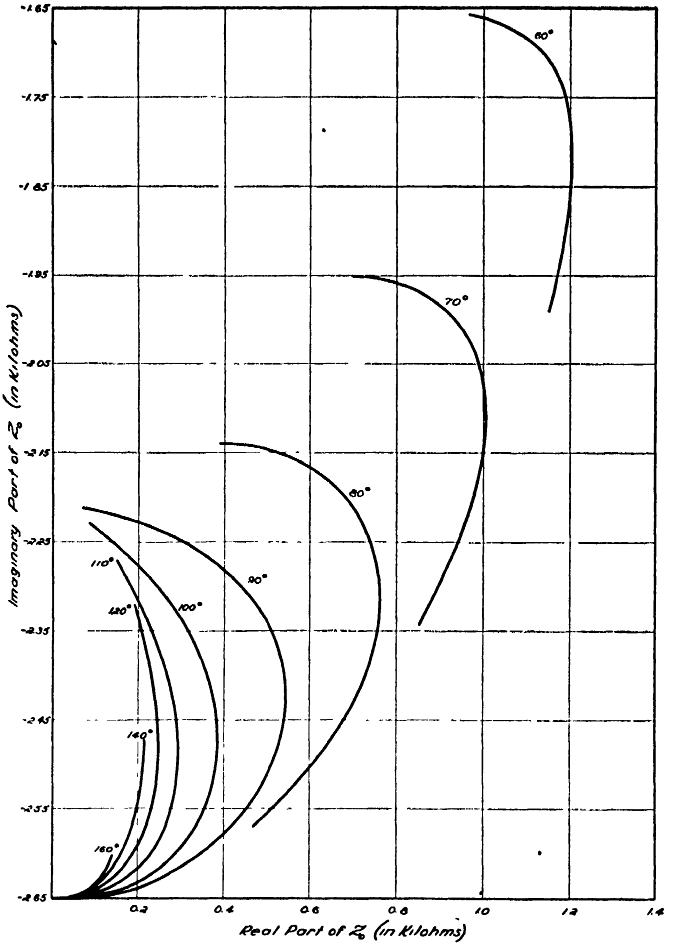


FIG. 27. Output impedances of phase-lag networks.
$C_1 = 0.2$ μfd., $C_0 = 1.0$ μfd., ζ, R_1, and R_0 as in Fig. 25.

6. OVERALL TRANSFER CHARACTERISTIC FOR BRIDGE "T" AND PHASE-LAG NETWORK.

From the form of the input and output impedances of the bridge "T" and of the parallel "T," it may be seen that the essential proportional-derivative numerator factor, $[\omega_0^{-2}p^2 + 2(n\omega_0)^{-1}p + 1]$, is preserved in the overall transfer characteristic for either of the phase-lag circuits of Fig. 28. This is true also for the methods of series input impedance or load impedance. In either case of Fig. 28, and for non-inductive series input or load impedances, the overall proportional-derivative characteristic is not very seriously disturbed since it is known that the denominator of the transfer characteristic for any R-C network can have no complex roots. For the parallel "T," the transfer characteristic for either circuit of Fig. 28 will be of the form:

$$\frac{E_o}{E_i} = \frac{(U_3p + 1)\left(\frac{1}{\omega_0^2}p^2 + \frac{2}{n\omega_0}p + 1\right)}{(U_4p + 1)(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)}, \tag{5}$$

where $\zeta_1 > 1$ and $\zeta_2 > 1$. For the bridge "T," the transfer characteristic will be of the same form with $U_3 = U_4 = 0$.

When two networks are connected in cascade, as for example the parallel "T" and phase-lag networks in Fig. 28, there will be no phase shift due to the connection of the output of the first network to the input of the second, if the angle of the output impedance of the first network is equal to the angle of the input impedance of the second. This of course does not mean that the overall transfer characteristic is the product of a constant gain $\leqq 1$ and the separate transfer characteristics for each network. In fact if $B(p)$, $B'(p)$ are the separate transfer characteristics for each of the two networks, Fig. 29, the overall transfer characteristic is:

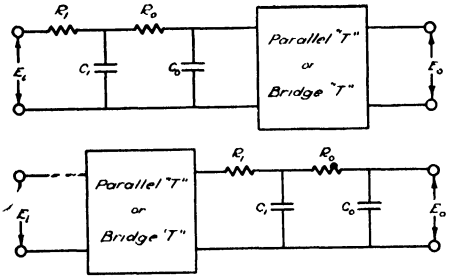


FIG. 28.

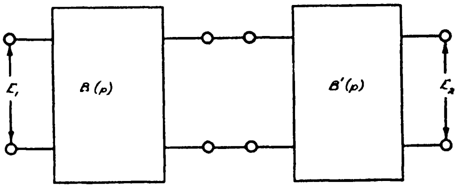


FIG. 29.

$$\frac{E_2}{E_1} = \frac{B(p)B'(p)}{1 + \frac{z_{22}}{z_{11}'} - [B(p)]^2\frac{z_{11}}{z_{11}'}} = \frac{z_{11}'}{(z_{11}' + Z_o)}B(p)B'(p), \tag{6}$$

where z_{11}, z_{11}' are the separate input impedances, and $z_{22} = Z_o + z_{11}[B(p)]^2$ is the impedance looking into the output of the first network with the input open-circuited. This follows from the transfer characteristic of Fig. 3, where $Z = 0$; or immediately by Thevenin's theorem.

For the combination of phase-lag network and bridge "T" as in Fig. 30, the transfer characteristic is:

$$\frac{E_o}{E_i} = \frac{\frac{1}{\omega_0^2}p^2 + \frac{2}{n\omega_0}p + 1}{Ap^4 + Bp^3 + Cp^2 + Dp + 1}, \tag{7}$$

where

$$A = \frac{T_0T_1}{\omega_0^2} = \frac{T^2}{\omega_0^2}$$

$$B = \frac{(T_0 + S_1)}{\omega_0^2} + \frac{2T_1T_0}{m\omega_0} + \frac{T_1(R_0/R_2 + 1)}{\omega_0^2} = \frac{2\zeta T}{\omega_0^2} + \frac{2T^2}{m\omega_0} + \frac{T^2}{R_2C_0\omega_0^2},$$

$$C = \frac{2(T_0 + S_1 + T_1)}{m\omega_0} + \frac{(R_0/R_2 + 1)}{\omega_0^2} + \frac{R_1}{R_2\omega_0^2} + T_0T_1 + \frac{2T_1R_0/R_2}{n\omega_0}$$

$$= \frac{4\zeta T}{m\omega_0} + \frac{1}{\omega_0^2} + \frac{(2\zeta T - T_1)}{R_2C_0\omega_0^2} + T^2 + \frac{2T^2}{R_2C_0n\omega_0},$$

$$D = (T_0 + S_1 + T_1) + \frac{2(R_0 + R_1)/R_2}{n\omega_0} + \frac{2}{m\omega_0}$$

$$= 2\zeta T + \frac{2(2\zeta T - T_1)}{R_2C_0n\omega_0} + \frac{2}{m\omega_0}.$$

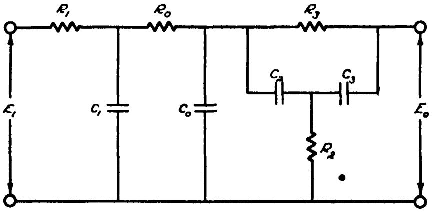


FIG. 30.

The transfer characteristic $B(p)$ of the phase-lag network is:

$$B(p) = \frac{1}{T^2p^2 + 2\zeta Tp + 1} = \frac{1}{T_1T_0p^2 + (T_0 + S_1 + T_1)p + 1},$$

where $T_1 = R_1C_1$, $T_0 = R_0C_0$, $S_1 = R_1C_0$. The transfer characteristic $B'(p)$ of the bridge "T" is:

$$B'(p) = \frac{\frac{1}{\omega_0^2}p^2 + \frac{2}{n\omega_0}p + 1}{\frac{1}{\omega_0^2}p^2 + \frac{2}{m\omega_0}p + 1} = \frac{T_2T_3p^2 + (T_2 + S_2)p + 1}{T_2T_3p^2 + (T_2 + S_2 + T_3)p + 1},$$

where $T_2 = R_2C_2$, $T_3 = R_3C_3$, $S_2 = R_2C_3$.

The transfer characteristic (7) is of the form (5) with $U_3 = U_4 = 0$. The denominator consists of a product of four linear factors:

$$(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)$$

$$= \left(\frac{\mu_1}{\omega_0}p + 1\right)\left(\frac{\mu_2}{\omega_0}p + 1\right)\left(\frac{\mu_3}{\omega_0}p + 1\right)\left(\frac{\mu_0}{\omega_4}p + 1\right).$$

$\left(\text{The phase lag } \phi_i \text{ at } \omega_0 \text{ due to a linear factor } \left(\frac{\mu_i}{\omega_0}p + 1\right) \text{ is given by } \tan \phi_i = \mu_i.\right)$ Since there are several combinations of products of pairs of the linear factors, U_1, ζ_1, U_2, ζ_2 of course are not unique for specified A, B, C, and D. The relations between U_1, U_2, ζ_1, ζ_2, and A, B, C, and D are:

$$\begin{aligned} A &= U_1^2U_2^2, \\ B &= 2U_1U_2(\zeta_1U_2 + \zeta_2U_1), \\ C &= 4\zeta_1\zeta_2U_1U_2 + U_1^2 + U_2^2, \\ D &= 2(\zeta_1U_1 + \zeta_2U_2). \end{aligned}$$

The procedure now will be indicated for answering the following two questions: (1) Given a bridge "T," and a phase-lag network either determined experimentally or designed by the method of a previous section to give the appropriate carrier phase shift, what are the values of U_1, ζ_1, U_2, ζ_2? (2) Given a bridge "T," what should be the values of the components of a phase-lag network in order to realize a specified set of values of U_1, U_2, ζ_1, ζ_2 or A, B, C, and D?

To answer (1), the values of A, B, C, and D may be calculated by the above formulae, and the roots for $x = (p/\omega_0)$ of the equation

$$A\omega_0^4x^4 + B\omega_0^3x^3 + C\omega_0^2x^2 + D\omega_0x + 1 = 0$$

found by numerical methods. Each of the four quantities μ_i in the factored form of the denominator then is given by $\mu_i = -1/x_i$, where x_i is one of the roots. The effect of the phase-lag network on stability of the bridge "T" servo then may be determined by referring to Fig. 32 in the next section.

To answer (2), we solve the second of the above sets of expressions for A, B, C, and D for T, ζ, $S_2 = R_2C_0$, $T_1 = R_1C_1$ in terms of A, B, C, and D. Since the bridge "T" is given, R_2 is known, and S_2 determines C_0. Formulae for the design of an unloaded phase lag network to realize any T, ζ have been given in section 4; from the formula for R_1, it follows that the ratio $r = C_0/C_1$ may be found from T_1 by solving the quadratic equation:

$$T_1^2r^2 + (2T_1^2 - 2\zeta TT_1 + T^2)r + (T^2 - 2\zeta TT_1 + T_1^2) = 0.$$

The values of C_0 and r determine C_1; and R_0 is given by the formula of section 4 for R_0 in terms of ζ, T, r, and C_0. Thus if T, ζ, S_2, T_1 to realize a given set of A, B, C, and D are known, the phase-lag network components R_1, R_0, C_1, C_0 are determined.

The solutions for T, ζ, S_2, T_1 in terms of A, B, C, and D are as follows:

$$T = \omega_0 A^{\frac{1}{4}},$$

$$\zeta \text{ is a root of } a\zeta^2 + b\zeta + c = 0,$$

where

$$a = \left[\frac{8T^2}{m\omega_0^2}\left(\frac{4}{m} - \frac{4}{n} - n\right)\right],$$

$$b = \left[\frac{4nTD''}{m\omega_0^2} + \frac{16TB'}{mn} + \frac{2TC''}{\omega_0}\left(\frac{4}{n} - \frac{8}{m} + n\right)\right],$$

$$c = \left[2(C'')^2 - \frac{n}{\omega_0}C''D'' - \frac{4\omega_0}{n}B'C''\right],$$

$$C'' = C - \frac{1}{\omega_0^2} - T^2, \quad D'' = D - \frac{2}{m\omega_0}, \quad B' = B - \frac{2T^2}{m\omega_0};$$

$$T_1 = 2\zeta T + \frac{2T^2D'\omega_0}{(nD' - 2C'\omega_0)},$$

$$S_2 = \frac{2T^2D'}{(2C'\omega_0 - nD')C'\omega_0} + \frac{2T^2}{nC'\omega_0},$$

where

$$C' = C'' - \frac{4\zeta T}{m\omega_0}, \qquad D' = D'' - 2\zeta T.$$

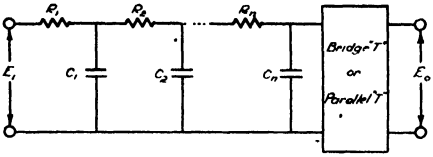


FIG. 31.

7. MULTI-SECTION PHASE-LAG NETWORKS. GAIN-PHASE MARGIN DIAGRAMS.

The transfer characteristic for an n-section R-C phase-lag network followed by a bridge "T," or for a bridge "T" followed by an n-section phase-lag network, is of the form:

$$\frac{E_o}{E_i} = \frac{\dfrac{1}{\omega_0^2}p^2 + \dfrac{2}{n\omega_0}p + 1}{(U_1p + 1)(U_2p + 1)\cdots(U_np + 1)(T_1p + 1)(T_2p + 1)}.$$

Excluding the proper proportional-derivative factor

$$(\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)(n\omega_0/2p),$$

the phase lag at any frequency ω of the remaining portion of the transfer characteristic is:

$$\phi = \Sigma \arctan U_i\omega + \arctan T_1\omega + \arctan T_2\omega - 90°.$$

Clearly, as $\omega \rightarrow \infty$, $\phi \rightarrow (n + 1)90°$, so that for a phase lag ϕ_0 at ω_0, it is best to keep the number of sections n as small as possible, in order to minimize the change of phase with frequency in the vicinity of $\omega = \omega_0$. The frequency characteristic, on the other hand, is most flat, and the gain is largest, when the number of sections is large. For n *R*-*C* sections of equal time constant, as $n \rightarrow \infty$, the transfer characteristic $1/(Tp/n + 1)^n \rightarrow \exp(-Tp)$. Thus the limiting frequency characteristic is $|\exp(-j\omega T)| \equiv 1$ and the limiting phase lag is $\phi = \omega T$.

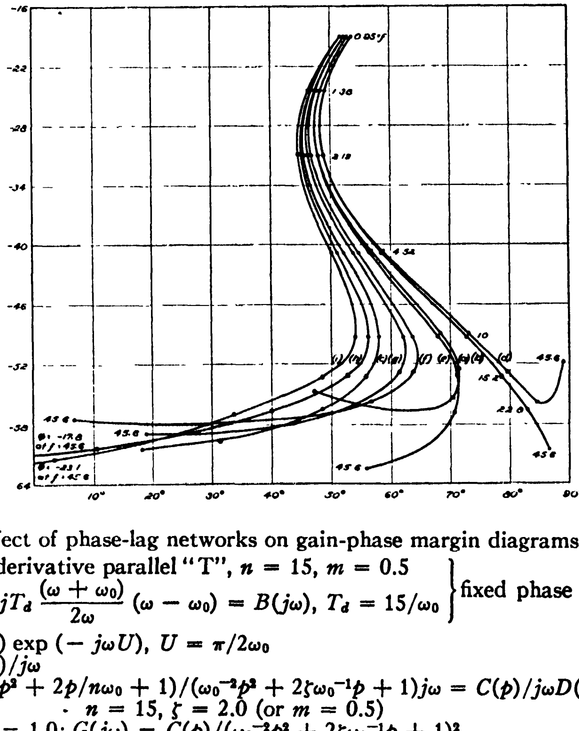


FIG. 32. Effect of phase-lag networks on gain-phase margin diagrams.

(*a*) Proportional-derivative parallel "T", $n = 15$, $m = 0.5$

(*b*) $G(j\omega) = 1 + jT_d \dfrac{(\omega + \omega_0)}{2\omega}(\omega - \omega_0) = B(j\omega)$, $T_d = 15/\omega_0$

(*a*), (*b*): fixed phase 90° from carrier

(*c*) $G(j\omega) = B(j\omega)\exp(-j\omega U)$, $U = \pi/2\omega_0$

(*d*) $G(j\omega) = B(j\omega)/j\omega$

(*e*) $G(j\omega) = (\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)/(\omega_0^{-2}p^2 + 2\zeta\omega_0^{-1}p + 1)j\omega = C(p)/j\omega D(p)$, $n = 15$, $\zeta = 2.0$ (or $m = 0.5$)

(*f*) $\zeta = 2.0$, (*i*) $\zeta = 1.0$; $G(j\omega) = C(p)/(\omega_0^{-2}p^2 + 2\zeta\omega_0^{-1}p + 1)^2$

(*g*) $U_1\omega_0 = 2.68$, $U_2\omega_0 = 0.37$, $\zeta_1 = \zeta_2 = 2.0$,

(*h*) $U_1\omega_0 = 0.87$, $U_2\omega_0 = 1.73$, $\zeta_1 = \zeta_2 = 1.0$;

$$G(j\omega) = \frac{(\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)}{(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)}$$

(*c*)–(*h*): fixed phase 0° from carrier applied to $G(j\omega)$

The gain-phase margin diagrams in Fig. 32, together with the *M* and *E* curves of Fig. 6, Part I, show the effect on error and stability of various methods of obtaining 90° phase lag, for a servo using the simplified motor with time constant $T = 0.2$ sec. Curve (*a*) is the decibel-phase margin plot for the proportional-derivative parallel "T" transfer characteristic $(\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)/(\omega_0^{-2}p^2 + 2\zeta\omega_0^{-1}p + 1)$, where $n = T_d\omega_0 = 15$, $\zeta = m^{-1} = 2.0$. (This is the same as curve (*d*) in Fig. 9, Part I.) Curve (*b*) is the decibel-phase margin plot for

$G(j\omega) = 1 + jT_d \dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)$. (For curves (*a*) and (*b*) the fixed phase excitation is 90° out of phase with the carrier; for curves (*c*) through (*i*) the fixed phase excitation is in phase with the carrier applied to $G(j\omega)$.) Curve (*c*) is the plot for $G(j\omega) = \left[1 + jT_d \dfrac{\omega+\omega_0}{2\omega}(\omega - \omega_0)\right] e^{-i\omega U}$, where $U = \pi/2\omega_0$. This corresponds to a 90° phase-lag network having a very large number of sections and gain nearly 1. Curve (*d*) is the plot for $G(j\omega) = \left[1 + jT_d \dfrac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right]\dfrac{1}{j\omega}$; curve (*e*) for

$$\left[1 + jT_d \frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right] \Big/ \left[1 + jl \frac{\omega + \omega_0}{2\omega}(\omega - \omega_0)\right] j\omega$$
$$= (\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)/(\omega_0^{-2}p^2 + 2\zeta\omega_0^{-1}p + 1)j\omega.$$

The latter characteristic is approached by a single-section R-C network as its phase lag approaches 90° and its gain approaches 0. Curves (*f*) and (*i*) are the plots for $G(j\omega) = (\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)/(\omega_0^{-2}p^2 + 2\zeta\omega_0^{-1}p + 1)^2$, where $\zeta = 2.0$ for (*f*), $\zeta = 1.0$ for (*i*); and curves (*g*) and (*h*) for

$$G(j\omega) = \frac{(\omega_0^{-2}p^2 + 2p/n\omega_0 + 1)}{(U_1^2p^2 + 2\zeta_1U_1p + 1)(U_2^2p^2 + 2\zeta_2U_2p + 1)},$$

where $U_1\omega_0 = 2.682$, $U_2\omega_0 = 0.373$, $\zeta_1 = \zeta_2 = 2.0$ for (*g*), and $U_1\omega_0 = 3^{-\frac{1}{2}}$, $U_2\omega_0 = 3^{\frac{1}{2}}$, $\zeta_1 = \zeta_2 = 1.0$ for (*h*). The first factor produces 60°-phase lag, and the second factor 120°-phase lag, so that with the numerator lead of 90°, the net phase lag is 90°.

8. TOLERANCE REQUIREMENTS FOR PHASE-LAG NETWORKS.

To illustrate the method of calculation of tolerances on the components of a phase-lag network, in this section we state the results of the derivation of the variational conditions for a phase-lag network to precede the bridge "T" indicated in Fig. 33 (same as Fig. 12 in section 15 of Part II). Suppose that the desired phase lag is 120°.

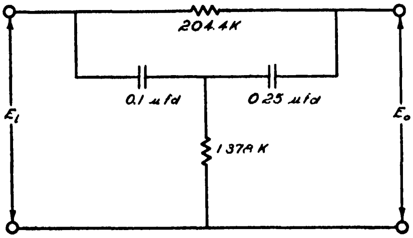


FIG. 33.

The input impedance to the bridge "T" at 60 cycles is replaced by an equivalent parallel condenser $C = 0.099$ μfd. and resistor $R = 0.149$

megohm. A network to give 120° phase lag and a gain of 0.203 to the bridge "T" input is as follows: input resistor $R_2 = 4.76k$, $R_0 = 27.2k$, input condenser $C_2 = 1.0\ \mu$fd., output condenser $C' = 0.1\ \mu$fd. For this phase-lag network, the tolerance requirements for phase shift of $-120°$ correct to within $\pm 5°$ are:

$$-0.168 < 0.29\Delta + 0.85\Delta_0 + 0.89\Delta_2 + 0.91\epsilon_0 + 0.95\epsilon_2,$$
$$0.24\Delta + 0.43\Delta_0 + 0.45\Delta_2 + 0.47\epsilon_0 + 0.51\epsilon_2 < 0.121,$$

where $C_0 = C' + C$, $\epsilon_0 = \Delta C_0/C_0$, $\Delta = \Delta R/R$. If Z is the impedance of R and C in parallel, $Z = R/(Tj\omega + 1)$, where $T = RC$, and

$$\frac{dZ}{Z} = \frac{dR}{R} - \frac{T\omega^2 dT}{(1 + T^2\omega^2)} - \frac{j\omega dT}{(1 + T^2\omega^2)}$$
$$= -4.45\Delta - 5.45\epsilon - 0.985j(\Delta + \epsilon).$$

From this and the expression for $\Delta Z_i/Z_i$ in section 15 of Part II, assuming 0.6 per cent tolerance on the bridge "T" components, it follows that $(\Delta C/C_0) = (a - 4.45b)/1.99$, $\Delta = (5.45b - a)$, where a and b in per cent may have any values in the intervals $-0.507 \leqq b \leqq +0.507$, $-1.327 \leqq a \leqq +1.327$. Thus $\Delta C/C_0$ cannot exceed 1.8 per cent in numerical value, and Δ cannot exceed 4.09 per cent. (These are the bounds when a and b vary independently in the expressions for $\Delta C/C_0$ and $\Delta = \Delta R/R$; we do not pause for closer examination of the expressions and of the conditions for phase lag of 120° ± 5°, to determine the actual bounds.) Substituting $(\Delta C/C_0) = 1.8$ per cent and $\Delta = 4.09$ per cent in the conditions for phase lag of 120° ± 5°, we obtain the following tolerance conditions on the components of the phase-lag network:

$$-0.140 < 0.85\Delta_0 + 0.89\Delta_2 + 0.457\epsilon' + 0.95\epsilon_2,$$
$$0.43\Delta_0 + 0.45\Delta_2 + 0.236\epsilon' + 0.51\epsilon_2 < 0.103.$$

These conditions require a uniform tolerance of -4.45 per cent, $+6.33$ per cent on R_0, R_2, C_2, C'; or, for example, if R_0, R_2 are held to within 0.1 per cent, the appropriate tolerances on C', C_2 are, respectively, -15.1, $+21.6$ and -7.27, $+10.0$ per cent. Thus if the bridge "T" components are held to ± 0.6 per cent, to insure phase shift through the bridge "T" within $\pm 10°$, these tolerances on the phase-lag network components will insure that the phase lag through the network preceding the bridge "T" is 120° ± 5°.

ACKNOWLEDGMENT.

The author wishes to express his appreciation of the work of Miss Virginia Driscoll. She carried through the required computations, and prepared or directed the preparation of all of the figures and charts, in the three parts of the paper. Without this assistance the publication of the paper would not have been possible.

Tube Expected to Make Fluoroscope 500 Times Brighter.—The fluoroscopic X-ray image amplifier is based on information gained in four years of intensive study resulting in the actual amplification of an X-ray image. Basically it will consist of a high vacuum tube that electrostatically focuses and accelerates an electron stream. When perfected, the device will take the form of an attachment for standard fluoroscopic equipment.

The conditions that confront the physician in present day fluoroscopy are even worse than those encountered in finding a seat in a movie after leaving the bright sunshine. To see the fluoroscopic image he has to dark-adapt his eyes for twenty minutes. Even then with the low brightness available in most difficult cases, such as a large abdominal thickness where the brightness may be as low as 0.00005 millilamberts, a separation of $\frac{1}{4}$ in. is necessary before contours can be distinguished. Furthermore this $\frac{1}{4}$ in. figure applies only in measurements where the contour lines separated regions with a contrast of 100 per cent, as in contours between black and white. While this is an extreme case it compares with 30 millilamberts brightness and 0.001 in. contour separation as normal conditions for observing X-ray plates. A brightness of 0.001 millilamberts with a required contour separation of $\frac{1}{32}$ in. is about the center of the fluoroscopic range.

The new image amplifier increases the brightness of the fluoroscopic image after the X-rays have passed through the patient. This solution to a long felt problem is necessary because the X-ray intensities are already at the patient's tolerance level and is possible because the sensitivity of the physician's eyes is the main limitation to effective fluoroscopy today. Fluorescent screens now have a gross efficiency of about 3 per cent so that even a theoretically perfect screen would be only about thirty times as bright. Thus screen improvement alone cannot achieve the desired brightness gains.

With the image amplifier tube attached to his equipment the physician will step into his dark room and see at once an image in appearance almost as bright as a movie screen. He may still have to dark-adapt his eyes, though for only 3 or 4 minutes, before he can see all the details. What he sees after that will be a clear, easily discernable feature presentation of his patient's organs at work.

Wheat Clean-up.—In the early 1930's more than half of the cars loaded with wheat that reached terminal markets in the Pacific Northwest were graded "smutty," says Dr. M. A. McCall of the U. S. Department of Agriculture. As a result of the breeding of resistant wheats in the program in which the Bureau of Plant Industry, Soils, and Agricultural Engineering cooperates with the States, new wheat varieties were introduced. Ten years later, in the early 40's, only a little more than one car of wheat in a hundred was graded "smutty."

THE CONESTOGA WAGON.

BY

THOMAS COULSON.[1]

Have you any ideas upon the origin of the expression "I'll be there with bells on"? Or why a certain form of cigar came to be called a "stogie"? There would appear to be no common denominator for the two, and both seem far remote from the history of land transportation, yet both are derived from the drivers of the Conestoga wagon in the golden age of wagoning.

In the days of the colonies the state of the roads provided the drivers of the wagons with a rich variety of minor hazards to spice their journey. Whenever one driver had to go to the aid of another in distress on the rough, slippery, or water-logged roads, it was customary to reward him with a gift of the luckless driver's hame-bells. Thus, to assure anyone that he would arrive at his destination with bells complete was an expression of absolute assurance in his ability to surmount all difficulties without outside aid.

As for the stogie, it was a long cheap cigar (sometimes a foot in length) of rough rolled leaf made especially for the taste and purse of the Conestoga wagoner. The word is simply a corruption of "Conestoga."

No form of transportation has left a deeper impression upon national life than has the "covered wagon." Although it is most popularly associated with the great days of pioneering over the wide prairies, that was not the age of its greatest splendor. Its real glory grew with the colonies as they spread out from Pennsylvania into Ohio in what was probably the most romantic period of American transportation.

Its popularity waxed with the development of the road and waned with the conquering onrush of the steam locomotive. Since the trail has been swept by the winds and beaten with the rains of more than a century, it is not surprising that it grows dimmer with the passage of the years. Nevertheless, its memory deserves to be preserved. It is not only a very significant link in the chain of transportation growth, but it merits remembrance as an example of early American craftsmanship at its stage of highest achievement.

We cannot say with any degree of assurance when the Conestoga design emerged from the more primitive straight body lines of the earliest wagons. The diarists of Colonial days were content, like George Washington, to write of "the wagons" they owned without

---

[1] Director of Museum Research, The Franklin Institute, Philadelphia, Penna.

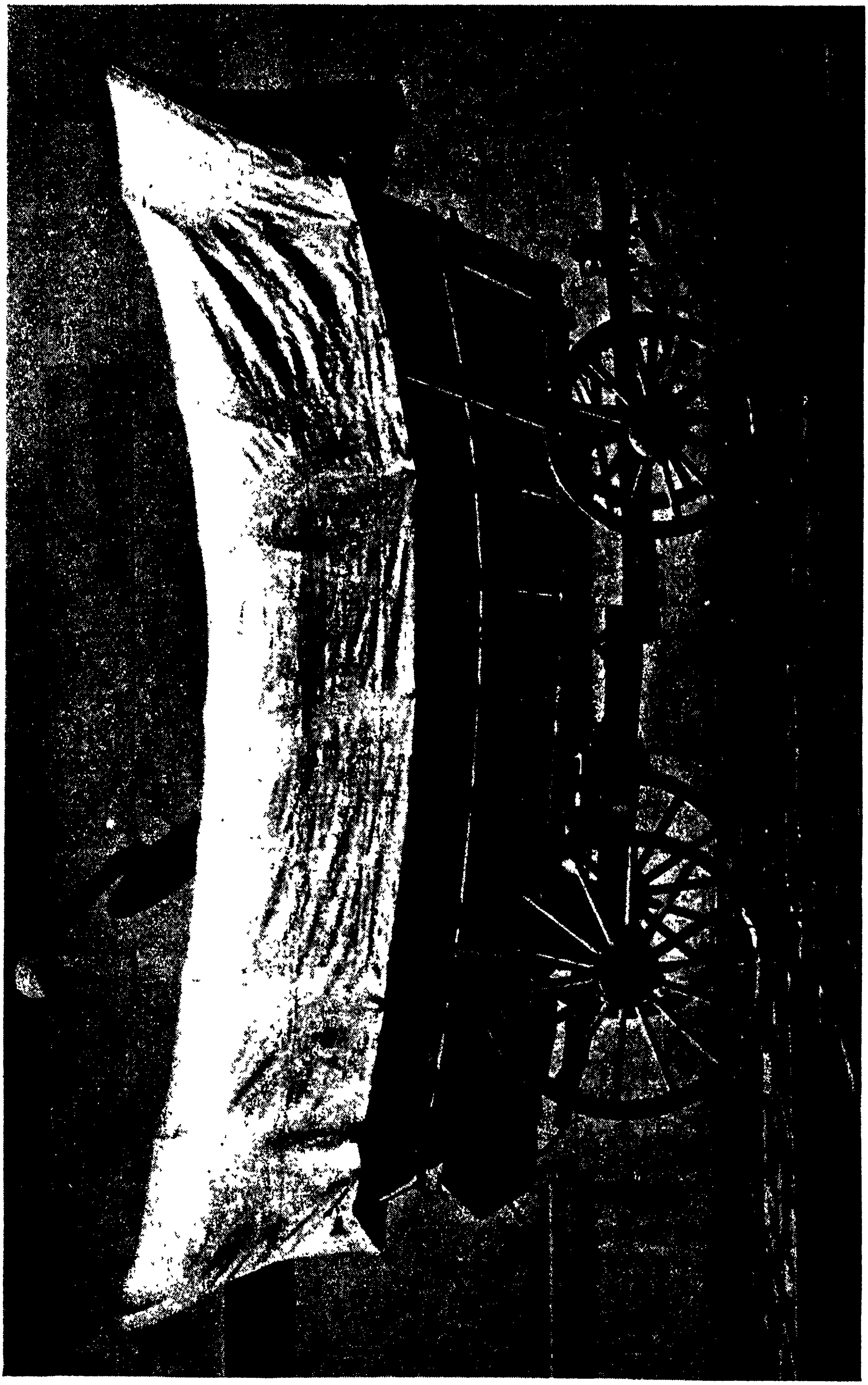


FIG. 1. The Conestoga wagon. The specimen in The Franklin Institute Museum, built about 1830.

specifying in any way the distinctive features. The first hint that any change had taken place in design comes from later writers who allude to "Dutch" and "English" wagons. Doubtless the first word applied to the popular but misapplied name given to the Germans who were the early settlers of the Conestoga Valley in Lancaster County, Pennsylvania, who made extensive use of this type of wagon.

The application of the word "English" to the wagon is not so obvious, but it probably has more justification. The design of the hay-wain has remained unaltered in England for more than 300 years. Early examples have the curved bed that is characteristic of the Conestoga. However, this feature is not peculiar to the English vehicle, as it is seen in pictures appearing in French manuscripts of the fifteenth century. The curved bottom is merely a commonplace where merchandise was transported in barrels.

What is stronger evidence of the English origin is a detail which seems to have been completely overlooked. This is the notched edge and chamfered decoration which re-appears on the end gate of the American form of wagon. This simple form of ornamentation (which may be seen on the specimen in The Franklin Institute Museum) is a traditional feature of the English hay-wain.

Nor can we find when the term "Conestoga" was first applied to the wagons of this design. The first mention in print occurs in the *Pennsylvania Gazette* of February 26, 1750, which refers to a tavern "at the sign of the Conestoga Wagon." This tavern was situated on Market Street above Fourth, in Philadelphia. Still another tavern of the same name was situated in Lancaster. It is mentioned in 1789, but as it is then spoken of as a "famous old tavern," it had probably displayed the name for several years before the year mentioned. However, we have evidence that the name was established at both ends of the turnpike which is associated with the origin of the wagon, by the middle of the eighteenth century.

In those days, Philadelphia drew the bulk of its food supply from Lancaster County. Faris says[2] that the Lancaster Turnpike was called the Conestoga Road because it was the favored route of the freight carriers. In 1794, the turnpike, 24 ft. broad and surfaced with broken stone, was opened as the first toll road in the country. It extended from Philadelphia to Lancaster, 62 miles away. This was the main artery for the city's food supply. Some twenty or thirty white-hooded wagons would follow one another along it in stately procession. But the traffic was not all in one direction.

From Lancaster the trail ran west through Columbia and Marietta over the Alleghenies to the Forks of the Ohio, or northwest to Lake Erie. Along the former route went the pack-trains and the Conestoga wagons as the country began to develop and the young nation expanded.

---

[2] J. F. FARIS, "Old Roads out of Philadelphia," 1917.

It must have been a rather terrifying experience to drive one of the cumbersome vehicles over the five mountain ranges between Pittsburgh and Philadelphia, especially on the steep grades. On such journeys it was almost beyond the power of the driver to manage his horses and manipulate the clumsy brake. It became a common occurrence to offer a lift to a pedestrian who would undertake to discharge the duties of brakesman.

At the time of Commander Perry's operations on Lake Erie the ammunition trains made up of Conestoga wagons carried gunpowder from the Dupont factory on the Brandywine. Being reluctant to return to Lancaster County, where freight carriers abounded,[3] the drivers turned aside and made their way to Vermont and New Hampshire where there was need for carriers, and where their wagons were well adapted to the existing roads. From that time, the Conestoga wagon became a familiar sight in that region.

The constructional details of the wagon were largely traditional. An unwritten law decreed that the wheels and removable side boards should be colored vermilion, while the running gear should be blue. The nature of the woods employed, the types of iron braces and plates, the location of the tool-box, the feed-box, and the lazy board were all uniform. About the only thing in which an owner exercised his individual taste dwelt in the height of the wheels and the number of bows to support the cover.

Since the best of roads was none too good in rainy weather, and streams were rarely bridged, the wagon had to be sturdily built. On the average they weighed 3500 pounds when empty. The woods employed in the structure had to be as light as practicable without sacrificing strength. White oak was most generally used for the framing, black or sour gum for the hubs, hickory for the axles and singletrees, and poplar for the boards. The modern engineer, with his careful selection of alloys for specialized applications of metals, is carrying on the tradition of the old wheelwright.

Two skilled workmen were needed in the construction—the wheelwright (or wagon-maker) and the blacksmith. To complete a wagon with its equipment of feed and tool boxes, tar-bucket, jacks, bows, etc., would provide these two workers and their helpers with work for the greater part of two months, and the cost would be about $250.

Why should a wagon-builder be called a wheelwright? Well, a wagon will stand or fall by its wheels. Since Conestoga wagon wheels had to contend with a continuous succession of stiff obstacles they were often made heavier and stronger than would seem necessary. One of the peculiarities of these wheels is that their plane inclines outward from the hub to the rim, forming a dish shape. This feature is sometimes thought to be original in the Conestoga wagon, but some of the very earliest spoked wheels were built in this way. The purpose was to bring the lower spokes into a vertical position when a wheel sank into mire or on a cambered road or in any other circumstances where additional weight was thrown upon one wheel. Strength required that the spokes be upright. The tremendous side thrust of the heavy body of a Conestoga wagon would have caused an ordinary wheel to fold up like an accordion, when a turning was made, and the dishing of the wheel provided supplementary resistance to this side thrust.

---

[3] Although every farmer had a Conestoga wagon and was a potential carrier, we must accept with reservation the statement in Omwake's excellent book "Conestoga Six-horse Bell Teams in Eastern Pennsylvania," that there were 10,000 such wagons around Philadelphia. In the year 1794, the city had only 827 wheeled vehicles, other than wagons.

One does not have to be an engineer to realize the uncommon skill required to set the spokes into the hubs and rim at precisely the correct angle.

Many of the Lancaster wagons were equipped with six wheels. When the wagon was to make a trip to Philadelphia or to make a journey along a good road surface, the largest pair of wheels were mounted in the rear and the second largest in front. For use on the farm, the tallest wheels were discarded, the forward wheels were put in their place, and the smallest pair was put in front.

Certain features of the construction give us a clue to the purpose for which the wagons were built. Those of the commercial carrier can be distinguished by two features. They had high sides with three adjustable chains across the top to provide the sides with additional support. The bottom had a more pronounced swell, because the barrel was the principal merchandise container. Since most things in trade—tar, sugar, tobacco, or flour—were transported in this sort of container, a wagon's cargo consisted of a variety of barrels, hogsheads, or casks. As a consequence it was elementary wisdom to take precautions to prevent them from rolling around and battering down the wagon sides. The curved bottom encouraged the containers to work toward the middle rather than against the sides of the wagon.

Lighter but bulkier loads of charcoal had less tendency to roll, so that the bottoms were not so curved but the sides were higher and more guard chains were provided. The wagon intended to convey fuel to a furnace had less swell and lower sides than the carrier's. Those used for transporting ore had still stronger and lower sides. Occasionally we find coal and ore wagons with a sliding panel or trap-door underneath to facilitate unloading. The later wagons gradually lost these distinguishing features since their use became more general and less specialized and sacks were substituted for barrels in packaging.

The example in The Franklin Institute Museum is typical of the solid construction of Conestoga wagons built about the year 1830. The wheels are characteristic of those employed in farm work, but marks of rubbing on the side indicate the fact that larger wheels were used at some time or other. The larger pair of wheels are 4 feet high, and the smaller are 3 feet 6 inches. The body is 6 feet 8 inches high, and the bed is 13 feet 10 inches long. The over-all length is twenty-five feet nine inches. The cover has nine bows and stands ten feet from the ground. With its six-horse team this wagon would cover about sixty-five feet of road.

Although one thinks of the wagon as being the product of the wheelwright, the work of the smith is much too important to be dismissed without mention. Ever since the discovery of iron, the smith had occupied a prominent position in the community, and he lost none of his dignity through his work upon the Conestoga wagon. At first, the iron tires were not made in one piece. Instead, they were made in pieces of the same length as the felloes (the segments of the rim that supported the spokes). These were butted in the middle of the felloes to bind the joints. Examples of these pieced tires are very rare today.

The smith's skill improved rapidly as more workers could be spared from essential labor on the land and competitive labor produced better craftsmanship. The common tires came to be made from two pieces of a single bar, each long enough to go half way around the wheel, and welded at both joints. The tires varied in width from two to ten inches, but the average was four inches. At one time there was advanced a claim that the broadest tired vehicles should be exempted from payment of tolls on the turnpike, on the theory that their effect was to roll and smooth the surface rather than to wear it.

Tire making was compounded of the arts and sciences. It must have called for no little skill to handle the 150 pounds of iron forged in a brick forge, and a considerable amount of craft was required to obtain the necessary "twist." Owing to the dish shape of the wheel the inner edge of the felloes had a greater diameter than the outer edge, and the iron tire had to be twisted to compensate for this. Having regard to the tools available at the time, this must have been a ticklish job.

When the tire had been hammered to shape and welded, the time came to attach it to the wheel. This was an occasion for a ceremony which no village child willingly missed. The tire was laid flat on iron or stone blocks and a wood fire was built under it. The wooden wheel, meanwhile, stood ready on a trestle. When the tire reached the right temperature (a matter of purely empirical judgment) it was whisked out of the fire, fitted around the wheel, and hammered to a snug fit. This latter operation had to be performed at top speed and, as soon as it was completed, cold water was thrown on it to shrink the metal on the rim and to prevent charring the wood. Sometimes the wheel was suspended over a water trough, into which it was instantly plunged, and rotated in the water to hasten cooling. Speed in the operation was a matter of prime necessity because the danger always existed that it would char and ruin the woodwork. To prolong the process might render the whole work on the wheel wasted energy, and to overdo the haste in chilling the tire exposed it to the risk of breaking. Tire making was no easy task.

Small rectangular holes were punched in the face of the tires to correspond with the ends of the spokes, and into these were driven wedges to prevent the side slip of the tires.

This did not comprise the entire contribution of the smith to the wagon. Much might be written upon the ironwork distributed over the vehicle. Since the wood had to be planed and pared to compromise between weight and strength, it was necessary to provide reinforcement at various points. All frictional surfaces had to be provided with iron rubbing plates. But not all the ironwork is of this severely utilitarian nature. Some of it gave expression to the smith's sense of decoration.

It is worth observing that the file was never used in this work. All the ornamentation was done with hammer and chisel. This called for a considerable amount of skill in the handling of the tools, as did the beating out of the metal to the required thickness.

American owners rarely followed the English example of denoting ownership by means of a small wooden board, bearing the owner's name and address, attached to the side of the wagon. The American practice was to have private marks fabricated of iron. Sometimes a Conestoga wagon is decorated with the owner's initials wrought in iron, and occasionally one finds a date. This is not always significant. Ironwork was not infrequently transferred from one wagon to another, so that, while the date is undoubtedly that of the ironwork, it may not apply to the wagon's woodwork. Dates were not scratched or scribed; they were either cut or punched with a chisel. Dates are also found at times on the wagon-jacks and wagon bodies.

Wagon-jacks were standard equipment, usually hung on the back axles. They were in frequent demand on journeys,, not merely to lift the wagon over obstacles, but also because the wheels not infrequently had to be removed for oiling the bearing parts.

A certain amount of ironwork is to be found on the feed-boxes, but it is more of a practical nature, being intended to prevent the horses from gnawing on the wood. The tool-box, on the other hand, is the main object of ornamentation. The design employed is frequently the heart and tulip which appears in all forms of the Pennsylvania Dutch decoration.

In the older type of wagon the tool-boxes were large, and were located on the side of the wagon. In these boxes the wagoner carried all the small tools he might need on the journey for effecting running repairs—pincers, tongs, wrench, nails, spare straps, etc. The inevitable axe had a separate socket. The later wagons, intended to be restricted to farm use, had the tool-box mounted in front, a position they could not have occupied when a full tool-kit had to be carried.

Another conspicuous feature of the Conestoga equipment was the bells. These were usually small open bells suspended from flat hoops, with the open ends dangling down. The number of bells varied from three to six, but four were customary. Most likely the bells were adopted for exactly the same reason which had led to their earlier use in Europe, to warn travellers of the team's approach. The saddle horse did not carry bells.

All other vehicles were driven from the right, but the Conestoga wagon was driven from the left side, either from a saddled wheel horse or from a lazy board pulled out from under the left side. So strong was the influence of this type of wagon upon road transportation that the practice of driving vehicles from the left persists to this day in the country.

Such a wagon deserved a noble horse, and the Conestoga found one. Or, as Dunbar[4] would have us believe, which is not improbable, the wagon took its name from a breed of horses that were already well known. Like the wagon itself, the Conestoga horse is now only a memory. Where it came from no one appears to know, but it certainly was not the result of selective breeding upon a scientific plan. It is reported that William Penn had three Flemish stallions which he sent into Conestoga Valley, and that they were bred with Virginian mares. There is a strong resemblance between the Flemish horse and the strong, heavy, slow Conestoga. Many of the latter were black, but mixed breeding produced bays (perhaps from the Suffolk Punch strain) and, later, dappled grays. These docile animals stood over 16 hands high at the withers, and weighed about 1800 pounds at normal weight. The wheel horses were usually the heaviest and the best pair, since they performed the duties of backing and turning the wagons. When wagons declined in popularity so did this breed of noble animals. They faded before the advance of the hardy Western horse, which was lower in price and less expensive to maintain. But during their day, the Conestoga, both horse and wagon, filled a chapter in American history of transportation that we shall turn back to again and again with unfailing interest.

As the frontier was pressed steadily westward, the white-hooded wagon was a familiar sight in every migration. There was majesty in its leisurely advance. It must have been an impressive sight, that of the great wagon lumbering behind its six superb horses half covered with bear skins, or gayly decorated with fringed housings, stamping beneath their sweet-toned bells. Heroic sagas might be written about the wagoners who conducted these wagons over the rough trails that are today our automobile highways. And while they were so quickly fading into the obscurity of the past, the wheel tracks of their wagons could still be traced on the prairies. They were the pioneers of an advance that has known no retrogression.

---

[4] S. DUNBAR, "History of Travel in America," Vol. I. Indianapolis, Bobbs, 1915.

SCIENCE IN NATIONAL SECURITY.*

BY

SCOTT B. RITCHIE.[1]

The problem of National Security is of concern to every patriotic American. No subject today is of greater importance. To preserve our cherished way of life it is imperative that we develop and maintain adequate preparedness. This will be our greatest guarantee of peace. This means a strong healthy industry, a trained military manpower, and a sound program of research and development, to insure that in case fate should decree another national emergency, we will have in ample quantities the best possible weapons that can be produced under the existing state of the arts and sciences. In other words, our defense forces—Science, Industry, Army, Navy and Air Forces—working as a team, must be kept at maximum efficiency until peace is assured in this troubled world. Each of these is of transcending importance; none can be neglected without jeopardizing the others.

I am glad for this opportunity to discuss these matters briefly with you who are in the engineering profession, who have done so much for preparedness, and upon whom rests a responsibility to continue to fight for those things you know and I know are needed and against those things repugnant to the Nation we love. It is with the scientific phase I should like to deal, and since this subject is broad, I shall be able in this brief period with you to touch upon only a few of the more important aspects.

The primary aim of science is the welfare of our people. Our ability to protect ourselves depends upon our scientific progress. This, in turn, is dependent upon the intelligence of our people, which means that a tremendous responsibility now rests upon our educational systems to reduce the shortage of scientists and to build up a balanced force of highly intelligent people in all the essential professions. One of the greatest resources of our Nation is the intelligence of its people.

During the last war we drew heavily from the scientific knowledge accumulated by many years of basic fundamental research. Our store of knowledge needs to be replenished and enlarged by new knowledge, else we shall find ourselves wanting. We used to benefit greatly from the basic research programs carried out abroad, notably by the Germans.

---

* A talk presented before the Engineers Society of Pennsylvania, in Harrisburg, April 7, 1948.

[1] Colonel, Ordnance Dept. U. S. A. (Ret.); formerly Director of Projects Procurement for The Franklin Institute Laboratories; now with United States Pipe and Foundry Co., Burlington, N. J.

These sources are now largely closed to us. Research is not only desirable for the progress of the nation, it is imperative now for national security, security not only today, but in the long years ahead. Fundamental research creates scientific capital essential for progress. The nation which can produce the greatest quantity of the most destructive weapons will win in the end, and science will determine the degree of destruction. It is well to remember too that no one—no nation—now has a monopoly in science.

Many of our most important developments took years of painstaking research to bring them to fruition. An outstanding scientist engaged in industrial research recently stated that in his industry it took on the average seven years from an idea to a finished new product. Under the stress of war and with all the money we needed and all the talent we could muster, it took on the average 300 to 350 days to design, test and standardize a new tank or a piece of artillery. Radar and the atomic bomb were the results of years of fundamental, painstaking scientific effort, much of which was done abroad. With all the magnificent things that have been achieved by science and technology, I believe we have only scratched the surface in the things to come which may or may not be good for humanity, depending upon how we use them.

Many of the things developed for military purposes since the beginning of our nation have repaid their cost many times over in their use by industry and in improving our standards of living. Likewise, improvements by industry enhance our preparedness. The introduction of alloy steels, the development and improvement of manufacturing processes, interchangeability of parts, mass production, fixation of nitrogen, transportation, communication; these are a few of the many things that have originated with or have been greatly advanced by military requirements. Our success in war has been gaged by the quality of our fighting equipment—if you please—by the research back of it.

Unfortunately, the place of science in our national security, the importance of research and development to keep ahead of our potential enemies has not in the past received the attention it deserved. In World War II for the first time science and engineering were mobilized in the national interest. In the first World War we had our first experience in industrial mobilization, and due to our great industrial effort at that time the old German, Von Hindenberg, said that "America understood war." World War II was to science and technology what World War I was to industrial mobilization.

From both, one thing stands out preeminently—the value of *time*. We entered both unprepared, but in each case we escaped sudden devastation for we were thousands of miles of ocean from the immediate front and our allies were holding the enemy in check. Now all is rapidly changing, and if war should come again I doubt that we would have the time to mobilize our scientific, industrial and manpower forces as we have had in the past. This emphasizes the urgency of doing the necessary things for national preparedness *now*, while time may yet be available.

In the last emergency, science came into its own and played a major role in the outcome. Our military people and our scientists learned to work together as a team which was an exceedingly important development, not only in the past war but for any which may come in the future.

Fortunately for us, in Japan there was a great gulf between the scientist and the military. The military people were extremely skeptical, in fact hostile to their civilian scientists. Unfortunately, the Germans were better coordinated than the Japs, yet they failed to handle their research people to their greatest advantage.

Nevertheless, even though we developed new and secret weapons never before visualized, and took a commanding lead toward the end of the war, we had strong competition from our enemies in many categories. In fact, some of the German weapons were superior to ours in the early days. They had started on their developments almost before the ink was dry on the Treaty of Versailles, while we cut back at that time so far that we scarcely had enough money to keep our most essential arsenals in standby condition. For example, in the Ordnance Department, which was the Technical Service of the Army responsible for research, design, development and procurement of the weapons for the army, the funds available for research and development averaged about $1,000,000 per year between the two wars. This is not large when it takes hundreds of thousands of dollars to develop one new tank, or a new piece of heavy artillery; or when we compare it to the more than $81,000,000 spent by Ordnance alone for research and development in fiscal year 1944, or to the $749,000,000[2] spent by all Government agencies on research and development in that year.

But, in spite of limited funds we were able to do a number of things between the two wars which were godsends when the storm broke. Let me mention a few specially concerned with Ordnance development and production.

1. We had developed the centrifugal casting process for making cannon which helped us, *especially in the early days of the war*, to meet our requirements in artillery. About one-third of all the cannon used by our armed forces were centrifugally cast.
2. The cold working or auto-frettage method for making cannon with improved strength permitting lighter weight and higher muzzle velocities.
3. Research and development at Watertown Arsenal had pioneered in the development of molybdenum high speed tool steels, thus making this country immune to a possible paralysis of industrial manufacturing due to shortage of tungsten.

---

[2] This figure does not include atomic bomb research.

4. The application of welding to Ordnance material, a major development resulting in weight reduction, improvement in design, greater ruggedness and economy in material, time and money.

5. The development of testing techniques such as macro-etching, radiography and impact, which contributed to improved quality and safety of performance.

6. We had designed and developed, or had drawings and specifications ready for production, such items as the .50 caliber machine gun which was 98 per cent of the gun armament of the Army airplane, and which the Commanding General of the Army Air Forces stated was the most outstanding aircraft gun of World War II; the 105-mm. Howitzer which was the workhorse of the Army artillery and which fired more rounds of ammunition than any other artillery weapon; the .30 caliber semiautomatic Garand rifle, MI, which gave our men a fire power superiority over all others, and which the enemy soldiers would risk their lives to steal from our soldiers. In a letter of 26 January 1945, General Patton wrote, "In my opinion, the MI rifle is the greatest battle implement ever devised." Many important items such as I have mentioned had been developed between the two wars and were ready to go into production by industry in the fall of 1940 when the Ordnance Department received 1¼ billion dollars for the manufacture of equipment.

As the war clouds began to gather and funds were made available to the armed services, the tempo of research and development was progressively increased as the war days went by. Cooperative programs with science and industry for research and production were established by the armed services and constituted the ground work for the war effort. At the peak, the Ordnance Department alone had some 1500 facilities—research and engineering organizations, university and commercial laboratories—and thousands of sub-contractors engaged in its research and development program. The National Defense Research Committee was established in 1940 by the President to mobilize the Nation's scientific talent and to assist the armed services in research and development projects. The Ordnance Department sponsored more than 200 projects with NDRC, of which approximately one-half had been completed by VJ-Day. Notable in these cooperative projects were rockets and the VT fuze about which I shall speak later. The NDRC was established none too soon as the general unpreparedness of our country had given the Axis nations such a tremendous initial advantage. Perhaps Hitler made one of his greatest mistakes, possibly costing him the loss of the war, when he ordered research and development stopped and production concentrated on the then standard weapons at the time when his greatest successes led him to think there would be an early decision. About 4 months later he revoked this order. We, however, after a not too early start, foresaw the fact that the great struggle ahead would as never before be a race between scientific giants to achieve mastery. This, as you know, culminated in the most devastating weapon of all time, the atomic bomb. Even before it was used, the outcome was evident. Germany had collapsed and the Jap homeland was being devastated by bombs from our B-29s. Let me mention only a few of the developments which made possible our success.

Military aviation, the submarine, and the armored tank, used in World War I to a limited extent, came into full use by both sides in the last conflict. Radio, used so effectively for a variety of military purposes, came into use before the war and was greatly expanded during the war. To these were added many other new weapons of destruction such as recoilless rifles, amphibious vehicles, VT or proximity fuzes, radar, new tanks, rockets, bazookas, which for the first time in history placed weapons of artillery power in the hands of the foot soldier, shaped charge ammunition, high velocity sintered carbide projectiles which can penetrate several times their caliber of armor, guided missiles, new and vastly more powerful artillery weapons, new fire control instruments which made bombing and artillery extremely accurate, and special fuzes for artillery shell which greatly aided in the demolition of concrete fortifications. We developed the greatest line of high-powered, speedy self-propelled artillery of any nation. Many of you are familiar with the 105-mm. howitzers, the 155-mm. Long Toms, the 240-mm. howitzers, not to name a multitude of other calibers which helped to give us a tremendous firepower superiority. Ranges of our artillery in general were increased 40 to 50 per cent over that in World War I. For the first time artillery was installed in our aircraft and was used with deadly effect on shipping and other opportune targets.

Our high explosives which helped to destroy the German and Jap cities were 20 per cent more powerful than those used in World War I; and certain individual new explosives were developed much beyond this figure. None of the propellent powders of 25 years ago was in use by the end of the war. They had been replaced by new and improved types. In this connection, prior to World War II, silk cloth was used as a container for propellant powder for all semi-fixed and separate loading ammunition. Silk was a strategic material because the Japs controlled practically all raw silk production and the new artificial silks, or rayons, of industry were not satisfactory due largely to the marked difference of the physical characteristics of extruded filaments and those of natural fibers. After much research Picatinny Arsenal solved this problem and provided a suitable cotton cartridge cloth to replace silk.

A development of extreme importance was the means to permit our automotive equipment, including armored tanks, to wade and swim in water. Those of you who participated in the Normandy landings or the operations in the Pacific know how important that was. The DUKW, for example, was built to run on land and in the sea and was an indispensable amphibious vehicle.

When the Japs took over the sources of natural rubber, the ingenuity of American science and industry came into play and after some difficulties, and with extensive tests carried out by the Ordnance Department in cooperation with industry, synthetic rubbers were developed which in some ways even excelled the natural rubbers. Since our armies moved on rubber, that development was a must.

Fuels and lubricants were among the three most essential items used in the war. They are the life blood of a mechanized army and received much attention. Great stress was placed on the need for a single all-purpose fuel for use in all combat and transport equipment. There was also need for a single purpose year-round recoil oil for all artillery, and for preservative oils and rust preventive compounds to protect our equipment stored and used under global conditions. One of our great problems was due to the fact that we had to provide equipment suitable for the Arctic, the desert and the tropical jungle. Innumerable problems arose under these conditions, and extensive tests were carried out in the North, in the desert and in the jungle, to solve them. These tests are still continuing.

It has frequently been said that the last war was one of metals. Certainly steel played a dominant role. Other metals were also exceedingly important and many substitutes had to be used. Our 70–30 brass, of which copper is the major element, had long been our standard for cartridge cases, but shortages of copper forced the development of steel cartridge cases. The light weight metals, aluminum and magnesium, found many applications in war equipment, but steel was the great tonnage item and many metallurgical improvements were made under the stress of war.

Prior to the emergency the metallurgy of armor and armor piercing projectiles was very much of a mystery and the few commercial companies engaged in their peace-time manufacture held the processes secret. During the war the producers shared this secret information to make the mass production of these items possible by plants scattered throughout the country.

The demand for rolled armor plate for tanks and armored vehicles could not be met and a program for developing and manufacturing cast steel armor on a grand scale was inaugurated. Starting with practically no knowledge in this country on cast armor, American foundries pooled their efforts and soon developed cast armor of suitable quality. Without this our tank program could not have been met. It might be of interest to note that body armor supplied to our aircraft crews resulted in a 50 per cent reduction in deaths and 80 per cent reduction in wounds.

I mentioned previously the centrifugal casting of cannon. To further assist in meeting the tremendous demand for cannon, the rolling of cannon barrels up to and including 75 mm. on seamless tubing mills was successfully accomplished. During the first World War the yield strength usually prescribed for cannon was around 65,000 psi. By the end of World War II we were procuring cannon with yield strengths as high as 150,000 psi. This is a measure of metallurgical improvement, yet I am sure we have only touched the potentialities of metals.

Due to the shortage of alloys normally used in guns and armor, the alloy content had to be drastically reduced. In some cases it was cut down to one fourth or less, yet with no reduction in performance.

In the huge program of developing and producing arms and equipment, welding played a major role. We used it in mass production and to a far greater extent than did our enemies. One of the notable metallurgical achievements in this war was the development of suitable techniques for welding armor plate and armor castings which permitted the change from riveted to all-welded structures in our fighting vehicles and gave increased production and improved performance.

These were a few of the remarkable metallurgical achievements for which the Ordnance engineer, the metallurgist and the production engineer, working as a team, deserve much credit.

Great progress was made in the war against disease. This is evidenced by the fact that the death rate for all diseases in the army was reduced from 14.1 per thousand in the first World War to 0.6 per thousand in World War II. You all know what the great discoveries of sulfa drugs and penicillin have meant, and the lives saved by surgical techniques and the use of blood plasma. Research in food and clothing also did much for the comfort and welfare of our troops.

It would be inappropriate not to mention at least two research tools of great importance which were developed during the war and the use of which is now increasing, namely—the ENIAC (Electronic Numerical Integrator and Computer) and the Supersonic Wind Tunnel. Both are in operation at Aberdeen Proving Ground. These are products of research sponsored by the Ordnance Department. The ENIAC is among the foremost scientific achievements of the war. It contains some 18,000 electronic tubes and solves intricate problems a thousand times faster than any machine ever built—problems that are hopelessly beyond the scope of other methods, which opens up new fields of research to all branches of science. The other research tool, the Supersonic Tunnel, installed at Aberdeen Proving Ground, is an invaluable facility for ballistic and aerodynamic research on high velocity projectiles, bombs, guided missiles, and supersonic aircraft. The Germans also used wind tunnels extensively during the war in the development of projectiles, aircraft, and V-weapons. The value of such tools for future research is incalculable.

I have mentioned only a few of the many scientific developments which helped us to win the war. It has been said that the three greatest scientific achievements were VT fuzes, radar, and the atomic bomb. It has also been said that our greatest secret weapon was the unexcelled production capacity of American industry. Certainly the things that won the war were produced in great quantities, and the developments alone would have been of little or no avail to crush the enemy had they not been rapidly translated into huge quantities of finished items to give the necessary fire power. A few figures on Ordnance materiel will illustrate what I mean.

In Army Ordnance alone, in World War II, there were some 1800 major items and over 600,000 principal components whose development and manufacture embraced every phase of the arts and sciences. In war things move swiftly as evidenced by the fact that 75 per cent of these 1800 major items were newly designed or radically improved after the beginning of the emergency. During the last three years of the war, more than 1000 new items were developed and brought into production. The expenditures for Ordnance materiel were ten times as much as in World War I. In weight the ratio was twenty-five to one. The Army Air Forces dropped 1,500,000 tons of bombs on western Europe between August 1942 and victory in May, 1945. At the peak of fighting 293,000,000 rounds of small arms ammunition were expended in one month. Each fighting infantryman needed 35 pounds of arms and ammunition a week. Of the $45,000,000,000 spent for Ordnance, $38,000,000,000 went for tanks, small arms, trucks, ammunition and accessories. Nearly $5,000,000,000 went into construction and equipment of government owned, contractor operated manufacturing plants. Nearly a billion dollars went for lend-lease machine tools shipped to foreign countries. Another billion was spent for additional items, including schools, research, and establishment of the Ordnance supply chain. Ordnance had the biggest segment, about 50 per cent, of the total supply task of the Army Service Forces. Roughly, Ordnance accounted for 20 per cent of the total United States war munitions effort including air forces, navy vessels and merchant marine production. There were 88,000 tanks, 133,000 miscellaneous combat vehicles, 50,000 self-propelled weapons, two and one-half million trucks, 600,000 artillery pieces and millions of such items as rifles, carbines, bayonets, helmets and grenade launchers. This was truly the most ponderous array of armed might this world has ever seen, and it was used by soldiers unexcelled anywhere, at that time.

Looking ahead, we may expect further developments in VT fuzes, radar, and the atomic bomb. Guided missiles and pilotless aircraft to operate at transonic speeds and extreme altitudes are already here. Means to combat these things must be and are being developed. Muzzle velocities and ranges of weapons two to three times those now used are in the offing. Rates of fire will be multiplied in some cases by the thousands. Transport of troops and their fighting equipment by air in another emergency will be what movement by motor was in the last. The future may see flying tanks, ships that sail under the water by atomic power, and death rays even more deadly than the atomic bomb. Who knows what the future will bring forth? Our only safety is in scientific progress, more and more military research to be sure that we keep in the front. God only knows what would have happened to us if the Germans had beaten us to the VT fuze or the atomic bomb. The VT fuze in their hands could have wiped our air forces out of the sky. On the other hand, it has been authoritatively stated that the VT fuze saved London from the German buzz bomb, and that new apparatus for airplane detection developed by British physicists saved England. This permitted the English Spitfires to go up in time to shoot down the attacking German bombers. Your imagination is perhaps better than mine as to what an atomic war head in the German V-Rockets would have done to England and our forces in Europe. We were brought perilously close to the brink in many cases.

One of the great dangers facing us now is over-confidence. Stock piling of strategic materials is vital, and where there may be shortages, research is imperative to develop adequate substitutes or proper conservation measures. Another world war may find us seriously crippled in many essential raw materials. We must be careful not to depend upon false security which might be given by great stores of weapons, good today but obsolete tomorrow. Time moves on. France depended upon her excellent standard weapons of World War I to start World War II, while the Germans started from scratch and therefore had the most effective modern weapons, which contributed to their early success. All of you know the terrible cost of France's mistake. We cannot afford similar passive complacency.

Yet, we must not neglect our tested and tried weapons with which we ended the last war. We must always strive to be first in the development of new things, but we should not unduly extol the virtues of the new and novel and advocate the wholesale obsoletion of the weapons of the past until we are certain that the new weapons of the future are more versatile and powerful than the ones they replace, that they are suitable for use under the extreme and drastic conditions of warfare anywhere in the world and that even in the hands of an unskilled man they are more dangerous to the enemy than to ourselves. Furthermore, they must be capable of large scale manufacture to the necessary limits of precision under wartime conditions.

I am glad that the preeminent importance of research and development has finally been recognized in the National Security Law as a division of the defense organization, reporting directly to the Secretary of National Defense. This should mean a sound, coordinated program of research and development, prevent undue duplication of effort in security research, and foster the teamwork among the armed services, science, and industry which is vital for national security. This teamwork will help to provide that preparedness *now* necessary to make the aggressor take heed—the preparedness which will be our greatest assurance of the peace we all desire.

Developed Water Power of the World.—The total capacity of water-power plants of the world at the end of 1947 was 86,900,000 hp., as determined by the Geological Survey, United States Department of the Interior. The total capacity for different years is shown in the following table:

Year (December)	Total Capacity of Water-Power Plants, hp.	Comparison with 1920, Per Cent Increase
1920	23,000,000	—
1923	29,000,000	26
1926	33,000,000	43
1930	46,000,000	100
1934	55,000,000	139
1936	60,000,000	161
1938	63,900,000	178
1940	69,400,000	202
1941	71,600,000	211
1945	77,800,000	238
1947	86,900,000	278

The table shows an increase in installed capacity of water wheels from 23,000,000 hp. in 1920 to 86,900,000 hp. in 1947, a very large growth in the short period of 27 years. The increase appears to be continuing, taking the world as a whole. The shortage of fuels (oil, gas, and coal) and their increase in price make water power appear more desirable than ever. Although there was practically no increase in the United States during the last two years, in 1948 and the years immediately following there is indicated a possibility of an increase of about 1,000,000 hp. a year. The Union of Soviet Socialist Republics is reported to be building many new plants, some of large capacity. Increased activity is reported also in Canada, France, Norway, Sweden, New Zealand, and India.

Total capacities of water-power plants for the different countries are obtained from consular agents, from information in the Yearbooks published by foreign countries, from publications of the Department of Commerce, the Federal Power Commission, the International Union of Producers and Distributors of Electric Energy, and from other sources.

Because of the restrictions on the release of information and delays in publication of official Yearbooks due to the war, accurate figures for recent years of the total capacity and of the construction and destruction of water-power plants in some European and Asiatic countries have not been made available. There is considerable conflicting information on some countries, but an effort has been made in these cases to select data from the most reliable sources. The figures of capacity of water wheels used for such countries may, therefore, be considered conservative and for others, such as Germany, Korea and Taiwan, may be high. The Dnieper station in the Union of Soviet Socialist Republics was completely destroyed, but three units of 100,000 hp. each have been replaced. The figure, 86,900,000 hp. for the world at the end of 1947, should be reasonably correct.

A THEORY OF THE INTERNAL BALLISTICS OF THE "HOCH-UND-NIEDERDRUCK KANONE."

BY

J. CORNER, Ph.D.[1]

SUMMARY.

During the war the Germans developed guns with a constriction between chamber and bore, so that the pressure in the chamber was substantially higher than that in the bore. It is shown that a simple theory of such guns can be devised, which is adequate for analysis of firing trials and for testing the claims advanced for this idea. The claim of a high piezometric efficiency is not supported by this theoretical analysis.

1. INTRODUCTION.

Up to the end of the war it was taken for granted in this country that the pressure in the chamber of a gun was almost as that in the bore and at the shot; the small difference of pressure was that needed to accelerate the gases, and was therefore of the order of 10 per cent, except in hyper-velocity guns. Waves of pressure had occasionally been observed traveling up and down in guns, but their consequences were not welcome, and it was accepted that all well-behaved guns had nearly the same pressure at the shot base as at the breech. So far as is known to the writer, no one in Britain or America had even considered the possible advantages of deliberate differences of pressure between chamber and bore.

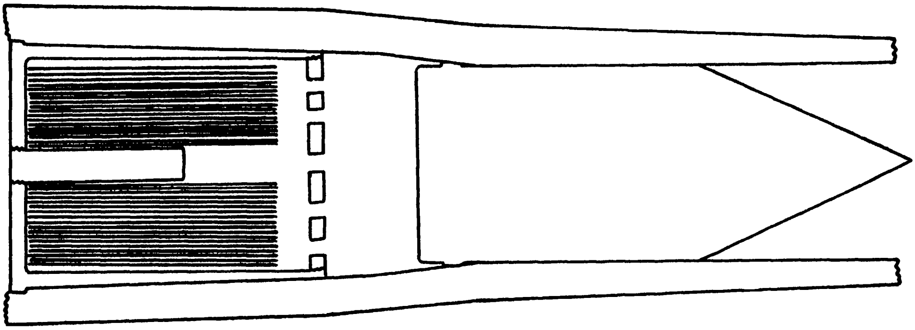


FIG. 1. A typical "hoch-und-niederdruck kanone" in section.

Investigations in Germany in 1945 revealed that this radical innovation had been studied by German engineers, and had been used in two light anti-tank guns, of which one, the 8-cm. PAW 600, was all but in service. The idea of separating bore from chamber by a nozzle, to keep up the pressure in the chamber (Fig. 1), was apparently due to Dr. Herrmann (Rheinmetall), and was patented early in the war. The first application to a service weapon was made in 1944, when a demand arose for a light but accurate projector for an 8-cm. hollow-charge projectile. Now the penetration of a hollow-charge of given diameter is insensitive to the strength of the case; thus a low bore pressure means a light-walled projectile and therefore a low muzzle-energy for a given penetration at a given range. On the other hand, a low-pressure gun of conventional type needs a big chamber, a bulky cartridge case, and a charge somewhat bigger than normal. Furthermore, the regularity of burning is asserted (by German engineers) to fall off as the chamber pressure is reduced. A plate held in the front of the cartridge case, carrying nozzles to throttle the flow, gives the advantages of a low bore pressure without the faults of a low chamber pressure. The aim of the "high-low pressure principle" (abbreviated to "H/L" in this paper), in its application to the 8-cm. PAW 600, was therefore the production of a light weapon with good regularity.

---

[1] Armament Research Department, Ministry of Supply, England.

There is no suggestion in this argument that the H/L piece can be expected to be substantially lighter for a given muzzle-energy. Comparing the 8-cm. H/L gun with earlier German anti-tank guns, there is no improvement to be seen in regard to muzzle-energy per unit of total weight.

German high-low pressure guns were developed by Rheinmetall without any theory other than the application of similarity rules to firings in existing guns. Dr. Bloch (Krupps) used a modification of the solution of Schmitz (**6**),[2] leading to a rather laborious process and sometimes a substantial overestimate of the muzzle-velocity obtained from a given peak pressure.

The theory given here was intended as a simple and easily-applied method for the calculation of H/L ballistics. The method can be used also for certain other weapons which are not covered by orthodox ballistic theory; one example is the mortar firing a projectile with primary charge housed in the tail-shaft in the conventional manner, and another is the rocket discharged from a closed projector.

2. THE PROBLEM AND THE APPROXIMATIONS MADE.

Let A be the bore area of the gun. The cordite is contained in a first chamber of volume U_1, and before the shot begins its motion there is a total volume $U_1 + U_2$ behind it. Let S be the throat area of the venturi or nozzles connecting the cordite chamber to the bore. Let the weights of projectile and charge be W and C. Frictional resistance is included by using an effective shot mass W_1. Let δ be the density of the cordite. Let η be the covolume of the propellant gases, whose equation of state is therefore

$$p(v - \eta) = nRT,$$

where p is the pressure, v the volume per unit mass, and n the mean molecular weight.

---

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Let $V(x)$ be the velocity of the projectile at a travel x from its initial position. Let $C\phi(t)$ be the amount of cordite burnt up to time t and let $CN(t)$ be the mass of gas in the cordite chamber.

Let $P_1(t)$ be the pressure in this chamber, $P_2(t)$ the space-mean pressure in the bore. The pressure is assumed to be uniform throughout the cordite chamber. The conventional Lagrange correction for pressure-gradient between breech and shot-base makes the pressure accelerating the shot $P/(1 + C/3W_1)$, where P is the space-mean pressure. The hydrodynamic conditions in the new type of gun are rather different, but if the same relation is used the error is likely to be no greater than that incurred by using the conventional pressure gradient in orthodox ballistics. This error is much greater than one would expect from a comparison of various theoretical solutions, which are very similar and all at variance with recent experimental findings. This is a matter which cannot be discussed further in this paper. However, in a "high-low pressure" gun the low bore pressure guarantees a low value of C/W, so that the pressure gradient is of small importance and errors in it of still less moment.

For the law of burning of the propellant it is assumed in this paper that

$$D\frac{d\phi}{dt} = \beta P_1. \tag{1}$$

This is approximately correct for propellant in tube and ribbon shapes, where D is the annulus or thickness, respectively. With D about 25 per cent greater than the least distance between perforations, (1) gives a reasonable fit to the ballistics of orthodox guns firing 7-hole multitubular chopped grains of the American type. Only for charges in cord form is (1) inadequate.

β is the rate of burning of the propellant. The effects of the resistance to engraving of the driving band of the projectile will be simulated by an increase in β. If this increase is chosen to give the same effect on peak pressure as physically more plausible representations, the effects on the muzzle velocity are nearly the same in both models; this is true whether the initial resistance is represented by a shot-shot pressure falling to zero immediately the projectile moves, or by a resistance which is a continuous function of the travel, with an initial peak. Empirically, the use of an effective β, varying with the charge conditions, is always possible, and the variation of β is in fact much more special and certainly more convenient than could have been expected: the effective rate β is a function of the peak pressure only, for given gun and projectile.

The equations of internal ballistics are greatly simplified by the approximation that the temperature of the propellant gases is constant during the period of burning. In practice there is a fall due to the work done on the shot. This decay is roughly linear in the travel, and attains about 10 per cent when the propellant is completely burnt. The approximation of constant temperature was introduced by Moisson (4), who applied it also to the adiabatic expansion after the gunpowder had burnt, a period in which the approximation is very rough; Mata (3) also used the isothermal approximation. It has not been used much in the Continental literature since that time, perhaps because the approximation is useful rather than elegant. The isothermal model has been much used in the British school of ballistics, as, for example, by Clemmow (1). This is partly because it simplifies the analysis at a point which is not usually of primary interest in these investigations; another reason will appear in a moment.

Three important approximations have been introduced up to this point: (*a*) the burning law (1), linear in the pressure; (*b*) the simulation of the effects of initial resistance by a change in β; (*c*) the isothermal approximation. A theory based on these three approximations has been used in Britain for many years for the routine estimation of charges for orthodox guns. If the rate of burning is chosen to give the observed peak pressure the theoretical muzzle-velocity comes within a few per cent of the fired value. This agreement is sufficiently close to enable the error to be predicted from existing firings of a similar type, while empirical rules have been developed for the prediction of effective rates of burning. Thus a ballistic scheme has been evolved, in which a mass of ballistic data is tied together by this simple approximate theory. In the present paper the same fundamental approximations are used in a theory of the "high-low pressure gun," and it is assumed that the general behavior of such guns will be reproduced with the same accuracy as in the analogous theory for orthodox guns.

The pressures at inlet and exit of the nozzle are P_1 and P_2. Let γ be the ratio of the specific heats at constant pressure and volume for the propellant gases. If

$$\frac{P_2}{P_1} < \left(\frac{2}{\gamma + 1}\right)^{\gamma/\gamma-1} \tag{2}$$

then the rate of flow is settled by P_1 alone. This is a familiar result in the one-dimensional theory of nozzles. There should, as a matter of fact, be a covolume correction to this critical ratio, the magnitude of which has been worked out by Rateau (5) and which amounts to about 7 per cent for $\gamma = 1.25$ when $P_1 = 25$ tons per square inch, towards the upper limit of the ballistic range. For this value of γ, the condition (2) is $P_2/P_1 < 0.555$.

The covolume also alters the rate of flow at a given pressure; it can be shown that the error is less than 7 per cent for pressures up to 25 tons per square inch. In this paper the covolume effects on the flow between the two chambers will be neglected, but *not* the direct effect on the equation of state.

For P_1 and P_2 satisfying (2), the rate of flow is

$$\psi \frac{SP_1}{\sqrt{\lambda}}, \tag{3}$$

where ψ is a numerical factor which depends on γ but lies within 1 per cent of 0.66 for all Service propellants. A correction for friction and heat losses in the nozzle should be added to ψ; a 5 per cent reduction, which is reasonable, makes ψ about 0.63.

If P_2/P_1 is greater than the limit mentioned in (2), the rate of flow is

$$\frac{\chi SP_1}{\sqrt{\lambda}} \left(\frac{P_2}{P_1}\right)^{0.8} \left[1 - \left(\frac{P_2}{P_1}\right)^{0.2}\right]^{\frac{1}{2}} \tag{4}$$

where χ is 3.162 for $\gamma = 1.25$ and no friction losses. With allowance for friction one may take $\chi = 3.00$. Another way to express the difference between (4) and (3) is by their ratio, which can be regarded as the factor representing the effect of back-pressure on the flow. Table I shows some typical values, and is useful in approximate calculations when P_2/P_1 exceeds 0.55.

TABLE I.—*The Back-pressure Factor.*

P_2/P_1	Factor	P_2/P_1	Factor
0.56	1	0.94	0.507
0.6	0.995	0.96	0.419
0.7	0.948	0.98	0.300
0.8	0.840	0.99	0.214
0.85	0.755	0.997	0.117
0.9	0.638	1	0

It will be assumed that P_2/P_1 remains throughout at less than the critical value (2). The great simplicity which this introduces into the mathematics will be seen later. The German guns actually built did satisfy this restriction, though they had been considering types for which (2) would not hold. Provided P_2/P_1 and hence the "back-pressure factor" do not vary too much during the firing, the theory of this paper can be used as a guide when P_2/P_1 exceeds 0.555. As P_2/P_1 approaches unity the solution given here becomes less and less accurate, regarded as a solution of the basic equations.

It is assumed that no unburnt cordite passes the nozzle. This is probably true in the later types of H/L guns with a number of small nozzles in parallel between the two chambers. With normal ignition no appreciable error is expected from this assumption.

3. EQUATIONS OF INTERNAL BALLISTICS, UP TO "BURNT."

3.1 *The Cordite Chamber.*

The equation of state for the gas in the first chamber is

$$P_1\left\{ U_1 - \frac{C(1-\phi)}{\delta} - CN\eta \right\} = C\lambda N \tag{5}$$

and for that in the second chamber and bore is

$$P_2\{U_2 + Ax - C(\phi - N)\eta\} = C\lambda(\phi - N), \tag{6}$$

where λ is written for the mean value of nRT during the period of burning of the propellant. The same λ applies to both the bore and the cordite chamber.

Also

$$\frac{dN}{dt} = \frac{d\phi}{dt} - \frac{\psi S P_1}{C\lambda^{\frac{1}{2}}},$$

which by (1) is

$$\frac{dN}{dt} = \left\{ 1 - \frac{\psi S D}{\beta C \lambda^{\frac{1}{2}}} \right\} \frac{d\phi}{dt},$$

with the initial condition $N = \phi = 0$. Hence

$$N = (1 - \Psi)\phi, \tag{7}$$

where

$$\Psi = \frac{\psi S D}{\beta C \lambda^{\frac{1}{2}}}. \tag{8}$$

This dimensionless parameter Ψ, which is of order 0.5, was found to play a fundamental part in the ballistics of recoilless guns (2), where S was the throat area of the venturi in the breech. Ψ plays an equally important part in the theory of the H/L gun.

Substituting from (7) into (5),

$$\begin{aligned} C\lambda(1-\Psi)\phi &= P_1\left\{ U_1 - \frac{C}{\delta} + \frac{C\phi}{\delta} - C\eta(1-\Psi)\phi \right\} \\ &= P_1\left[U_1 - \frac{C}{\delta} - C(1-\Psi)\phi\left\{ \eta - \frac{1}{\delta(1-\Psi)} \right\}\right]. \end{aligned} \tag{9}$$

Thus the pressure builds up in the first chamber as if it were a closed vessel with charge $C(1 - \Psi)$, covolume η, propellant density $\delta(1 - \Psi)$, and web-size D.

From (1) and (9),

$$\begin{aligned} \frac{d\phi}{dt} &= \frac{\beta C\lambda(1-\Psi)\phi}{D[U_1 - C/\delta - C\phi\{\eta(1-\Psi) - 1/\delta\}]} \\ &= \frac{\mu\phi}{1 + b\phi}, \end{aligned} \tag{10}$$

where

$$\mu = \frac{\beta C\lambda(1-\Psi)}{D(U_1 - C/\delta)} \tag{11}$$

and

$$b = \frac{C\{1/\delta - \eta(1 - \Psi)\}}{U_1 - C/\delta}. \tag{12}$$

Note that b may have either sign, and is usually smaller than the analogous "covolume term"

$$C\left(\frac{1}{\delta} - \eta\right)\Big/(U_1 - C/\delta)$$

for an orthodox gun. Taking the origin of time when $\phi = 1$, the solution of (10) is

$$\phi e^{b(\phi-1)} = e^{\mu t}. \tag{13}$$

The pressure in the first chamber is

$$P_1 = \frac{C\lambda(1 - \Psi)}{(U_1 - C/\delta)}\left(\frac{\phi}{1 + b\phi}\right), \tag{14}$$

whose maximum value always occurs at "burnt" and is

$$P_{1m} = \frac{C\lambda(1 - \Psi)}{(U_1 - C/\delta)(1 + b)}. \tag{15}$$

3.2 The Second Chamber and Bore.

The equations for the second chamber and bore are simply

$$W_2\frac{dV}{dt} = AP_2, \tag{16}$$

where

$$W_2 = W_1 + \tfrac{1}{3}C \tag{17}$$

and

$$P_2 = \frac{C\Psi\lambda\phi}{U_2 + Ax - C\Psi\eta\phi}. \tag{18}$$

Hence

$$\frac{dV}{d\phi} = \frac{\lambda C\Psi(1 + b\phi)}{W_2\mu\left(\dfrac{U_2}{A} + x - \dfrac{\eta C\Psi}{A}\phi\right)}. \tag{19}$$

If $U_2 \neq 0$, then for small ϕ

$$P_2 = \frac{C\Psi\lambda\phi}{U_2} \quad \text{and} \quad P_1 = \frac{C\lambda(1 - \Psi)}{U_1 - C/\delta},$$

so that initially

$$(U_2 \neq 0) \quad P_2/P_1 = \frac{\Psi}{(1 - \Psi)}\,\frac{(U_1 - C/\delta)}{U_2}. \tag{20}$$

Returning to (19) with

$$X = \mu\left(\frac{U_2}{A} + x\right)\left(\frac{W_2}{\lambda C\Psi}\right)^{\frac{1}{2}}, \tag{21}$$

then

$$\frac{d}{d\phi}\left[\frac{\phi}{(1+b\phi)}\frac{dX}{d\phi}\right] = \frac{1+b\phi}{X-\nu\phi}, \tag{22}$$

where

$$\nu = \frac{\mu\eta}{A}\left(\frac{W_2C\Psi}{\lambda}\right)^{\frac{1}{2}}. \tag{23}$$

b and ν will both be small. It is easy to show that ν is roughly $2\eta\bar{P}/\lambda$, where $\bar{P}$ is the space-mean pressure on the shot up to "burnt." For a gun working at $\bar{P} = 8$ tons per square inch, which is not likely to be exceeded in H/L guns, ν is less than 0.3. The value of b increases with the density of loading, and for a density of loading of 0.8 gm. per cubic centimeter the limits of b are -0.6 and $+1$. For practical values of Ψ the range of b values is -0.5 to $+0.5$. One may expect, therefore, that for H/L guns b will usually be small but larger than ν.

The solution which passes through $X = 0$ at $\phi = 0$ is

$$X = 2\phi^{\frac{1}{2}} + \frac{\nu}{5}\phi + \left(\frac{4b}{5} + \frac{4\nu^2}{125}\right)\phi^{\frac{3}{2}} + \frac{44}{425}b\nu\phi^2 + \frac{19}{325}b^2\phi^{5/2} + 0(b\nu\phi^{5/2}) + 0(\nu^3\phi^2). \tag{24}$$

This is the solution for the special class of guns with $U_2 = 0$. In terms of X and ν, for all solutions

$$P_2 = \frac{\mu(W_2C\Psi\lambda)^{\frac{1}{2}}}{A}\left(\frac{\phi}{X-\nu\phi}\right). \tag{25}$$

For the particular solution (24),

$$P_2 = \frac{\mu(W_2C\Psi\lambda\phi)^{\frac{1}{2}}}{2A}\left[1 + \frac{2\nu}{5}\phi^{\frac{1}{2}} + \left(\frac{18\nu^2}{125} - \frac{2b}{5}\right)\phi - \frac{158}{425}b\nu\phi^{\frac{3}{2}} + \frac{17}{130}b^2\phi^2 + \cdots\right]. \tag{26}$$

The maximum of P_2 occurs at $\phi = 1$ unless $b > \frac{5}{6}$, and even then only if ν is small. For a density of loading of 0.9 gm. per cubic centimeter, b is greater than $\frac{5}{6}$ only if $\Psi > 0.74$, while for lower densities of loading Ψ must be greater still; for example, at a density of loading of 0.7 gm. per cubic centimeter the peak of P_2 always occurs at $\phi = 1$. Hence it is only in the most extreme cases that $P_2(\phi = 1)$ will not be the maximum pressure P_{2m}, and even here they will agree closely. Therefore

$$P_{2m} = \frac{\mu(W_2C\Psi\lambda)^{\frac{1}{2}}}{2A}\left[1 + \frac{2\nu}{5} + \frac{18\nu^2}{125} - \frac{2b}{5}\right] \tag{27}$$

and

$$\frac{P_{2m}}{P_{1m}} = \frac{\beta(1+b)(\lambda C W_2 \Psi)^{\frac{1}{2}}}{2AD}\left[1+\frac{2\nu}{5}+\frac{18\nu^2}{125}-\frac{2b}{5}\right]. \tag{28}$$

For values of ϕ less than unity, P_2/P_1 is nearly proportional to $\phi^{-\frac{1}{2}}$. On going back in the solution towards smaller values of ϕ, one arrives always at a state in which the basic assumption $P_2/P_1 < 0.555$ is violated. This means that the particular solution (24) is not a physically acceptable solution for small ϕ, though it is of course an exact solution of the equation (22). The paradox that $P_2/P_1 \to \infty$ as $\phi \to +0$ is of no physical significance; this behavior would be eliminated in a real gun by Ψ tending to zero as P_2/P_1 approached unity.

The velocity V corresponding to (24) is

$$\begin{aligned} V &= \frac{\phi}{(1+b\phi)}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}\frac{dX}{d\phi} \\ &= \left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}\left[\phi^{\frac{1}{2}}+\frac{\nu}{5}\phi+\left(\frac{b}{5}+\frac{6\nu^2}{125}\right)\phi^{\frac{3}{2}}\right. \\ &\qquad\qquad \left.+\frac{3}{425}b\nu\phi^2-\frac{7}{130}b^2\phi^{5/2}+\cdots\right] \end{aligned} \tag{29}$$

and the velocity at "burnt" is

$$V_B = \left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}\left[1+\frac{\nu}{5}+\frac{b}{5}+\frac{6\nu^2}{125}+\frac{3b\nu}{425}-\frac{7b^2}{130}+\cdots\right]. \tag{30}$$

The particular solution obtained up to the present applies only if $U_2 = 0$. A series solution can be found if $U_2 \gg Ax_B$. This solution is

$$\begin{aligned} X = X_0 + \frac{\phi}{X_0} + \left[\frac{3b}{4X_0}+\frac{(\nu X_0-1)}{4X_0^3}\right]\phi^2 + \left[\frac{(\nu X_0-1)^2}{9X_0^5}-\frac{(\nu X_0-1)}{36X_0^5}\right. \\ \left.-\frac{b}{12X_0^3}+\frac{5b(\nu X_0-1)}{18X_0^3}+\frac{b^2}{6X_0}\right]\phi^3+\cdots. \end{aligned} \tag{31}$$

The corresponding pressure is

$$\begin{aligned} P_2 = \frac{\phi\mu(W_2C\lambda\Psi)^{\frac{1}{2}}}{AX_0}&\left[1+\frac{(\nu X_0-1)\phi}{X_0^2}\right. \\ &+\left\{\frac{(1-\nu X_0)}{4X_0^4}+\frac{(1-\nu X_0)^2}{X_0^4}-\frac{3b}{4X_0^2}\right\}\phi^2 \\ &+\left\{\frac{(\nu X_0-1)^3}{X_0^6}-\frac{7(1-\nu X_0)^2}{12X_0^6}+\frac{b}{12X_0^4}+\frac{16b(1-\nu X_0)}{9X_0^4}\right. \\ &\qquad\left.\left.-\frac{b^2}{6X_0^2}\right\}\phi^3+\cdots\right] \end{aligned} \tag{32}$$

and the velocity is

$$V = \frac{1}{X_0}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}\left(\frac{\phi}{1+b\phi}\right)\left[1+\left\{\frac{3b}{2}+\frac{(\nu X_0-1)}{2X_0^2}\right\}\phi \right.$$
$$+\left\{\frac{(\nu X_0-1)^3}{3X_0^4}+\frac{(1-\nu X_0)}{108X_0^4}-\frac{b}{4X_0^2}\right.$$
$$\left.\left.+\frac{5b(\nu X_0-1)}{6X_0^2}+\frac{b^2}{2}\right\}\phi^2+\cdots\right]. \qquad (33)$$

For $X_0 \geqslant 5$ this solution is convenient and accurate. For non-zero values of X_0 such that these series do not converge sufficiently rapidly, it is best to integrate (22) numerically. The boundary conditions are that $\phi \frac{dX}{d\phi} = 0$ and $X = X_0$ at $\phi = 0$. Since for the analysis of routine firings one is interested only in the pressure, velocity, and travel at "burnt," $\phi = 1$, the most useful results can be tabulated very simply, as in Tables II, III, and IV. Here are given X and $dX/d\phi$, both at

TABLE II.—*Values of $X/(2+X_0)$ at "Burnt," as a Power-series in b and ν.*
(The coefficients should be interpolated linearly with respect to X_0.)

X_0					
0	1.000	$+0.400b$	$+0.032b^2$	$+0.100\nu$	$+0.05b\nu$
0.05	0.964				
0.1	0.928				
0.15	0.894				
0.2	0.861	$+0.355b$			
0.25	0.830			$+0.095\nu$	
0.3	0.801				
0.35	0.773				
0.4	0.747	$+0.308b$			
0.45	0.724				
0.5	0.703		$+0.030b^2$	$+0.085\nu$	$+0.05b\nu$
0.55	0.687				
0.6	0.674	$+0.262b$			
0.65	0.662				
0.7	0.651	$+0.239b$			
0.75	0.642			$+0.065\nu$	
0.8	0.634	$+0.218b$			
0.9	0.621	$+0.198b$			
1.0	0.611	$+0.181b$	$+0.025b^2$	$+0.045\nu$	$+0.03b\nu$
$1\frac{1}{8}$	0.604	$+0.162b$			
$1\frac{1}{4}$	0.600	$+0.146b$		$+0.033\nu$	
$1\frac{3}{8}$	0.600	$+0.132b$			
$1\frac{1}{2}$	0.602	$+0.120b$	$+0.020b^2$	$+0.025\nu$	
$1\frac{3}{4}$	0.609	$+0.100b$			
2	0.618	$+0.084b$	$+0.016b^2$	$+0.015\nu$	$+0.01b\nu$
$2\frac{1}{2}$	0.641	$+0.061b$	$+0.013b^2$	$+0.009\nu$	
3	0.665	$+0.047b$	$+0.010b^2$	$+0.005\nu$	
$3\frac{1}{2}$	0.688	$+0.037b$	$+0.007b^2$	$+0.002\nu$	
4	0.708	$+0.030b$	$+0.005b^2$	$+0.000\nu$	
$4\frac{1}{2}$	0.726	$+0.025b$	$+0.003b^2$		
5	0.743	$+0.021b$	$+0.002b^2$	$+0.000\nu$	$+0.01b\nu$

TABLE III.—*Values of* $\left(\frac{1 + X_0}{1 + b}\right)\frac{dX}{d\phi}$ *at "Burnt."*

(The coefficients should be interpolated linearly with respect to X_0.)

X_0					
1.1	1.425	$+0.515b$	$-0.081b^2$	$+0.401\nu$	$+0.07b\nu$
1.3	1.410	$+0.542b$		$+0.378\nu$	
1.5	1.392	$+0.560b$	$-0.070b^2$	$+0.350\nu$	$+0.125b\nu$
1¾	1.368	$+0.576b$			
2	1.344	$+0.585b$	$-0.054b^2$	$+0.285\nu$	$+0.12b\nu$
2¼	1.322	$+0.590b$			
2½	1.301	$+0.593b$	$-0.034b^2$	$+0.234\nu$	$+0.115b\nu$
3	1.264	$+0.592b$	$-0.016b^2$	$+0.200\nu$	$+0.10b\nu$
3½	1.234	$+0.587b$	$-0\ 008b^2$	$+0.174\nu$	$+0.09b\nu$
4	1.210	$+0.580b$	$-0.000b^2$	$+0.152\nu$	$+0.08b\nu$
4½	1.191			$+0.135\nu$	
5	1.175	$+0.564b$	$+0.037b^2$	$+0.123\nu$	$+0.05b\nu$

TABLE IV.—*Values of* $\frac{1}{1 + b}\frac{dX}{d\phi}$ *at "Burnt."*

(The coefficients should be interpolated linearly with respect to X_0.)

X_0					
0	1.000	$+0.217b$	$-0.070b^2$	$+0.210\nu$	$+0.00b\nu$
0.05	1.018				
0.1	1.027	$+0.226b$		$+0.235\nu$	
0.15	1.029				
0.2	1.025	$+0.234b$		$+0.253\nu$	
0.25	1.015				
0.3	1.000	$+0.241b$		$+0.264\nu$	
0.4	0.965	$+0.247b$		$+0.268\nu$	
0.5	0.924	$+0.251b$	$-0.056b^2$	$+0.265\nu$	$+0.01b\nu$
0.6	0.881	$+0.253b$		$+0.257\nu$	
0.7	0.837	$+0.254b$		$+0.247\nu$	
0.8	0.795	$+0.253b$		$+0.233\nu$	
0.9	0.754	$+0.252b$		$+0.220\nu$	
1.0	0.715	$+0.249b$	$-0.042b^2$		$+0.03b\nu$
1.1	0.678	$+0.245b$		$+0.191\nu$	

$\phi = 1$. From these can at once be obtained the maximum pressure in the bore,

$$P_{2m} = \frac{\mu(W_2 C\Psi\lambda)^{\frac{1}{2}}}{A(X_B - \nu)}, \tag{34}$$

the travel at "burnt" equals

$$\frac{(X_B - X_0)}{\mu}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}, \tag{35}$$

and the velocity at "burnt" equals

$$\frac{1}{1 + b}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}} \cdot \left(\frac{dX}{d\phi}\right)_B. \tag{36}$$

Linear interpolation leads to errors of, in the worst case, 0.002 in $X_B/(2 + X_0)$ and 1 in 400 in $(dX/d\phi)_B$. Other errors can arise from the approximate representation of the computed results, so that the possible errors quoted just above should be doubled. This accuracy is ample in view of the simple assumptions underlying this theory, and is greater than the accuracy with which the analogous theory represents the behavior of orthodox guns.

It has already been shown that if $U_2 = 0$ the maximum of P_2 occurs at "burnt," except in extreme cases. Now an increase of the chamber volume of a gun causes the maximum pressure to occur at larger values of ϕ. Hence it is seen, and this can also be verified from the numerical solutions, that it is only in the most extreme cases that the peak bore pressure does not occur at "burnt."

4. INTERNAL BALLISTICS AFTER "BURNT."

So long as $P_2/P_1 < 0.555$, the gas flow from the first chamber is the same as if P_2 were zero, and the rate of decay of pressure P_1 is given by Hugoniot's theory with Rateau's corrections for covolume. It is probable, however, that covolume corrections will be small in all practical "high-low pressure" guns. In such a case,

$$P_1 = \frac{C(1 - \Psi)\lambda}{U_1 - \eta C(1 - \Psi)} \left(1 + \frac{t}{\theta}\right)^{-2\gamma/\gamma-1} \tag{37}$$

and

$$N = (1 - \Psi) \left(1 + \frac{t}{\theta}\right)^{-2/\gamma-1} \tag{38}$$

where

$$\theta = \frac{2U_1}{(\gamma - 1)\psi S\sqrt{\lambda}}. \tag{39}$$

The gas in the second chamber and bore also expands adiabatically. The equations are

$$P_2 = \frac{C\lambda(1 - N)T'}{U_2 + Ax - C(1 - N)\eta} \tag{40}$$

and

$$W_2 \frac{d^2x}{dt^2} = AP_2 \tag{41}$$

together with the equation of energy

$$\frac{C\lambda}{\gamma - 1} \{\Psi - (1 - N)T'\} = \tfrac{1}{2}W_2 \left\{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dx}{dt}\right)_B^2\right\}, \tag{42}$$

where T' is written for the ratio of the gas temperature to the mean value which it had during burning. Suffix B refers to conditions at “burnt.” Differentiating (42) and using (40) and (41),

$$T'\frac{dN}{dt} - (1 - N)\frac{dT'}{dt} = \frac{(\gamma - 1)(1 - N)T'Adx}{\{U_2 + Ax - C(1 - N)\eta\}dt},$$

which integrates to

$$\log\left\{\frac{(1 - N)T'}{\Psi}\right\} = -(\gamma - 1)\log\left\{\frac{U_2 + Ax - C(1 - \bar{N})\eta}{U_2 + Ax_B - C(1 - \bar{N})\eta}\right\}. \qquad (43)$$

To obtain the muzzle-velocity, therefore, it is possible to guess $\bar{N}$ for the period after “burnt,” determine $(1 - N)T'$ from (43), and the muzzle-velocity from (42). The mean velocity after “burnt” gives the epoch of shot-ejection and so, from (38), the value of N at that instant. It is then verified that the first $\bar{N}$ was sufficiently accurate.

Finally one must insert the value of $(1 - N)T'$ in (40) and the estimate for the time of shot-ejection in (37) and examine whether P_2/P_1 is still less than 0.555 at the muzzle.

Although (38) and (39) apply only if P_2/P_1 is less than 0.555, the equations (42) and (43) are always true. If P_2/P_1 is greater than 0.555 for part or all of the period after “burnt,” ψ in (39) should be multiplied by an appropriate “back-pressure factor” from Table I. (38) can still be used, as an approximation, with the new and larger value of θ, when finding $\bar{N}$ for the period after “burnt.”

5. SUMMARY OF THE WORKING FORMULAE.

In practice one is usually interested only in certain salient features of the ballistic solution. The first of these is the peak pressure in the cordite chamber:

$$P_{1m} = \frac{C\lambda(1 - \Psi)}{(U_1 - C/\delta)(1 + b)}, \qquad (15)$$

where

$$\Psi = \frac{\psi SD}{\beta C\sqrt{\lambda}} \qquad (8)$$

and

$$b = \frac{C\left\{\frac{1}{\delta} - \eta(1 - \Psi)\right\}}{U_1 - C/\delta}. \qquad (12)$$

If P_2/P_1 is found to be greater than 0.555 for all or part of the time, ψ is multiplied by an appropriate “back-pressure factor” from Table I.

One can calculate

$$\mu = \frac{\beta C\lambda(1 - \Psi)}{D(U_1 - C/\delta)} \qquad (11)$$

and from the initial volume U_2 of the second chamber

$$X_0 = \frac{\mu U_2}{A}\left(\frac{W_2}{\lambda C\Psi}\right)^{\frac{1}{2}}, \tag{21}$$

where

$$W_2 = W_1 + \tfrac{1}{3}C.$$

One needs also

$$\nu = \frac{\mu\eta}{A}\left(\frac{W_2 C\Psi}{\lambda}\right)^{\frac{1}{2}}. \tag{23}$$

From Table II are obtained X and $dX/d\phi$ at "burnt." Then the peak pressure in the bore is

$$P_{2m} = \frac{\mu(W_2 C\Psi\lambda)^{\frac{1}{2}}}{A(X_B - \nu)}, \tag{34}$$

the travel at "burnt" is

$$x_B = \frac{(X_B - X_0)}{\mu}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}, \tag{35}$$

and the velocity at "burnt" is

$$\frac{1}{1+b}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}\left(\frac{dX}{d\phi}\right)_B. \tag{36}$$

The muzzle-velocity follows by the process described at the end of the preceding section.

6. THE MAXIMUM POSSIBLE PIEZOMETRIC EFFICIENCY.

As an illustration of the kind of problem which can be investigated by the methods of this paper, one may take the question of the maximum possible piezometric efficiency of a high-low pressure gun. By the piezometric efficiency is meant the ratio of the mean accelerating pressure during shot travel to the peak pressure, the term being due to Zaroodny. The interest in this particular problem arises from certain German engineers having claimed that a H/L gun can be designed to give a higher piezometric efficiency than can be obtained from any orthodox gun. This claim was not put forward by the inventor nor is it supported by the guns which were built, though as these were designed for high regularity the low piezometric efficiency is only to be expected.

The theoretical maximum piezometric efficiency obtainable from orthodox guns depends to some extent on the details of the theory used, and, in particular, on the type of resistance which is assumed to oppose the motion of the shot. Various theories give results between 85 and 90 per cent. It is clear that in any event 85 per cent is possible, so that the room for improvement by the H/L principle is small.

Consider now an H/L gun for which $P_2/P_1 < 0.555$ throughout the firing. The maximum piezometric efficiency of a given gun obviously occurs when "burnt" coincides with shot-ejection. The muzzle-energy of the shot is then

$$\frac{\lambda C\Psi}{2(1+b)^2}\frac{W_1}{W_2}\left(\frac{dX}{d\phi}\right)_B^2. \tag{44}$$

The maximum value of W_1/W_2 is unity. Also the peak bore pressure is

$$P_{2m} = \frac{\mu(W_2C\Psi\lambda)^{\frac{1}{2}}}{A(X_B - \nu)}$$

and the travel is

$$X_B = \frac{(X_B - X_0)}{\mu}\left(\frac{\lambda C\Psi}{W_2}\right)^{\frac{1}{2}}$$

so that the work done by P_{2m} acting through a distance X_B would be

$$\frac{\lambda C\Psi(X_B - X_0)}{(X_B - \nu)}. \tag{45}$$

From (44) and (45), the piezometric efficiency is

$$\text{P.E.} = \frac{(X_B - \nu)}{2(1+b)^2(X_B - X_0)}\left(\frac{dX}{d\phi}\right)_B^2. \tag{46}$$

For $b = \nu = 0$, this is a function of X_0 alone. The P.E. starts at 0.5 for $X_0 = 0$, rises to a maximum of about 0.6 near $X_0 = \frac{1}{4}$, and then falls to 0.5 again as X_0 tends to infinity. Increase of b increases the upper limit of the P.E. For $b = 0.5$, which is not likely to be exceeded in practice, the maximum P.E. is still less than 0.75. ν has very little influence on this limit, a value of $\nu = 0.2$ changing the piezometric efficiency by only 0.01.

Hence for H/L guns in which P_2/P_1 is always less than 0.555, the piezometric efficiency cannot exceed 0.8, which is less than can be obtained from orthodox guns. The analogous theory for orthodox guns gives a maximum piezometric efficiency of 0.88.

Although these equations can be applied as a guide when P_2/P_1 exceeds 0.555 for part or all of the solution, they become increasingly inaccurate as this ratio approaches unity; indeed, these solutions do not include the orthodox gun as a special case, in spite of having the same basic approximations. This arises from the parameter Ψ being assumed constant during the solution. An orthodox gun can, if one likes, be regarded as having a cordite chamber and a vanishingly small second chamber. The equations for such a system are included in this fundamental set, but as the variation of Ψ is important this method of solution fails.

If any H/L gun has a better piezometric efficiency than can be obtained from an orthodox gun, then the ratio P_2/P_1 must be greater than 0.555 for a large part of the motion. Although it cannot be said that this disproves the claim of a high piezometric efficiency, this claim is certainly made to seem unlikely. It is possible that the claim refers to the conversion of some existing gun into a high-low pressure gun by using a special cartridge case. If the gun were relatively small-chambered the piezometric efficiency might well be raised in this change.

ACKNOWLEDGMENT.

I am indebted to Professor N. F. Mott, F.R.S., for his encouragement of this work, to Mrs. H. N. Wilkinson for computation of the solutions underlying Tables II–IV, and to the Chief Scientist, Ministry of Supply, for permission to publish this paper.

REFERENCES.

(1) C. A. CLEMMOW, *Phil. Trans. Roy. Soc.*, Vol. A227, p. 345 (1928).
(2) J. CORNER, *Proc. Roy. Soc.*, Vol. A188, p. 255 (1947).
(3) O. MATA, "Balística Interior," Madrid (1890).
(4) A. MOISSON, "Pyrodynamique," Paris (1887).
(5) A. RATEAU, *Comp. Rend.*, Vol. 168, p. 330 (1919).
(6) O. SCHMITZ, *Artill. Monatsh.*, Vol. 84, 482; Vol. 86, p. 85 (1913).

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

NEW CRYSTALS FOR INFRARED SPECTROMETRY.

For the past two years scientists of the National Bureau of Standards have been engaged in a study of the thallium halide crystals to determine the feasibility of developing prisms of these substances for use in long-wavelength infrared spectrometry. While it has been known for some time that such materials possess optical properties of value for measurements in the infrared region, crystals of sufficient size for this purpose had never been produced.

During the recent war, mixed crystals of thallium bromide-iodide (generally referred to as KRS-5) were grown at Jena, Germany, and lenses were actually made from this material for use in field instruments. Early in 1946, under the direction of Dr. Francis Phelps, the Bureau began a program of growing crystals of the thallium halides, and a single mixed crystal of high purity was successfully grown. This crystal, which contained about 42 per cent thallium bromide and 58 per cent thallium iodide, was large enough for making a prism for use in an infrared spectrometer.

Crystals of the thallium halides are soft and semi-plastic, so that only slight pressures will set up considerable internal strains; hence, great precaution must be taken in annealing, cutting, and polishing. Edgar Robertson of the Bureau's Optical Shop succeeded in developing methods for grinding and polishing such crystals. He is now able to produce prism faces that are flat to within a few fringes of yellow monochromatic light. The prism which was made for use in the spectrometer has faces 1¾ by 2½ in. Because of the high index of refraction, it was cut so that the refracting angle is 26 deg.

Measurements of the index of refraction of this prism have been made throughout the spectral region extending from the yellow mercury lines in the visible cut to 39.2 microns in the infrared, by L. W. Tilton and R. E. Stevens of the Optical Instruments Laboratory and E. K. Plyler of the Radiometry Laboratory. In the visible region, the value is approximately 2.6 while at 39.2 microns it is about 2.2. Changes in the index with temperature are large, so that constant temperature control is necessary in carrying out accurate spectroscopic work with this material.

Although prisms of other materials, such as potassium bromide, have been available for measurements to 24 microns, the new thallium bromide-iodide prism now allows observations to be made with a prism spectrometer all the way out to 40 microns. Making use of this prism in an infrared spectrometer, the Radiometry Laboratory is investigating the transmission of a great variety of materials from 24 to 40 microns.[1] It has been found that many compounds containing atoms of chlorine, bromine, and sulfur have bands in this region. Studies are also being made of the transmission of various solids to find materials having transmission characteristics that would make them suitable window materials for use in cells and in other applications in the 24- to 40-micron band. For example, it has been determined that thin layers of polystyrene and of polyethylene are very transparent out to wavelengths of 40 microns.

---

* Communicated by the Director.

Data obtained in this investigation will be of material assistance in designing lens systems and in determining optimum prism angles for use in the different regions of the infrared spectrum. Other applications of measurements in this portion of the spectrum, such as chemical analysis by infrared methods, have also been worked out as a result of this work. Additional crystals of thallium bromide-iodide of the same high purity have been successfully produced in the Bureau laboratories.

PROPERTIES AND USE OF EYE-PROTECTIVE GLASSES.

In keeping with general industrial progress in this country, protective glasses and goggles have been developed and improved until today there is a glass or goggle available for adequate protection in every occupation and avocation. Recently, an investigation was made by Ralph Stair at the National Bureau of Standards covering the spectral-transmissive properties of about 200 glasses intended for use as sun glasses or for special industrial operations where injurious amounts of ultraviolet, luminous, or infrared radiant energy are present.[2] Extensive consideration was given to the elimination of glare, the distortion of colors of objects, the use of glasses for driving at night, the standardization of glasses for out-of-doors and industrial purposes, and the spectral-transmissive properties of glasses for use under various types and intensities of radiant energy.

Conservation of eyesight is one of the foremost problems that confront safety engineers in America today. The eyes must be protected not only from mechanical injury, but from exposure to various sources of radiant energy. The natural protective mechanism of the eye is sufficient to adapt it to ordinary amounts of radiant energy encountered about the home, office, shop, or other place of business or recreation.

---

[1] For further technical details, see "Infrared Prism Spectrometry from 24 to 40 Microns," by Earle K. Plyler, *J. Research NBS*, Vol. 41, No. 2, August, 1948 (RP 1911).

[2] A comprehensive report of this work is being published as National Bureau of Standards Circular C471, "Spectral Transmissive Properties and Uses of Eye-Protective Glasses" by Ralph Stair.

However, in certain outdoor activities, for example, automobile driving and snow or water sports, and in industrial applications such as in steel mills, electric or gas welding, and around furnaces, glasses having low transmittancy to harmful radiant energy are required.

The principal reason for which most people wear dark glasses is to eliminate or reduce glare in connection with ordinary outdoor activities. Everyone, no doubt, has experienced the temporary blurring of vision, eye fatigue, or other unpleasant sensation caused by an excessive brightness—glare within the field of vision. The wearing of dark glasses simply reduces the brightness to a comfortable value such that the protective mechanism of the eye can assume complete control of the radiant energy reaching the retina. This result may be achieved through neutral, amethyst, red, amber, yellow, green, blue, or combination-color glasses so long as the optical density (shade number) is sufficient to reduce the luminous transmittance to the proper value for the light intensity encountered.

Glasses having sufficient opacity to give the eye reasonable protection from glare usually exhibit a considerable amount of selective absorbance and may appreciably alter the color of objects. The neutral-shade and smoke glasses have a roughly uniform absorbance throughout the visible spectrum and consequently cause the least distortion of color. On the other hand, since the maximum of visual response is in the yellow-green, for wear outdoors, glasses having a hue in the yellow-green or greenish-yellow portion of the visible spectrum are finding favor in aviation, automobile driving, and recreation. The slight distortion in object color appears to be of less importance than the gains obtained through elimination of radiant energy that serves no useful purpose.

Because of the serious risks involved, the use of dark glasses for relieving glare in night driving demands special attention. Most types of combinations of shaded glasses have been found to be unsafe as a means of protection against glare from automobile headlights. In general it is the belief that any advantage of reducing glare from automobile headlights by wearing dark glasses is more than counterbalanced by the extra hazard arising from the decreased visibility of objects. Tests by a number of observers indicate, however, some basis for claims of increased visibility and reduction of glare through the use of a yellow glass.

Present-day welding glasses having high opacity for the ultraviolet and infrared regions of the spectrum insure complete protection of the eyes when working with any source of radiant energy encountered in industry, provided the proper shade number for clear sight is employed. Since there are differences between individuals, the exact shade number for a particular operation must be chosen through trial by the operator himself for best results in his work.

In operations such as welding with coated rods, or in aluminum or magnesium welding and cutting, producing high radiant flex at the wavelengths of the sodium lines, glasses containing didymium (and sometimes additional materials to absorb more completely the red end of the spectrum) have been found very useful.

Special cobalt-blue glasses have been used and demanded by operators of open-hearth furnaces and similar installations, in the particular shade with which the operator is familiar, because of the contrast in brightness that is obtained between the molten metal and the interior of the furnace. In the production of steel and cast iron by the Bessemer process, a blue-amber glass is employed by the operator to estimate certain color changes of the flame as the impurities are burned off. When each impurity (primarily silicon, magnesium, and carbon) burns as the result of an air blast through the molten metal, a distinctive color appears, on which the operator depends in determining the proper point at which to stop the "blow" and to pour the metal.

Ordinary glass, with its high sodium content, gives off an intense yellow flare when heated. Because of this bright light (at 589.3 millimicrons) the glass worker has difficulty in seeing, without special glasses, the glass to bend or shape it into the desired article. It has, therefore, become the practice of glass blowers to wear didymium glasses which have a high total luminous transmittance and at the same time high absorptance at 589.3 millimicrons.

When glasses having a high optical density are worn for protection of the eyes in industrial operations the elimination of harmful ultraviolet rays must be given first attention. The infrared rays are given second consideration because, their action being thermal, the worker is generally forewarned by a burning sensation. Finally the optical density, or shade number, determines the suitability of a glass for a particular operation.

Through accurate determination of the spectral transmittances and application of luminosity factors for a complete radiator (black body) at a color temperature of 2848° K., standards of optical density have been set up in the form of samples of welding glass having luminous transmittances near the minimum and maximum densities for the various shades.

As a result of the Bureau's investigation it has been proposed that sun glasses be grouped into three or four shades, *A*, *B*, *C*, and *D*, having luminous transmittances of 60, 35, 25, and about 10 per cent, respectively, values that correspond roughly with those for the industrial shades No. 1.5, 2.0, 2.5 and 3.0.

RESIN BONDING—A NEW DEVELOPMENT IN PAPERMAKING.

Through the use of a synthetic resin which imparts unusual strength to paper, vast amounts of wood which have been of little use in the production of paper can now be added to the nation's critical supply of satisfactory pulp timber. This new technique in papermaking combines superior dimensional stability with high strength, when using the ordinary commercial wood pulps, and also makes it possible to produce suitable papers from relatively abundant wood species not now fully utilized. The development, growing out of a comprehensive research program at the National Bureau of Standards on offset printing papers, thus points to improved products and the conservation and extension of critical raw materials supplies. Furthermore, it does not involve either appreciable increase in costs or any important change in manufacturing methods.

Deciduous woods such as the maple, beech, birch, or poplar have until now been used only as "filler" in the manufacture of high grade printing paper. In 1946, for example, wood of this type accounted for only 16 per cent of the pulp consumed and in inventory. No matter how desirable their other characteristics may be, the deciduous woods do not produce the primary qualities of strength and resistance to surface pick, in the usual methods of processing. The development of strength through the use of the synthetic resin, melamine formaldehyde, will have the effect of materially increasing the use of the short-fibered pulps.

Paper is a felted sheeting of vegetable fibers, formed on a screen from a water suspension of the fibers. In conventional papermaking, the fibers are prepared for fabrication by mechanical beating in water. Beating causes the fibers to absorb water and form a gel-like film on their surfaces by a structural change called "hydration." This gel is the cement that bonds the fibers together to give paper of conventional manufacture its strength. Although this beating process has been necessary to provide sufficient fiber bonding for certain printing requirements, it also promotes some of the most troublesome behavior of paper in printing, namely, high expansion, excessive curling, slow oil absorption, and show-through of images.

The new technique consists essentially in substituting synthetic resin bonds between the fibers for the gel-like bonds formed by hydration. The resin bonding gives optimum strength with only a fraction of the beating required to develop comparable strength by beating alone, and produces a superior paper by elimination of the adverse effects of hydration. It is this feature of resin bonding that makes possible the satisfactory sheet strength from pulps of the deciduous woods that do not lend themselves to hydration. Resin-bonding thus opens up an entirely new reservoir of fibrous raw material.

Since paper was invented by the Chinese nearly 2000 years ago, there has been an almost unceasing search for fibrous raw materials. The use of wood, introduced some 75 years ago, brought a breathing spell after periods of near fiber famine. Under the present-day demand, however, the available supply is running low. The magnitude of the fiber requirements for paper can be visualized roughly from the startling fact that all of the cotton grown in the world would not make one-quarter of the paper used in the United States.

In 1944, the Paper Laboratory of the National Bureau of Standards developed a high-wet-strength paper primarily for war maps, but made from conventional pulps. The unusual strength was acquired by substitution of resin bonding for the usual gel-like bonds occurring naturally during manufacture. The present development, involving experimental manufacture on a semicommercial scale, is essentially an extension of this work to the short-fibered pulps that possess every characteristic needed for good printing paper except the important one of strength. Widely different combinations of commercial wood pulps were investigated to determine the advantages of resin-bonding in printing papers. A series of papers was made from each combination with controlled variations in beating, with and without synthetic-resin bonding. In all instances, the resin-bonded papers were superior with respect to curling, oil absorption, folding endurance, resistance to surface pick, and expansivity. Several types of synthetic resins have been used, with melamine-formaldehyde resin giving the best results to date. Surprisingly small amounts of this resin are required, usually less than 3 per cent by weight.

Resin-bonding as a means of utilizing the deciduous woods is being further explored through the use of experimental pulps made specifically for investigations by the Forest Products Laboratory of the U. S. Forest Service. Included in these species are aspen, paper birch, and black gum. Pulps for this research have been prepared by various chemical processes and by mechanical grinding. Sulfite-cooked birch and sulfate-cooked aspen pulps have produced excellent printing papers with the aid of resin-bonding, whereas they can be used only in small amounts as fillers by conventional processing. These particular woods are among the most neglected in so far as paper making is concerned.

Some specific accomplishments with melamine resin-bonding are of interest. For normal commercial fiber combinations, it was possible to develop as much strength with 2 per cent of resin and no beating as with 9½ hr. of beating without resin. Far better strength and resistance to surface picking were obtained with 3 per cent of resin than could be achieved by hydration alone, even with the most drastic beating. Moreover, well-beaten papers were unsuitable for printing stock. In the use of deciduous-wood fibers such as birch sulfite and aspen sulfate, the addition of 1 to 3 per cent of melamine formaldehyde resin increased the resistance to surface picking and folding endurance more than ten times in some instances. Surface fibers must resist the pull necessary to transfer tacky ink from the rubber blanket to the paper without lifting or being themselves transferred to the blanket. A fiber or bundle of fibers loosened by the pull of the ink not only leaves a blemish in the printed image on the sheet where the "picking" occurs, but it may cling to the blanket where it becomes moistened and repels ink so that it prints white spots on succeeding sheets.

All of the resin bonded papers showed low expansivity, essential, for example, in the printing of maps which may be run through the press as many as fifteen times. Expansivity was tested not only mechanically but in a series of printing tests using six colors.

An increase in the use of the short-fibered woods would have its most marked effect in the northeastern region of the United States. In this section, especially, the short-fibered woods have been increasing because past emphasis has always been on the use of the conifers, leaving behind the deciduous woods in logging operations. The ratio of conifers to other types has decreased to the point where the elimination of the short-fibered trees has actually become a problem in forest management.

With the United States using more paper than ever before in its history and the wood pile getting smaller, this new method should aid in making larger stocks available and provide another economic use for the non-coniferous trees. The Social Committee of the United Nations Economic and Social Council has warned that paper shortages are obstacles to freedom of the press. With no large new fiber sources available, the conservation and extension of our fiber resources becomes a matter of serious concern to all.

THE FRANKLIN INSTITUTE.

MUSEUM NOTES.

RUBBER EXHIBIT.

Science and technology know no geographical boundaries. The horizons are limited only by man's ingenuity and the materials with which he has to work. One of the most striking examples of what ingenious man can do to emulate and even surpass nature in the provision of useful things is exemplified in the rubber industry.

The Museum has been fortunate in securing the industrial cooperation of the Lee Rubber & Tire Company in equipping and decorating a room so that the comprehensive story of rubber may be told.

Beginning with a brief pictorial history of natural rubber and a superb wall map showing the distribution of its sources, the exhibits proceed to show the molecular structure of synthetic rubbers, and the significance of rubber in our modern world.

The people of the United States consume more than three-quarters of the world's supply of rubber, and the greater bulk of this goes to the manufacture of automobile tires. Since this application leaves relatively few families in the country unaffected, special attention is devoted to the manufacture of the component parts of the automobile tire.

An illustrated and illuminated flow chart shows the process of manufacture from crude rubber to finished tire, while a recorded talk assists the visitor to follow the process, reminding him that a tire is something more than a piece of molded rubber.

The Museum staff has always been concerned with the provision of dynamic exhibits which would enable the visitor to participate in the demonstration. In the case of this room devoted to rubber, opportunity has been taken to secure visitor participation by the provision of operable exhibits and demonstrations. For example, the simple push of a button will set in operation a device by which the rubber latex is coagulated and the usable base of commercial rubber is produced. By the exercise of a little strength, the visitor may learn the lesson of danger in a skidding tire and the safety furnished by a well treaded tire.

Still other demonstrations illustrate the properties of rubber. Everyone knows rubber's astonishing degree of passive resistance, how it will permit itself to be stretched, buffeted, pummelled, and mauled. Rubber's capacity to "take it" is due to its ability to absorb a blow and to return it. Therein lies most of its usefulness to man. But its other properties are not so well known. Its resistance to abrasion and its adhesive qualities when vulcanized are not so fully recognized, so that emphasis is placed upon these in operable demonstrations. The special properties of synthetic rubbers, such as the acid resistance and dielectric qualities, are similarly shown.

As admirable examples of the modern decorator's art, the rubber flooring and foam rubber "upholstery" of the seats further illustrate how the rubber goods manufacturer is taking advantage of the material which modern science is offering for his use.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

DRAKE, ROLLEN H. Aircraft Engines. 1948.
WERNER, EARL F. Aircraft Instrument Maintenance. 1948.

ARCHITECTURE AND BUILDING.

HUSBAND, JOSEPH AND HARBY, WILLIAM. Structural Engineering. Fifth Edition. 1947.
JOSELIN, E. L. Ventilation and Air Conditioning. Second Edition. 1947.
MAGNUS, EDWARD R. AND MARLOTT, GRACE D. The Refrigeration Serviceman's Manual. 1948.
SLEEPER, HAROLD REEVE. Architectural Specifications. 1940.

ASTRONOMY.

NASSAU, JASON JEAN. Practical Astronomy. Second Edition. 1948.
NIETHAMMER, THEODOR. Die Genauen Methoden der Astronomisch-Geographischen Ortsbestimmung. 1947.

BIOGRAPHIES.

BELL, A. E. Christian Huygens and the Development of Science in the Seventeenth Century. 1947.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

GAYDON, ALFRED GORDON. Dissociation Energies. 1947.
GODDARD, F. W. AND JAMES, E. J. F. The Elements of Physical Chemistry. 1946.
KOWALKE, OTTO LOUIS. Fundamentals in Chemical Process Calculations. 1947.
NOYES, W. A., JR. Chemistry; a History of the Chemistry Components of the National Defence Research Committee. 1940–1946. 1948.
PIGMAN, WILLIAM WARD AND WOLFROM, M. L., Editors. Advances in Carbohydrate Chemistry. Volume 3. 1948.
PIGMAN, WILLIAM WARD AND GOEPP, RUDOLPH MAXIMILIAN. Chemistry of Carbohydrates. 1948.
VERWEY, E. J. W. AND OVERBEEK, J. T. G. Theory of the Stability of Lyophobic Colloids 1948.

DICTIONARIES.

GOLDBERG, MORRIS. English-Spanish Chemical and Medical Dictionary. 1947.

ELECTRICITY AND ELECTRICAL ENGINEERING.

BEWLEY, LOYAL VIVIAN. Two Dimensional Fields in Electrical Engineering. 1948.
CARR, THOMAS HENRY. Electric Power Stations. Volume 1. 1947.
HODGES, E. S. Electricity and Fire Risk. Second Edition. 1947.
MOLLOY, E.; SAY, M. G. AND WALKER, R. C., Editors. "Electrical Engineer" Reference Book. Third Edition. 1948.

GEOLOGY.

SHAND, SAMUEL JAMES. Eruptive Rocks. Third Edition. 1947.
UMBGROVE, JOHANNES HERMAN. The Pulse of the Earth. Second Edition. 1947.

INDUSTRIAL MANAGEMENT.

BLOCKER, JOHN G. Cost Accounting. Second Edition. 1948.
COLUMBIA UNIVERSITY. STATISTICAL RESEARCH GROUP. Sampling Inspection. 1948.
DALLA VALLE, JOSEPH MARIUS. The Industrial Environment and Its Control. 1948.
GIPSON, HENRY CLAY. Films in Business and Industry. 1947.
MAZE, COLEMAN L. Office Management. 1947.

MANUFACTURE.

KRIEBLE, R. H; HOLMES, J. B. AND WILCOXON, B. H. Phenol Manufacture in Germany by the Chlorination, Sulfonation and Raschig Processes. No Date.

MATHEMATICS.

MURRAY, FRANCIS J. The Theory of Mathematical Machines. Revised Edition. 1948.
WALL, H. S. Analytic Theory of Continued Fractions. 1948.
WIDDER, DAVID V. Advanced Calculus. 1947.

MECHANICAL ENGINEERING.

STEEDS, W. Involute Gears. 1948.

METALLURGY.

LORIG, C. H. AND ADAMS, R. R. Copper as an Alloying Element in Steel and Cast Iron. 1948.

PATENTS.

TUSKA, C. D. Patent Notes for Engineers. 1947.

PHYSICS.

BOAS, W. An Introduction to the Physics of Metals and Alloys. 1947.
LITCHY, LESTER C. Thermodynamics. Second Edition. 1948.

RADIO.

COLLINS, GEORGE B., Editor. Microwave Magnetrons. 1948.
MORENO, THEODORE. Microwave Transmission Design Data. 1948.
RAGAN, GEORGE L., Editor. Microwave Transmission Circuits. 1948.
RIDER, JOHN FRANCIS. Television; How It Works. 1948.
SMITH, WOODROW. Antenna Manual. 1948.

STEAM ENGINEERING.

COPP, DOUGLAS. Power and Process Steam Engineering. 1947.

BOOK REVIEWS.

FATTY ACIDS AND THEIR DERIVATIVES, by A. W. Ralston. 986 pages, 14.5 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $10.00.

Dr. Ralston, who is assistant director of the Chemical Research and Development Department of Armour and Co., has written in this volume an excellent comprehensive treatise on the chemistry of the fatty acids and their derivatives.

The first five chapters of his book describe the synthesis, occurrence in nature, and physical properties of those aliphatic acids containing six or more carbon atoms. The preparation of fatty acids from their natural sources, and processes for separating and purifying these acids are also described. The next six chapters discuss the historical background, preparation, physical and chemical properties of the major fatty acid derivatives. Esters, amides, nitriles, amines, quaternary ammonium salts, alcohols, ethers, mercaptans, sulfides, sulfonates, and other sulfur-containing compounds, acid anhydrides, acid chlorides, aldehydes, ketones, metallic soaps and other derivatives are discussed. One chapter describes compounds produced by modifying the long hydrocarbon chain. Another chapter is devoted to a consideration of long chain hydrocarbons; these may not ordinarily be considered as derivatives of the fatty acids. Commercial uses for each class of derivatives are briefly mentioned.

The rapidity with which this important field of chemistry is now developing is illustrated by the fact that the last chapter is an "Addendum" apparently written after the first eleven chapters and intended to bring them more nearly up-to-date.

The viewpoint of the book is academic and fundamental rather than industrial and technological. Naturally occurring glycerides are discussed only briefly. References to other books more adequately covering this subject are given. The very brief discussion of alkali metal soaps is inadequate in view of their great commercial importance and fundamental interest. References to work published during the past ten years on the polymorphic forms of soaps are not given. Phase studies of soaps are not mentioned and no reference made to the many publications on this subject.

The graphs, tables and figures are well done. However, much information now presented in sentences would be more easily understood and readily used if presented in tabular or graphic form.

The book is an authoritative and thorough summary of our present knowledge on the subjects discussed. Over 5000 references to original publications are given. In an effort to impart a good understanding of the field, the author has discussed, where he considered desirable, such allied subjects as some reactions of low molecular weight aliphatic acids, catalysis, theory of distillation, the reaction of carbon with fluorine, and the Stieglitz formulation of the Hofmann rearrangement.

Dr. Ralston received the 1946 Midwest Award of the American Chemical Society in recognition of his contributions to the chemistry of the fatty acid derivatives.

This book is highly recommended to those who may be interested in the subject. It should be widely used as a reference.

R. C. MERRILL.

ATOMIC ENERGY LEVELS, AS DERIVED FROM THE ANALYSES OF OPTICAL SPECTRA, Volume I, Section I, Circular 467 of the National Bureau of Standards, by Charlotte E. Moore. 75 pages, tables, 23 × 29 cm. Price, $0.50.

It is sometimes forgotten that quantum mechanics was the result of the knowledge of atomic spectra supplied by the spectroscopists of the nineteen twenties and thirties. The opportune publication in 1932 of Bacher and Goudsmit's "Atomic Energy States" showed clearly that, in a few years, the spectroscopists had provided the answer to most of the critical questions of atomic structure, but that they had only just begun the complete job of the analysis of the thousands of possible spectra. Since that time, a dwindling number of research workers have been attempting with some success to rectify the earlier errors, analyze more complicated spectra, and make the data of spectroscopy a reliable tool for the astronomers, the theoretical physicists and the users of spectro-chemical analysis.

One of the most active workers in the field has been the author of this book, Mrs. Sitterly, who in her incarnation as a spectroscopist is Miss Charlotte Moore, former assistant to Professor H. N. Russell at Princeton. When Professor Russell's retirement was imminent, Dr. Meggers of the Bureau of Standards was wise enough to recognize Mrs. Sitterly's unique value for their important work. No one could have been selected more competent to undertake the work which is now under way—the publication of a new version of Bacher and Goudsmit's "Atomic Energy States." That the time is opportune for such a revision was recognized by Dr. E. U. Condon when he became director of the Bureau of Standards after the war, and he has given the project every encouragement.

In May 1946, a meeting of the National Research Council Committee on Line Spectra discussed the project. That discussion and the answers to a questionnaire which was sent out afterwards, were used as a basis for deciding the details of the presentation. In the main, there was nearly complete agreement among the physicists consulted, but on a few points there was considerable disagreement. In particular, there was little argument over the question of the order of the spectra; Bacher and Goudsmit's alphabetical order, and all other suggested arrangements were rejected in favor of the more logical order of atomic number. The nomenclature for electron structures and for term types led to no disagreement, but on the notation to precede the term type there was much discussion. The difficulty comes from the complications introduced by different series limits and it has no completely logical and simple solution. The notation finally used in the section under review is a compromise which, like most compromises, will be disliked initially, and will later probably be used by everyone. I find it somewhat confusing that the unprimed letters such as $2p\,^2P^\circ$ and $3s\,^4P^\circ$ in F V refer to terms built on quite different limits.

During the preliminary discussions and since, I have disagreed wholeheartedly with one majority decision—the decision that all levels of a multiple term should be gathered together whether they are mixed in energy with other levels or not. This practice places more importance on the opinion of an investigator than on the one indubitable fact about a level—its numerical value. In the tables there is an excellent example of the distortion produced by this procedure. In the fluorine arc spectrum, the collection together of the components of $4d\,^4D$ and $4d\,^4F$ effectively conceals the really important fact that the seventeen levels of $2s^2\,2p^4\,(^3P)4d$ are arranged in three very narrow groups whose separations are almost exactly the separation of the ^3P limit term. They are in fact a perfect example of J-j coupling, concealed by an arbitrary and quite unnecessary rearrangement.

Such criticism does not, however, detract from the job that Mrs. Sitterly has done. When it was remarked above that she is competent in the analysis of spectra, it was a definite understatement. Rather it should have been said that she is probably the only person there who is technically competent and at the same time has the temperament and training for such an arduous task. And it is a much more arduous task than might be imagined. The published material is spread through twenty or more journals and the unpublished material is in the papers of many more spectroscopists; it is presented in a different way and with a different notation by every individual; it is based on wave-lengths measured on all sorts of scales and with all sorts of inaccuracy; and it is full of errors of judgment and understanding. From such material, Mrs. Sitterly must sift out the nearest approach to a correct term table for each spectrum. She must, in fact, be an analyst of character as well as of spectra. I am sure that if I had to do the job myself, I would end up as a subject for a psychoanalyst.

The decision to produce the work in sections arranged by atomic number was a happy one, because it has meant that the first section on the elements up to fluorine has had as a background the quite outstandingly complete work of Professor B. Edlén, now of the University of Lund. Edlén's thesis which is a collection of data on the light elements and additions by him to most of them, constitutes the only homogeneous body of data on any group of spectra. Since the publication of that thesis, he and others in Professor Siegbahn's school in Sweden have examined so thoroughly the spectra of the light elements that the volume under review is largely the result of their work. They have been very generous with their great accumulation of unpublished data.

Even with the assistance of Professor Edlén's work, it is surprising that Mrs. Sitterly has been able to turn out such a finished piece of work in so short a time. Dr. Condon and Dr. Meggers should congratulate themselves on their perspicacity in recognizing the necessity for publishing this set of tables, and, equally, on recognizing that Mrs. Sitterly was the person to do it.

A. G. SHENSTONE.

THE INTERPRETATION OF SPECTRA, by William Mayo Venable. 200 pages, drawings, tables, 16 × 24 cm. New York, Reinhold Publishing Corp., 1948. Price, $6.00.

In this book, Mr. Venable has collected the available data on the emission spectra of hydrogen, deuterium, helium, lithium, sodium, and neon and attempted to interpret these and other spectra on the basis of a theory which he proposes. His theory is in direct contrast to the currently accepted theory of the emission of spectra as developed by Bohr and others which assumes, among other things, that spectral lines are the result of electronic transitions between different energy levels in an atom. Venable's theory postulates that "under excitation atoms may be separated into component sub-atoms," and that "these sub-atoms recombine with one another to produce the molecules cooperatively responsible for the observed spectral lines." In support of such a theory, he attempts to point out "intimate relationships between spectra emitted by supposedly entirely different sources."

Chapter one is a discussion of the history, development and assumptions of the presently accepted theory of the interpretation of spectra and that which Venable proposes. Chapters

2 to 7 are detailed discussions of mathematical relations between spectral lines. The work of Richardson, Merton, Dieke and Hopfield, Jeppesen and others on the hydrogen spectrum is critically discussed in considerable detail, sometimes without giving adequate references to the original literature

In chapters 8 to 14 attempts are made to begin development of a theory of spectra that accounts for the experimental facts without the assumptions of the quantum theory or without any assumption contrary to electromagnetic theory. The final chapter contains some highly theoretical speculations about the structure of atomic nuclei and the relationship of matter to the medium in which light is propagated. The author admits that these speculations are beyond the range of our present ability to test by experiment. The book contains predictions regarding the spectra of isotopes and mixtures which may be carried out as tests of Venable's theories.

This book will be completely understood only by those who have a good background in atomic and nuclear physics and spectroscopy.

R. C. MERRILL.

THERMODYNAMICS, by Lester C. Lichty. Second edition, 341 pages, illustrations, tables, charts, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

Thermodynamics is often thought of as a very difficult, highly mathematical subject of little practical value. The present book is an attempt to disprove this. Many examples of applications are given and mathematical derivations are held to a minimum.

The title is misleading. Far from being a treatment of the general subject it is restricted to an introductory course in mechanical engineering thermodyamics. Heating of materials, change of state, fluid flow, and engines of various sorts are studied. Chemical reactions are very briefly treated in connection with combustion engines. No attempt is made to explain phenomena from a kinetic or statistical viewpoint.

The rigorous derivations of relationships between variables that loom so menacing in the early portions of most textbooks on thermodynamics are relegated to the appendix. However, entropy is introduced in the second chapter and used in connection with T-S diagrams, although the second law and reversible processes are not discussed until Chapter VII. Even here the explanation hardly seems adequate for what is always a difficult subject to beginners.

The chapters are of a convenient length, and contain many examples worked out. At the end of each chapter is a series of exercises of varying degrees of difficulty. The appendices contain many tables of useful information: properties of water, steam, ammonia, freon, mercury, carbon dioxide, energies and entropies of gases, and many useful charts on properties of gases and liquids.

NEWTON W. MCCREADY.

FUNDAMENTALS OF SOIL MECHANICS, by Donald W. Taylor. 700 pages, illustrations, 14 × 2ʐ cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.00.

Most structures rest on soil, and the role of soil as a foundation and construction material has always been one of high importance. However, soil mechanics was introduced to this country as a scientific discipline only in 1925. Its development since has been rapid.

The author, recognizing the extensive scope of the subject, has kept in mind primarily the graduate student when writing this book. Nevertheless, by omitting certain portions it may be used on the undergraduate level.

The theoretical material is described thoroughly and the emphasis of the book is on correlating the fundamental theories and their applications in the laboratory and the field.

Many aspects of soil mechanics are covered: the testing and classification, permeability, capillarity, seepage, settlement analysis, stability of slopes, retaining walls, pile foundations and many others.

The book is also well referenced and should prove of value to persons engaged in the field of foundation engineering.

H. N. MICHAEL.

ERUPTIVE ROCKS, by S. James Shand. Third edition, 488 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1947. Price, $7.50.

As in the previous editions of this pioneering work, Professor Shand bases his concept of rock types on the chemistry of silicate systems, and thus continues his fight for the abolishment of the "jungle of rock names" which often cripples the perspective of the student.

The book is written for the advanced worker. Its third edition introduces much new material in nearly every chapter and is conveniently divided into two parts. A rough outline of the contents of the nineteen chapters of Part I would include discussions on the fixed constituents of eruptive rocks, the fugitive constituents, the temperature, pressure, and walls of magma on compatible and incompatible phases and on eruptive rock complexes. The discussion on linkage between eruptive rocks and ore deposits, which has been omitted in the wartime edition, is re-introduced and expanded and now covers Chapters X, XI, and XII of Part I. This part is concluded by a classification of eruptive rocks.

The four chapters of Part II dealing respectively with oversaturated, saturated, undersaturated non-feldspathoidal rocks, express the author's system of classification.

Numerous references are appended to each chapter.

H. N. MICHAEL.

PREPARATION AND CHARACTERISTICS OF SOLID LUMINISCENT MATERIALS. Symposium held at Cornell University, October 24–26, 1946. Sponsored by The Division of Electron Optics of The American Physical Society. 459 pages, illustrations, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.00.

This monograph includes twenty-nine papers which were read at a conference on luminescence held in October, 1946, at Cornell University.

The eight introductory papers describe the general features of the phenomenon of luminescence and should prove informative to those who wish to acquaint themselves with this highly active field, and at the same time they should review, for the advanced, what has been accomplished in recent years. A brief note on the contents of these eight papers would include, besides the phenomenon of fluorescence itself, the most effective methods in preparation of solid luminescent substances called phosphors, the constitution and structure of phosphors, their storage and release of light, the infrared-sensitive phosphors, and the phenomena of excitation and emission found in these substances.

The remainder of the papers are contributions of original material, both experimental and theoretical, and are conveniently divided into three groups, dealing respectively with the factors affecting fluorescence characteristics, the storage of luminescence energy, and with miscellaneous aspects of fluorescence.

The first group included papers on the factors of temperature and composition, on effects of intensity variations of the exciting radiation, and some little-known aspects associated with the development of fluorescence in the glassy state.

In the second group, methods for the measurement of brightness decay of phosphorescent materials are described, and there are three papers which concern themselves with the measurement and interpretation of stored energy. The last paper of this group deals with the stimulation of zinc sulfide phosphors.

The third group contains eight papers. The topics covered here are diverse. Effects of ultraviolet and X-rays, absorption measurements, relation between molecular structure and ability to fluoresce, and an unusual study of the motions of electrons in silver-halide crystals in an electric field, are some of the phenomena discussed, and indicate the diversity of the subjects in this group.

The concluding twenty-five pages of the text are devoted to the topics which had arisen during the extemporaneous debate on the last day of the conference. They express the trend of thought in regard to experimental and theoretical research which may be undertaken in order to gain a more quantitative understanding of luminescence.

H. N. MICHAEL.

MINERAL RESOURCES OF THE UNITED STATES, by the Staffs of the Bureau of Mines and Geological Survey. 212 pages, maps, charts, and tables, 21 × 28 cm. Washington, D. C., Public Affairs Press, 1948. Price, $5.00.

This publication is the result of five years of intensive research by some fifty-three staff members of the Bureau of Mines and the Geological Survey of the U. S. Department of the Interior. It presents a quantitative appraisal of the mineral-resource position of the United States, and is the most comprehensive appraisal yet attempted. There are thirty-nine individual commodity chapters which estimate the reserves for the more important metals and minerals used in American industry. These commodity chapters are prefaced by several introductory ones which discuss the economic and technological factors from a national viewpoint as applied to the mineral industry as a whole. There is also an historical review and an interpretive summary of the reserve data given in the individual commodity chapters.

The report reveals that those minerals basic to the maintenance of our present kind of industrial society are not wanting, contrary to recent "scarehead" reports. All in all, our mineral outlook is favorable but, as Secretary of the Interior Krug puts it, "it is obvious that a dynamic program of research and exploration must be pursued if new sources are to be developed to supply future needs."

A simple enumeration of the commodities covered by this report will best indicate its comprehensiveness: antimony; arsenic; asbestos; bauxite; bismuth; cadmium; chromite; coal; cobalt; copper; industrial diamonds; fluorspar; gold; graphite; helium; iron ore; lead; magnesium raw materials; manganese; mercury; strategic mica; molybdenum; natural gas; nickel; nitrates; petroleum; phosphate rock; platinum metals; potash; quartz crystal; salt; silver; sulfur; tantalum; tin; titanium; tungsten; vanadium; zinc.

The numerous tables, charts, and maps indicating the trends of production, distribution of deposits, probable reserves, are among the most valuable features of this publication.

Although it is stated that the purpose of the report is to make the above-described information available to the legislative and executive branches of the Federal and State Governments, the very nature of this publication should make it a "must" as a reference in all college geography and geology departments and to other public schools in general.

H. N. MICHAEL.

CHYMIA, ANNUAL STUDIES IN THE HISTORY OF CHEMISTRY, Volume I. 190 pages, plates, 15 × 23 cm. Philadelphia, University of Pennsylvania Press, 1948. Price, $3.50.

Published under the auspices of the Edgar F. Smith Memorial Collection at the University of Pennsylvania, this new annual promises to become an important medium for the distribution of writings on the history of chemistry. With a strong list of consulting editors, as well as an editorial board headed by Tenney L. Davis, the project carries an authoritativeness which augurs well.

The lead article is by the late C. A. Browne on Frederick Accum, celebrated London chemist of the early nineteenth century. As the result of an earlier article on Accum, Dr. Browne had been able to get in touch with a great-grandson of Accum who supplied information on many previously obscure points. It is this new information which Dr. Browne has discussed in his present article.

Then follows a tribute to Dr. Browne, who had taken an active role in initiating *Chymia* and who served as its Editor-in-Chief until his death early in 1947. The tribute is supplemented by a bibliography of his historical writings, which number 148 published items and 17 unpublished manuscripts deposited in the E. F. Smith Memorial Library.

One of the more significant papers in relating the discovery of new facts is that by Sidney M. Edelstein entitled "Priestley Settles the Water Controversy." A recently discovered letter from Priestley to Sir Joseph Banks is quoted as revealing Priestley's belief that Watt had "the idea of water consisting of pure air and phlogiston . . . before I knew him." Mr. Edelstein then reviews the evidence in the controversy and shows that the leading proponents of the two main claimants, Watt and Cavendish, were agreed that Priestley could settle the matter. Thus this letter becomes a most significant piece of evidence in support of Watt's claim.

Among the other papers are "The Early Use of Potassium Chlorate in Pyrotechny" by T. L. Davis; "Factors Which Led Mendeleev to the Periodic Law" by H. M. Leicester; "The Early Chemical and Pharmaceutical History of Calomel" by G. Urdong; "Pierre Louis Dulong, His Life and Work" by P. Lemoy and R. E. Oesper.

Of special interest to historians of chemistry should be the paper on "Thomas Thomson, Pioneer Historian of Chemistry" by H. S. Klickstein. This is an interesting review both of the man's life and of his principal writings, concluding with a bibliography of his chief publications on the history of chemistry.

In all there are thirteen papers, most of which are accompanied by suitable illustrations, principally portraits. In variety of material and method of presentation this first volume of *Chymia* has set a high standard.

G. E. PETTENGILL.

CHRISTIAN HUYGENS AND THE DEVELOPMENT OF SCIENCE IN THE SEVENTEENTH CENTURY, by A. E. Bell. 220 pages, illustrations, 14 × 21 cm. New York, Longmans, Green & Co., 1947. Price, $4.50.

Among the great scientists Christian Huygens stands a somewhat neglected figure. Although his writings have been collected in a magnificent set of more than twenty volumes by the Société Hollandaise des Sciences, not much attention has been paid to an adequate evaluation of his work. This volume by Dr. Bell is an attempt to rectify the situation.

The book is divided into two main parts, the first consisting of a narrative account of Huygens's life. This treats of the personal details of his life as well as fitting into a chronological picture his principal writings.

The second part is devoted to a more particular analysis of Huygens's scientific work. The first chapter considers the state of science in the first half of the seventeenth century. Subsequent chapters consider various phases of Huygens's studies—collisions between elastic bodies, centrifugal force, statics, cause of gravity, optical studies, the wave theory of light Saturn and Cosmotheoros. Four chapters are devoted to the important treatise on the pendulum clock, the "Horologium Oscillatorium."

Many pertinent illustrations and diagrams are included. A section comprising biographical notes on some of the persons mentioned and a short bibliography are additional useful features.

Although Dr. Bell himself lays no claim to having written more than a beginning account in English, it is a thoughtful and well considered delineation of some of the principal aspects of Huygens's career and works.

G. E. PETTENGILL.

PHYSICS OF METALS AND ALLOYS, by Walter Boas. 193 pages, illustrations, 16 × 25 cm. New York, John Wiley & Sons, Inc. Price, $3.50.

Physical theory has been applied extensively to metallurgy during the past twenty-five years. Crystal structures of metals and alloys and the arrangement of crystals in a polycrystalline aggregate have been determined through the application of X-ray diffraction.

This book, introductory in nature, deals with the development of the physics of metals from a definite point of view. Its main principle could be expressed in the statement that "the properties and the arrangement of the crystals determine the properties of polycrystalline aggregates."

Simple crystallography and some methods of X-ray diffraction are taken up in the first part of the book. This forms a basis for the understanding of the remainder of the text which is concerned with the properties and deformation of metals and with phase-changes in alloys. The physical properties are explained in terms of the properties of the atoms. The concluding part of the book describes chemical and phase-changes produced by alloying and heat treatment. General rules derived from extensive studies of these changes, of their mechanism and causes, are given.

The general, introductory nature of the book should prove of value to metallurgists and chemists by forming a solid basis for further studies.

H. N. MICHAEL.

PUBLICATIONS RECEIVED.

An Introduction to Color, by Ralph M. Evans. 340 pages, 19 × 25 cm., plates and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $6.00.

Industrial Electronics Reference Book, by Electronics Engineers of the Westinghouse Electric Corp. 680 pages, 23 × 30 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $7.50.

Inorganic Process Industries, by Kenneth A. Kobe. 371 pages, 16 × 24 cm., tables, drawings and illustrations. New York, The Macmillan Co., 1948. Price, $6.00.

College Physics, by Henry A. Perkins. Third edition, 786 pages, 16 × 23 cm., tables, drawings and illustrations. New York, Prentice-Hall, Inc., 1948. Price, $6.65.

Essentials of Physics, by Carl F. Eyring. 422 pages, 14 × 21 cm., drawings, illustrations, and plates. New York, Prentice-Hall, Inc., 1948. Price, $5.00.

Sequence in Layered Rocks, by Robert R. Shrock. 507 pages, 16 × 23 cm., illustrations and drawings. New York, McGraw-Hill Book Co., 1948. Price, $7.50.

The Function and Design of Army Searchlights, by E. W. Chivers and D. E. H. Jones. 164 pages, 15 × 24 cm., tables, drawings and illustrations. London, The Illuminating Engineering Society, 1948. Price, 18/6d. (paper).

Science, Servant of Man, by I. Bernard Cohen. 362 pages, 14 × 21 cm., plates and drawings. Mass., Little, Brown & Co., 1948. Price, $4.00.

Mathematics Our Great Heritage, by William L. Schaaf. 291 pages, 14 × 21 cm. New York, Harper & Brothers, 1948. Price, $3.50.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Inhibitors for Ribonucleinase.—CHARLES A. ZITTLE. In preparing the nucleic acids from biological sources it is desirable to be able to inhibit the enzymes which act on the nucleic acids so that maximum yields of nucleic acid in unaltered state can be obtained. McCarty and Avery (1), for example, have obtained a five-fold increase in the yield of desoxyribonucleic acid (DNA) from pneumococci by inhibiting the DNA-ase with citrate.

For the isolation of ribonucleic acid (RNA) it would be desirable to inhibit the specific enzyme ribonucleinase (RNA-ase) which cannot be inactivated by heat treatment (2). Copper in a concentration of 0.006 *M* inhibited RNA-ase 80 per cent (3) but tissue substances (such as those containing sulfhydryl, for example) probably would counteract this inhibition and the copper would have to be removed after extraction of the nucleic acid since it is bound by nucleic acid (3). Zinc is somewhat less inhibitory than copper (3).

In the present studies formaldehyde, ninhydrin, phenylisocyanate, as well as NaCl in high concentration, have been found to be inhibitory to RNA-ase. None is highly specific or active in high dilution but some may justify further study.

EXPERIMENTAL.

Crystalline RNA-ase was used and its activity with and without inhibitors present was determined manometrically at pH 7.5 (4).

Since RNA-ase is a basic protein, having an isoelectric point of 8.0 (2), the influence of acidic substances on it was investigated. In these experiments about 30 micrograms of RNA-ase were introduced into the test system containing the acidic substance in a final concentration of 0.004 *M*. None of the following had any effect: picric, sulfanilic, salicylic and phthalic acids.

In addition, reagents combining with amino groups were investigated because of the basic nature of RNA-ase. These were inhibitory. Shown in Fig. 1, curve B, is the effect of formaldehyde on RNA-ase. In these experiments the formaldehyde was in contact with the enzyme before the enzyme was added to the substrate. Ninhydrin tested in the same way gave an identical inhibition curve.

Phenylisocyanate completely destroyed the activity of RNA-ase under the following conditions: 0.01 cc. of phenylisocyanate was stirred into 20 cc. of 0.025 *M* $NaHCO_3$ containing about 2.5 mg. of the enzyme. The precipitate of diphenylurea was removed by centrifuging and the supernatant fluid assayed for RNA-ase. Samples containing as much as the equivalent of 70 micrograms of RNA-ase had no activity.

The inhibitory effect of NaCl on RNA-ase was investigated by the procedure used for the acidic substances with the results shown in curve A, Fig. 1.

DISCUSSION.

The reagents formaldehyde, ninhydrin and phenylisocyanate react with amino groups, the first two compounds with other groups as well, and they probably inhibit RNA-ase in consequence of this ability to react with amino groups. Thus it can be concluded that amino groups are essential for the activity of RNA-ase as is the case with alkaline phosphatase (5). Further study would be required to determine whether any of these reagents would be useful for inhibiting tissue RNA-ase when RNA is being prepared. Unfortunately the reagents are only relatively specific and they would react with other tissue proteins as well so that the amount of the reagent required might be difficult to determine. The possibility of interaction with nucleic acid would have to be considered also. If changes in the nucleic acid did occur the possibility remains that the changes might be reversible.

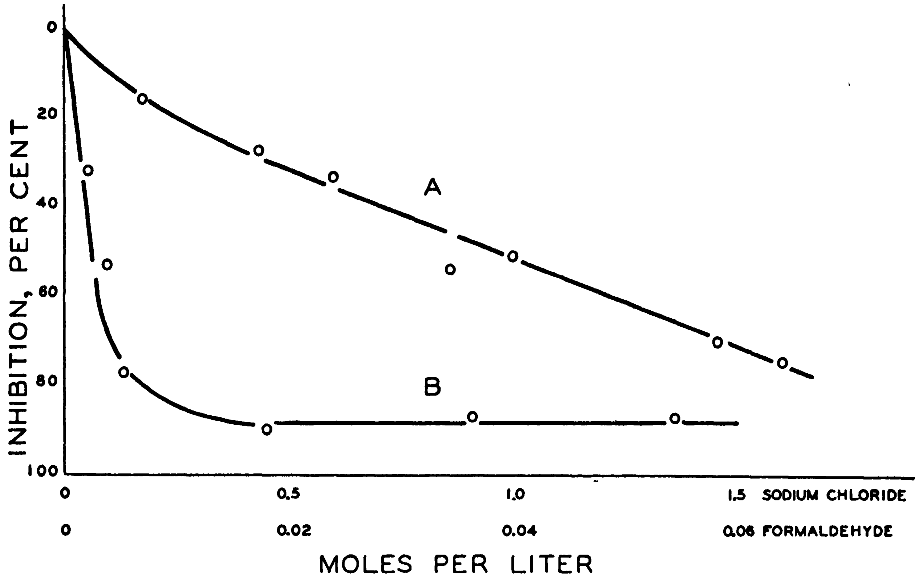


FIG. 1. Effect of formaldehyde and sodium chloride on the hydrolysis of ribonucleic acid by ribonucleinase.

Sodium chloride: Curve A, concentration shown by upper legend.

Formaldehyde: Curve B, concentration shown by lower legend.

Investigations of the effect of NaCl on RNA-ase were prompted by the observations of Mirsky and Ris (6) on chromosomes which were relatively stable in 2 *M* NaCl but unstable in 0.14 *M* NaCl in which RNA appeared in the supernatant fluid. The inhibitory effect of NaCl on purified RNA-ase, shown in Fig. 1, suggests that this enzyme may be involved in the instability of chromosomes. The less specific phosphodiesterase, prepared from calf intestinal mucosa (7), which acts on RNA, may be involved also for 0.25 *M* NaCl exerts a 30 per cent inhibition of this enzyme. In view of these effects of NaCl, extraction of RNA with concentrated salt solutions might be advantageous, for NaCl is more readily eliminated than the more inhibitory reagents discussed above. Carter and Greenstein (8) have reported a stimulating effect of salts (0.1 *M* and less) on RNA-ase but dilute solutions of RNA (1.0 mg. per 1.0 cc.) were necessary. In our experiments the concentration of RNA was 43 mg. per 1.0 cc. and in addition 0.03 *M* $NaHCO_3$ was present initially so the effect reported by these authors would not be observed.

SUMMARY.

Reagents reacting with amino groups (formaldehyde, ninhydrin, phenylisocyanate) are inhibitory to ribonucleinase. Sodium chloride is also inhibitory.

REFERENCES.

(1) M. McCarty and O. T. Avery, *J. Exp. Med.*, **83**: 97 (1946).
(2) M. Kunitz, *J. Gen. Physiol.*, **24**: 15 (1940).
(3) C. A. Zittle, *J. Biol. Chem.*, **163**: 111 (1946).
(4) C. A. Zittle and E. H. Reading, *J. Biol. Chem.*, **160**: 519 (1945).
(5) B. S. Gould, *J. Biol. Chem.*, **156**: 365 (1944).
(6) A. E. Mirsky and H. Ris, *J. Gen. Physiol.*, **31**: 7 (1947).
(7) C. A. Zittle, *J. Biol. Chem.*, **166**: 491 (1946).
(8) C. E. Carter and J. P. Greenstein, *J. Natl. Cancer Inst.*, **7**: 29 (1946).

CURRENT TOPICS.

Weather Towers Near Completion at Brookhaven Laboratory.—Two meteorological towers for weather observations in connection with nuclear research are nearing completion at Brookhaven National Laboratory. The taller tower, to be 420 ft. high, is comparable in height to a 35- or 40-story building, and will be the tallest structure on Long Island, laboratory officials believe. The second tower, 160 ft. high, is already complete, and the taller tower was complete with instruments the latter part of July. At five levels on the small tower, and at eight levels on the tall tower, will be platforms where observers may take readings from instruments, including anemometers to measure wind velocity and direction, and thermometers.

A main purpose of the meteorological towers will be to furnish information on wind currents in connection with safe operation of the nuclear reactor (chain-reacting pile) to be finished this Fall.

The nuclear reactor being built at Brookhaven will be cooled by air. Large fans will conduct the air away from the pile, through an air duct, and up a 300-ft. air-stack on a hill near the reactor. In the air will be minute quantities of radioactive argon, a well-known inert gas, which will be dissipated harmlessly into the upper air. Study of wind velocity and direction, and atmospheric pressure, during operation will permit control of the pile so that air emanating from the stack will not settle on or near the ground.

Instruments on the weather tower and elsewhere on and near the Laboratory site will give precise information on wind currents, and operation of the reactor will be controlled at a low level or stopped entirely if it appears that radiation from the cooling stack under unfavorable conditions would not be completely harmless. With complete information on wind currents at upper and lower levels, and a knowledge of weather conditions on Long Island, scientists are thus assured that operation of the reactor will be harmless to neighboring communities and plant and marine life in the vicinity.

Another feature of the tall weather tower will be a smoke stack which will carry only smoke created for the purpose of studying wind and weather. This is a 20-in. steel pipe running the height of the stack, and it will dissipate smoke produced by an army surplus M-1 smoke generator, similar to those used by naval vessels and beachhead troops in laying down smoke screens to conceal an operation. It produces a harmless oil fog smoke, 10,000 cu. ft. of smoke per minute, and at some time during tests it may create a small cloud over or around the Laboratory, which, under ideal weather conditions, might be visible for several miles.

Weather instruments on the towers will be mounted on beams which can be swung out away from the towers, and then pulled in for observations. In addition, the observations will be recorded electrically on an instrument panel in a building 900 ft. away from the towers. The building has to be this far away from the towers so that eddies created by winds passing over the building will not disturb observations made by instruments on the towers. Two electrified cables between the towers will also carry instruments to record temperatures and wind differences between them.

The meteorology group at the Laboratory will afford facilities for study and research by scientists of the permanent staff and by others who come on leave from their regular jobs elsewhere. The Weather Bureau, in addition to research, will be orienting and training U. S. Weather Bureau personnel, who will be assigned to similar work elsewhere.

Public Lands in Colorado and Utah Reserved for Uranium Drilling.—The U. S. Department of the Interior has advised the U. S. Atomic Energy Commission that in accordance with a request from the Commission it has approved an order (Public Land Order 494) withdrawing from entry and reserving for use of the AEC approximately 115 square miles of public land in Southwestern Colorado and Southeastern Utah.

The U. S. Department of Interior, at the request of the AEC, recently approved an order (Public Land Order 459) withdrawing from entry certain lands in San Miguel and Montrose counties in Colorado. The present order (Public Land Order 494) covers certain portions of lands located in Grand County, Utah, lying in Township 24S-R25E., T24S-R26E., T25S-R25E., T25S-R26E., and certain portions of lands located in Mesa County, Colorado, lying in T49N-R17W., T49N-R18W., T50N-R17W., T50N-R18W., T50N-R19W., T50N-R20W., T51N-R18W., T51N-R19W., T51N-R20W., T15S-R104W.; also included in the order are portions of lands in San Miguel County, Colorado, lying in T43N-R18W., T43N-R19W., T43N-R20W., T44N-R18W., T44N-R19W.

During this summer the Commission, with the cooperation of the U. S. Geological Survey, intends to make a comprehensive study of these areas and to test by diamond drilling for uranium bearing ores. Lands found to contain no uranium will be released from the withdrawal orders and will again be open for entry. It is expected that in furtherance of AEC policy the lands found to contain uranium will become available for development and mining by private interests under arrangement with the Government. No further withdrawals are contemplated at this time until the lands now withdrawn have been thoroughly examined and arrangements have been made for their development or relinquishment.

Wood Research Laboratory Facilities Enlarged.—Growing recognition of the value of research to the lumber industry has resulted in an expansion of plant and facilities of the Timber Engineering Company wood research laboratory.

Recent completion of a 60 by 60 ft. one-story addition to the laboratory makes possible additional services for the manufacturers and users of lumber and wood products. The laboratory, established in 1943, is engaged in product development and research on the chemical, physical, and mechanical properties of wood.

To expand markets for lumber, Timber Engineering Company pioneered to gain recognition of wood as an engineering material and developed the Teco connector system of wood construction. Facilities at the lab include equipment developed especially for testing full scale 50-ft. trusses—the only equipment of its kind in the country.

To improve the mechanical wear of railway ties is a new project that is jointly sponsored by associations of lumber manufacturers and the Association of American Railroads.

Another far reaching study just started is aimed at developing an economical and practical means of decreasing the shrinking and swelling of construction lumber and other wood products.

Finding economic uses for low-grade and waste hardwoods is a major project under way at the laboratory. Development of a hardwood pulping process has formed the basis for the production of many valuable new products out of material formerly considered unusable.

Users of wood office furniture will benefit from such Teco developments as burn and stain-proof desk tops and non-marring desk legs.

The new laboratory building itself is a tribute to the strides in wood engineering resulting from industry research. It is built with glued laminated timber girders (9½ in. by 24½ in. by 20 ft) which will support a second floor when further expansion is desired. The girders are built up of 15 laminations of 1⅝-in. material and were produced by Rilco Laminated Products, Inc., Wilkes-Barre, Pennsylvania.

New THER-MONIC Core Baking Tunnel.—The new THER-MONIC Electronic Core Baking Tunnel was exhibited in operation at the 52nd American Foundrymen's Association Congress and Show in Philadelphia, Pennsylvania. This unit represents the culmination of two years of intensive research and development by the Induction Heating Corporation, Brooklyn, New York, since they first presented Electronic Core Baking in 1946.

This model, with an average baking cycle of only a few minutes, has a capacity of 650 lb. of cores per hour and will produce a ton of baked cores at a power cost of only 92¢.

Production experience, during the last two years, in foundries throughout the United States has revealed many outstanding advantages for this new method of core baking. Electronic heating times are in the order of 20 seconds to a few minutes, whereas the minimum baking cycle in combustion ovens usually runs from 1 to 4 hr. Now cores can be baked as they are needed in the pouring room and changes in molding schedules can be effected without any thought as to what cores are in stock. The necessity for advanced scheduling of core-room activity is removed. This rapid baking cycle also prevents core sagging during the baking operation.

Green cores may be loaded, in the core-room, directly onto the THER-MONIC tunnel conveyor, eliminating all the racking and much of the handling of the cores in the green state.

After the baking cycle, the core plate emerging from the tunnel may go directly onto the inspection table. There is no cooling period involved and the cores are ready for immediate inspection and use. The conventional mhthod of loading and unloading large storage racks is entirely avoided. With conventional methods of core binders and baking, between 30 per cent and 40 per cent of the cores never find service in the foundry. Compared with this, by the use of THER-MONIC Electronic Core Baking and synthetic resin binders, these losses are cut to less than 10 per cent.

The THER-MONIC method makes it possible readily to adjust and control the green strength, hot strength, hardness, collapsibility, and other characteristics of the cores. This ready control makes it easy to overcome the problems of hot tearing, burning in, and poor finish of the casting. Shake-out is so complete that core residue may be removed without mechanical assistance.

Measure Honey in "Bee-Years".—The bee is a busy worker but not a big one. Bee specialists of the U. S. Department of Agriculture say the average bee collects several times her weight in honey in a season, but this is only about a quarter of an ounce on the average. So, if a bee could live that long it would take her about 64 years to make one pound of honey. But while the bee is making her quarter of an ounce of honey she is making untold wealth in the form of apples, melons, clover seed, and many other fruits and seeds through her pollinating activities.

United Nations Scientific Conference on the Conservation and Utilization of Resources (UNSCCUR).—United Nations Scientific Conference on the Conservation and Utilization of Resources will be held in the United States in May–June 1949 under the authority of the Economic and Social Council of the United Nations.

This Conference represents a new approach to one of the objectives of the United Nations, the promotion of economic development and higher standards of living throughout the world, by a free exchange of ideas and experience on the conservation and utilization of resources.

In its plenary meetings, comprising approximately half of the programme, the Conference will examine the contribution of improved techniques to the alleviation of shortages of mineral resources, fuel and energy, and food and forest products. A series of plenary meetings will be devoted to questions of particular importance to underdeveloped countries.

Specific problems and techniques will be placed on the agenda of specialized section meetings which form the other half of the programme. In these meetings, specialists will discuss Conference Papers on subjects such as forests, water, fisheries and wildlife, fuel and energy, power, mineral resources; and the interrelated group of subjects included under soils, crops, grazing and livestock resources.

The Conference will be scientific rather than policy-making. Technical developments will be selected for discussion according to the importance of their practical contribution, actual and potential, to economic development and will be treated so as to interest the economist as well as scientist, administrator and engineer.

Conference Papers submitted by experts as well as summary records of discussions will be published after the Conference, and it is expected that they will be a continuing and important contribution of the United Nations to better conservation and utilization of resources.

13 DEC 1948

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 246 OCTOBER, 1948 No. 4

RESEARCH WORK OF THE FRANKLIN INSTITUTE DURING WORLD WAR II ON AIRBORNE FIRE CONTROL APPARATUS.*

BY

RALPH H. McCLARREN.[1]

From 1824, when The Franklin Institute was founded, until today the Institute has participated actively in the advances of industry and science. Its past work in meteorology, boiler research, screw thread standardization, patent activities and other technical pursuits is known widely. But the increase in research and development done by the Institute during the war could not be made public until now. Security restrictions prevented such publicity. Indeed, much of the present work of the Laboratories is still held from publication by the security regulations.

However, some of the instruments developed during the war have been unclassified since the end of the war, and now it is possible to write of these instruments and to explain why they were vital to the victory.

The scientific and research investigations conducted at the Institute during World War II pertained, among other fields, to development of airborne fire control apparatus for military aviation. While the basis of a laboratory staff was assembled for this work, the Laboratories for Research and Development were not officially designated as a Department activity of The Franklin Institute until after the war. The original war contract with the Office of Scientific Research and Development (OEMsr-330) was negotiated in February 1942 and the technical work under this contract was supervised by Section 7.2 of the National

---

* Talk presented at a Joint Meeting of The Franklin Institute and the Instruments and Measurements Group of the American Institute of Electrical Engineers, January 7, 1948; released for publication by the Bureau of Ordnance, Navy Department.

[1] Director, Electronics and Instruments Division, The Franklin Institute Laboratories for Research and Development, Philadelphia, Penna.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute
New Delhi

Defense Research Committee. Projects undertaken concerned the analysis, development, design, and construction of instruments, computers, and sighting systems for directing the fire of torpedoes, bombs, bullets, and rockets, and also in the partial completion of a pilot's universal sighting system. A total of 24 projects was completed during the period 1 February 1942 to 30 September 1945. Of these, 16 were Naval Ordnance projects, 3 were for the Army Air Forces, 2 for the Bureau of Aeronautics, and 3 were exploratory NDRC projects. In addition to the 28 sets of experimental apparatus which were made and tested at service establishments, 8 volumes of technical reports including 125 project reports were completed as a result of the investigations. The research and development work was performed in the main building of The Franklin Institute. Facilities, personnel, laboratory space, purchasing, accounting, and all the necessary services required were provided upon a top priority basis. The Institute was fortunate in having a nucleus of scientific personnel, laboratories, and an excellent machine shop available to accomplish the initial phases of the contract work.

Preliminary contract negotiations were made in January 1942 by Dr. Henry B. Allen, Secretary and Director of the Institute, with Dr. S. H. Caldwell, then Chief of Section D-2, NDRC; plans were discussed with the staff, and on February 1st the original work was started. F. W. Schlesinger, Director of Astronomy and of the Fels Planetarium; I. M. Levitt, Assistant Director of the Planetarium; W. P. West, Assistant Associate Director of Electrical Engineering; and R. H. McClarren, Assistant Associate Director of Aviation and Mechanical Engineering, were transferred from regular Institute duties to the special research and development problems. Shortly thereafter, R. K. Marshall, Assistant Director of the Planetarium, and W. F. Ehringer of the electrical staff joined the research group. In addition to the scientific staff, valuable and necessary service was rendered by all the staff of the Institute: R. W. Lloyd, Vice-President, who acted as top administrator of the contract work; W. F. Jackson, Jr., Business Manager; S. J. Clevenger, Controller; L. G. Schlehner, Building Superintendent; J. H. Jackson, Purchasing Agent; and many others including the Guides, who performed guard duties.

As the number of employees grew and the requirements for laboratory areas increased, more and more space was provided from museum activity. However, this space was so chosen and segregated from the museum activities of the Institute that only those working on the project were aware of the secret nature of the work.

The first project assigned for investigation was the development of a special dive bomb sight, and the members of the newly formed staff looked forward to the early completion of an experimental unit so that they might soon have the feeling they were participating actively in the war. However, the design of an experimental sight was never started. About the time certain analytical and mathematical analyses of the problem were finished, in late February 1942, NDRC had a more urgent problem. Consequently, all work was stopped on the dive bomb sight, and the full capacity of the laboratory group was devoted to the study, design, and development of a computing-type of stabilized torpedo director.

TORPEDO DIRECTORS.

Within a few weeks the theory and analysis of the problem had been digested and a study model of the director, constructed. Dr. Caldwell and his Technical Aide, George H. Philbrick, were, in this as well as in all subsequent projects, of great help and assistance. Mr. Harry Jones and R. W. Pitman had been added to the staff. Mr. Pitman, formerly research engineer at the Underwood Elliott Fisher Co., designed the first experimental torpedo director. This marked the beginning of a series of developments of torpedo directors which occupied much of his time and the time of part of the staff throughout the war.

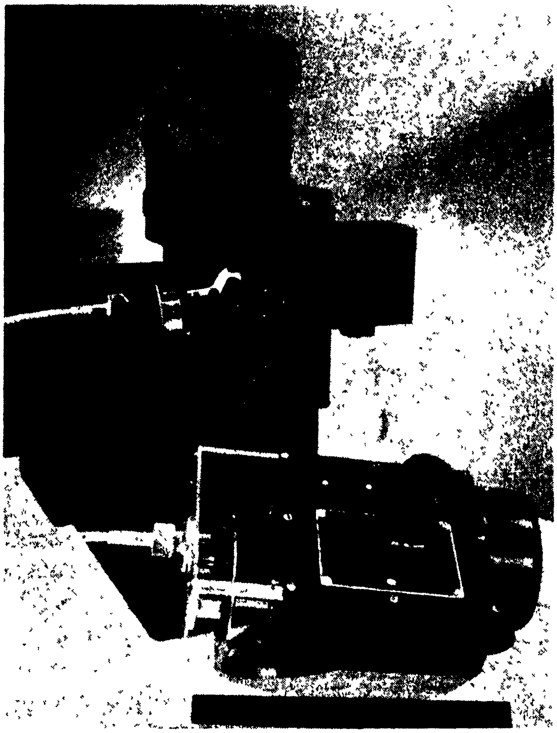


FIG. 1. The Mark 8 torpedo director adapted for automatic inputs and computation of lead angle.

The first experimental torpedo director was tested for principle of operation in a PBY airplane at Naval Air Station, Norfolk, Virginia. Two auxiliary pieces of apparatus were developed for use with this torpedo director. One, an a-c. controller and servomechanism, maintained a stabilized input to the director. The other was a two-phase stabilizer controller with a similar, yet different, servomechanism. During the time of test referred to above, a later development from the experimental model called the Mark 32 torpedo director was also tested at the Naval Air Station, Norfolk—this time in a torpedo bomber aircraft, TBM.

The second director is similiar to 25 such directors built by the International Business Machines Corp. in accordance with Franklin Institute designs and requirements. These directors were used by naval establishments for service tests conducted during the period February 1943 to June 1943. Later production models were installed on torpedo bombers for use in the Pacific theater of operation. Continued investigations produced, in the torpedo director development, many improvements over the first two models. During these investigations special attention was given to developing test equipment for use on motor torpedo boats. Toward the end of the war the latest improvements in radar and other fire control aids were being designed into an advanced type of torpedo director sight, as indicated in Fig. 1.

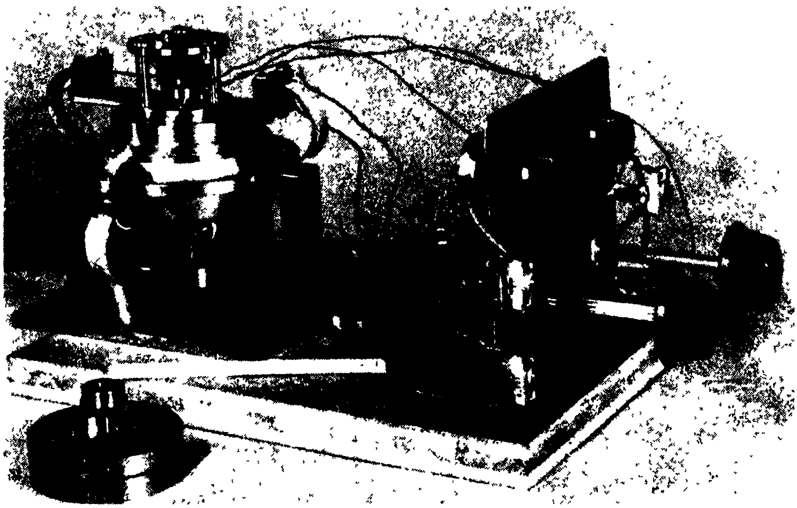


FIG. 2. Experimental gyro used in investigations of lead computing gun sights.

GUNNERY STUDIES AND SIGHTING SYSTEMS.

Shortly after initiating the investigations on torpedo directors the Institute was asked to perform a series of controlled experiments on the application of lead computing sights to the flexible guns used in bomber aircraft, to build apparatus suitable for conducting such tests, and to develop certain experimental gun sights. By this time the staff had been augmented by additions including H. A. Van Dyke, Foxboro representative in Philadelphia, Penna.; J. D. Eisler, Stanolind Research Laboratories in Tulsa; E. F. Allen, professor of Physics at Bryn Athyn Academy; Carl Sheppard, technician; U. C. S. Dilks, electrical engineer; and Frank Smith, mechanical designer. Figure 2 shows an experimental gyro constructed for laboratory studies of various methods for indicating gyroscopic precessional force. The advantage of gyroscopic gun sights was appreciated during 1940 by both British and American scientists, and several treatises on the subject were prepared by the Applied Mathematics Panel of NDRC. Fortified with their reports and with early experiments conducted on a British gyro gun sight, the Institute embarked upon a series of investigations and developments

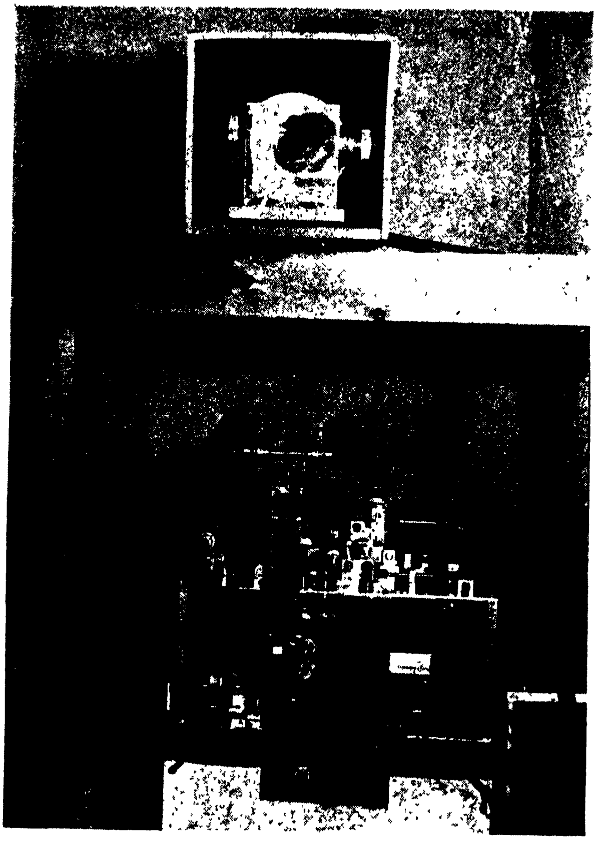


FIG. 3. Mechanical tracking mechanism used to study the behavior of lead computing sights.

which continued throughout the war. Figure 3 shows a mechanical tracking study mechanism designed by H. A. Van Dyke assisted by Professor J. B. Russell, of Section 7.2, NDRC. The device was built to study, under simulated conditions, the tracking problems involved in the use of a lead computing sight. The computing unit, with handlebar controls, and the interior of the target and reticle presentation unit are shown. Two recorders were used to obtain elevation and azimuth errors for both the sight and the guns.

The early success of the data obtained from a series of controlled experiments performed under the direction of consulting psychologists, Dr. Malcolm Preston, and Dr. Francis Irvin of the University of Pennsylvania, led to immediate changes in the designs and requirements for further production orders of turrets and flexible gun controls. The unquestionable advantage of specially controlled tracking and of the use of lead computing sights was demonstrated, and an additional tracking study mechanism was designed and built. The operator's station and the recording and control panels of this mechanism are shown in Fig. 4, and the electronic computing apparatus is illustrated in Fig. 5. In late 1944 the mechanism was revised to permit studies and the accumulation of data with the subject located in a turret assembly. Figure 6 shows the cathode-ray tube used for target and sight presentation and the other special equipment attached to the turret for the tests.

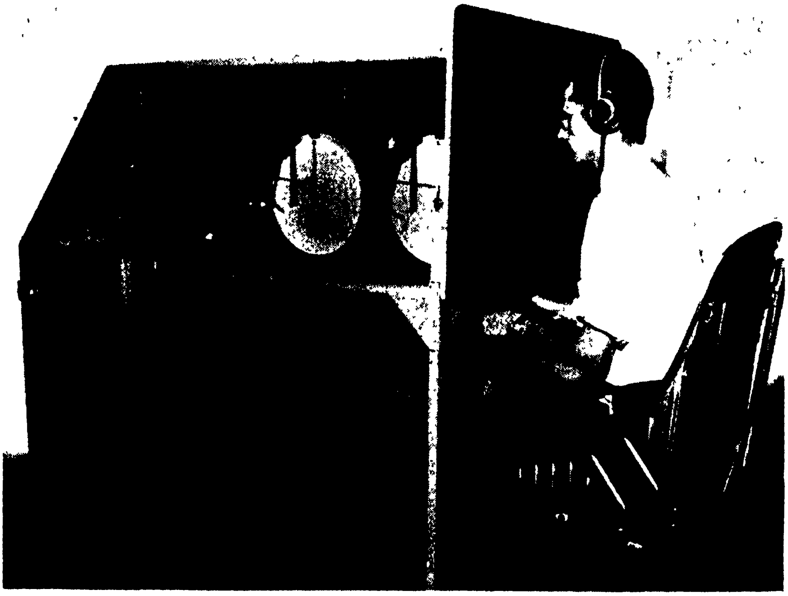


FIG. 4. Electronic tracking study mechanism showing the operators (subject) station and the control-recorder panels.

BOMB SIGHTS.

The development of special bomb sights was another major field of endeavor in which the Institute engaged. W. F. Schlesinger and I. M. Levitt took a leading part in this work. Figure 7 shows VASS Model 1 bomb sight, designed and built in nine days during August 1942 and tested by G. A. Philbrick of NDRC on the tenth day at the Anti-Submarine Development Unit, Atlantic Fleet, Quonset, R. I. So successful was the use of this experimental equipment that expansion of the development and another model were requested immediately. Within a period of one month Model 2, illustrated in Fig. 8, was developed and made available for tests. These sighting units were based upon a principle suggested by Commander A. B. Vossler, then Commanding Officer of the Anti-Submarine Development Unit, Atlantic Fleet. To incorporate desired improvements Model 3, shown in Fig. 9, was designed.

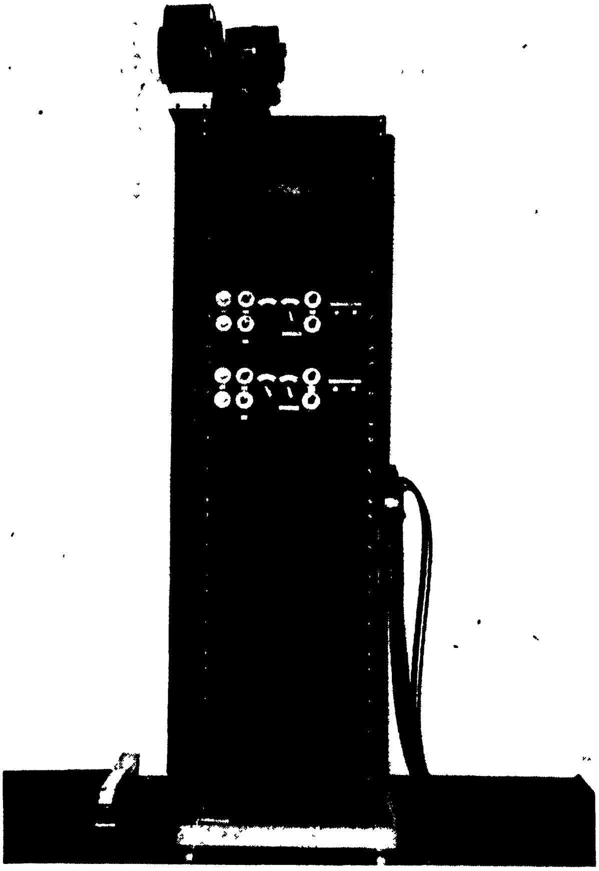


FIG. 5. Amplifiers, power supplies and electronic computer apparatus used with the tracking mechanism.

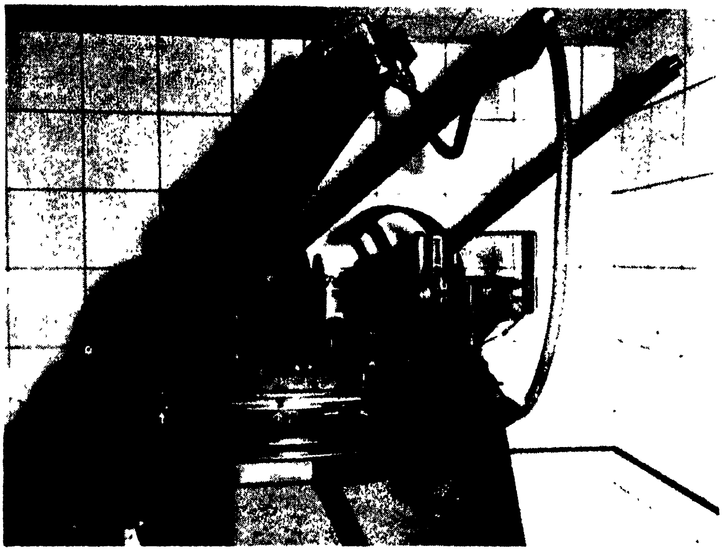


FIG. 6. A Martin upper turret equipped with target and sight presentation apparatus.

FIG. 7. The first experimental hand-held low-level bomb sight. VASS Model 1.

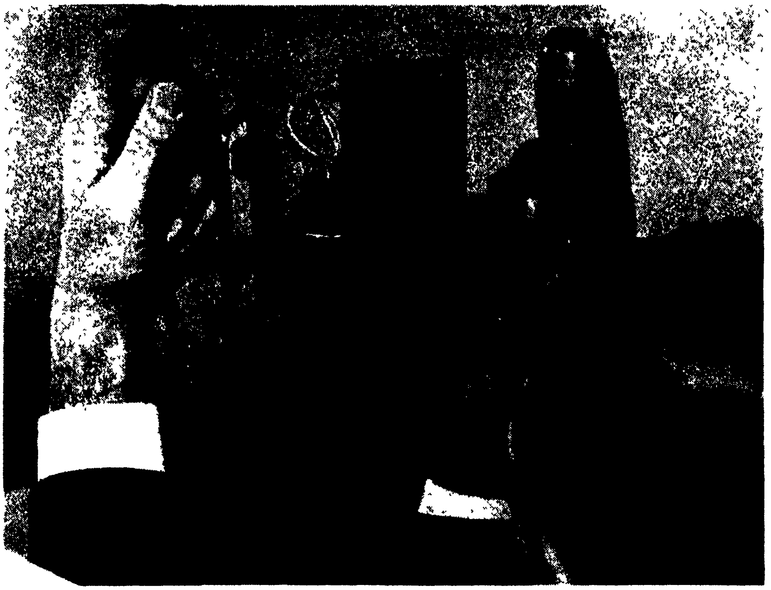


FIG. 8. Model 2 of the experimental VASS bomb sight.

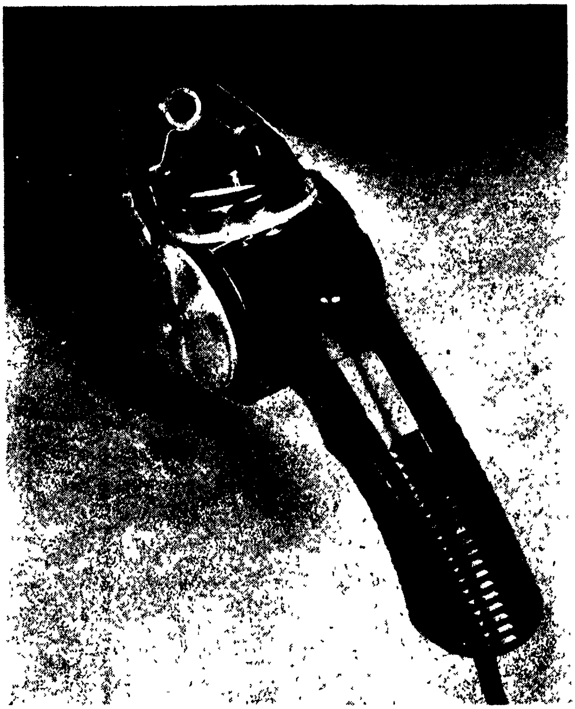


FIG. 9. Prototype production model of the VASS bomb sight.

Three Model 3 units were completed in February 1943 and demonstrated by NDRC to AAF, Wright Field, to the Anti-Submarine Warfare Unit in Boston, and to the Anti-Submarine Command, AAF, in New York. Airborne tests were made at ASDEVLANT, Quonset, R. I.

Model 3, though small, was rather a complicated mechanism, and to meet the requirements of the Navy Department, who desired 60 units for immediate service test and use in the North Atlantic, Model 4 (Fig. 10) was designed by R. W. Pitman. When the designs were completed and a prototype model constructed, no contractor could be found to build the 60 units. Consequently the research and development activity of the Institute was changed temporarily to a production activity.

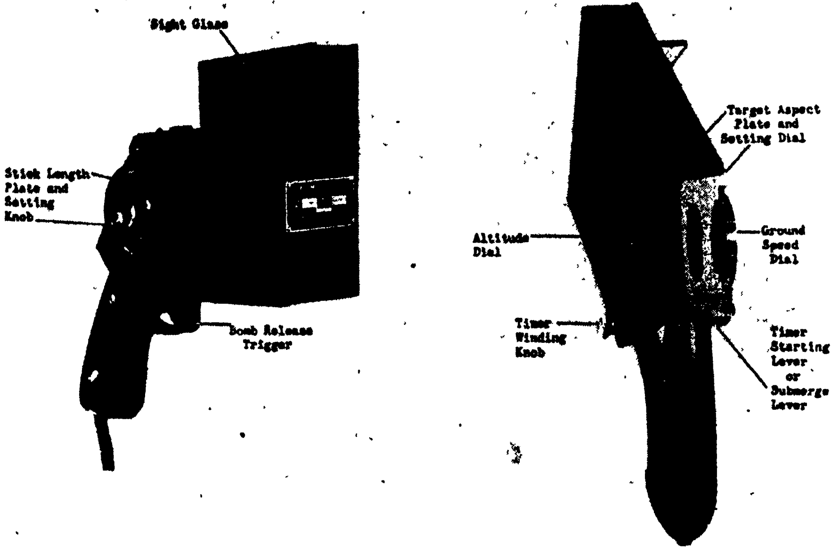

Sight Glass
Stick Length Plate and Setting Knob
Bomb Release Trigger
Target Aspect Plate and Setting Dial
Altitude Dial
Ground Speed Dial
Timer Winding Knob
Timer Starting Lever or Submerge Lever

FIG. 10. The Mark 20 hand-held low-level bomb sight.

Some 20 subcontractors were obtained to supply parts, and the units were assembled in the Institute's limited laboratory space. The aid of a considerable number of the secretarial and executive staff of the Institute was enlisted to check the parts, assemble them, and paint the radium dials. Even the wife of the vice-president was conscripted. The 60 sights were produced and delivered to the Navy within three months. Some of these sights, designated as the Mark 20 bomb sight, saw service in the North Atlantic.

Several auxiliary instruments were developed to improve the use and accuracy of the Mark 20 bomb sight. One of these, a direct, electric-drive, potentiometer-type servomechanism, developed by U. C. S. Dilks for use with a radio altimeter, is shown in Fig. 11. Another (Fig.

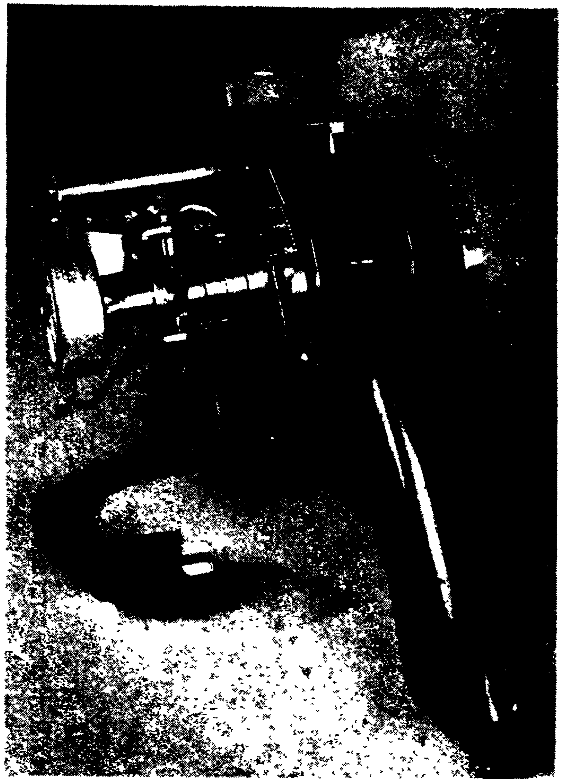


FIG. 11. Altitude servo unit attached to the Mark 20 bomb sight.

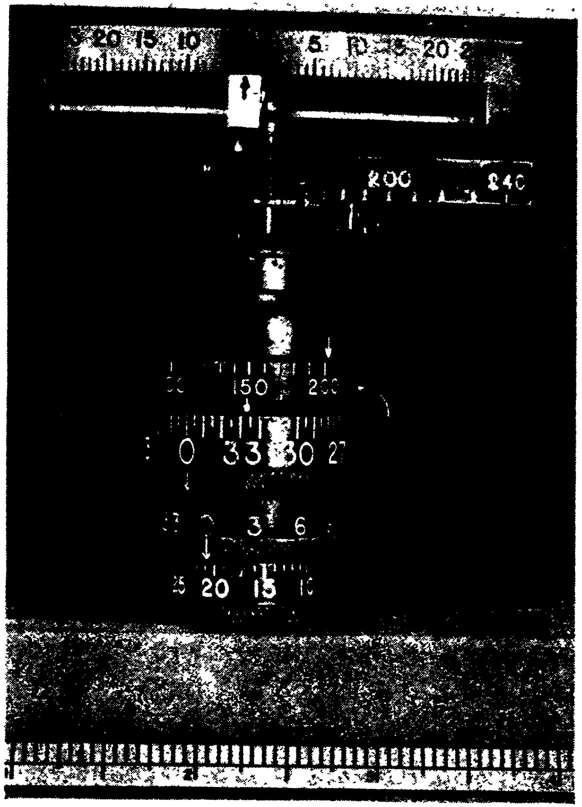


FIG. 12. Ground speed computer for use with the Mark 20 bomb sight.

12) was a small, compact, ground-speed computer for use with the Mark 20 bomb sight. In addition to showing ground speed, this computer would indicate the angle necessary for the pilot to establish a collision course with the target. The conception and development of hand-held bomb sights for use in low-level bombing maneuvers, as illustrated by the Mark 20, established a precedent for further work in the field. Several experimental models of the sights were constructed and tested. The example shown in Fig. 13 employs a rate principle of operation and does not need the vertical reference required by the Mark 20 bomb sight. Strain gages applied to flexure mounts were used as rate indicators connected to a bridge and amplifier unit. In this development a competitive sight, operating, however, on the same principles, was placed in production and used during the latter part of the war.

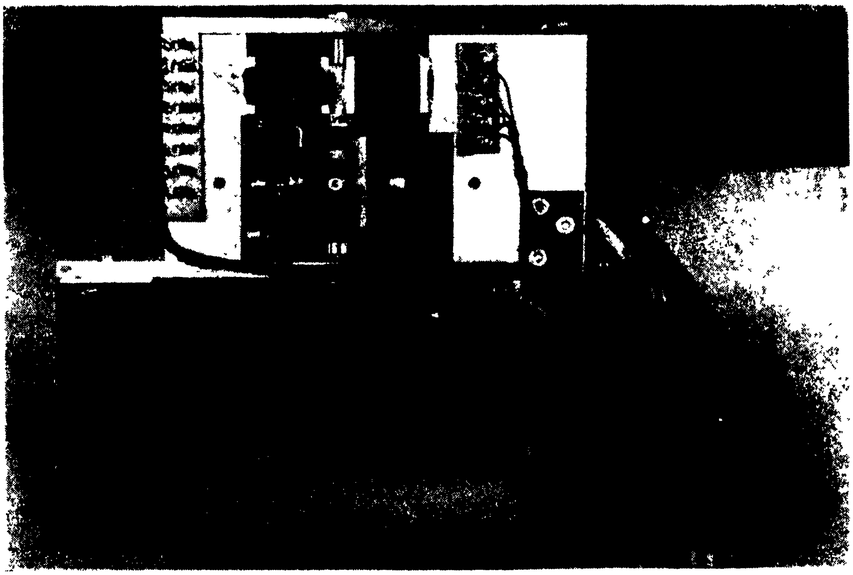


FIG. 13. An experimental rate type of sight developed for use in low altitude bombing.

The hand-held bomb sight in Fig. 14, designed especially for use on blimps, was tested at the Naval Air Station, Lakehurst, N. J., and at the Naval Air Station, Banana River, Florida. This sight proved very accurate for guiding the release of bombs from blimps. Mr. J. A. Bevan took a leading part in this work.

A slant-range computer (Fig. 15) was produced to allow the bomber a ready means for computing slant range from the aircraft to the target. Values of altitude and ground speed were set by turning the knob to the index line and reading slant range directly off the pointer. Means were provided also for rotating the range scale so the pointer would indicate the horizontal range, should this information be desired.

The inverse of a slant-range computer was developed to permit an observer at a potential target to determine at what position an airplane must release a bomb to hit the target, once the speed and altitude of the bomber had been determined. This item, referred to as an inverse bomb sight, was projected for use on surface vessels.

Figure 16 shows an interesting piece of apparatus designed and built to operate in conjunction with a bombing trainer. This item, called a bombing bug, could be remotely controlled to duplicate the motions of maneuvering surface vessels. Several of these units were built to assist the Navy in the training work at the Naval Air Station, Banana River, Fla.

FIG. 14. Special hand-held bomb sight developed for use on blimps.

During the latter part of 1943 the Institute was asked to investigate the possibilities of an instrument which would permit pre-set adjustment of the Mark 15 bomb sight. Mr. A. L. Ruiz of Section 7.2 NDRC and Mr. Bevan took a leading part in this activity. Figure 17 illustrates

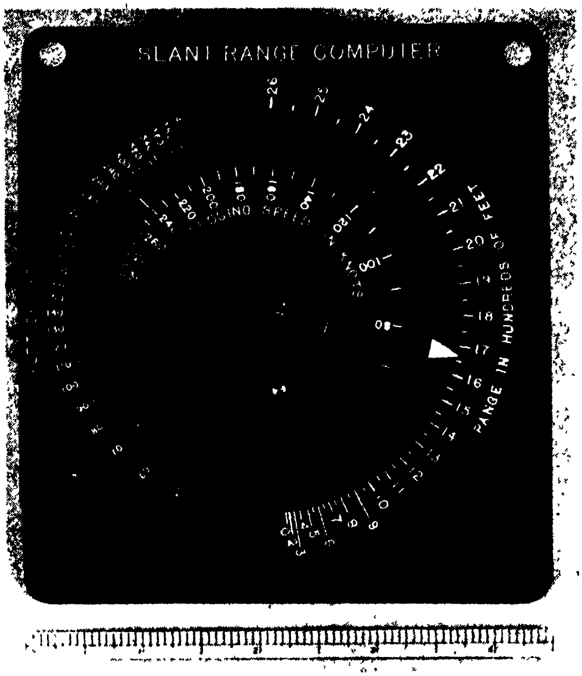


FIG. 15. Slant range computer.

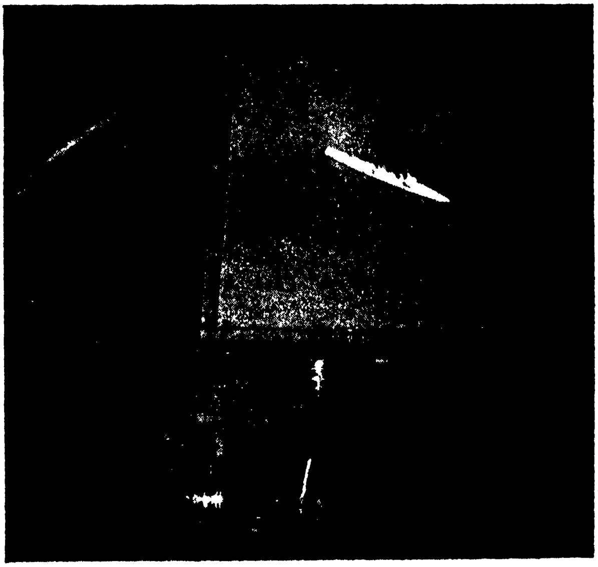


FIG. 16. Bombing bug developed for use with the high altitude bomb sight trainer.

the item delivered to the Naval Air Station, Banana River, Fla., in June 1944. To pre-set the bomb sight the bomber sets in values of air speed, wind speed, target speed, and wind, and stabilized target direction. Closing speed and drift angle are read and normal procedure is used in setting the values in the Mark 15 bomb sight. The unit was built to fit directly upon the stabilizer unit of the bomb sight. Test results showed that the bombing run could be reduced considerably by using the pre-set computer. During the development of this item four models were made. Model 1 consisted of a wood and metal study piece to check the theoretical investigations and to guide the design group; Model 2 was the first attempt to perfect an operating model; Model 3 was the item tested by both the Army Air Forces and the Navy Bureau of Aeronautics, and Model 4, which completed the work on this project, consisted of a circular slide-rule arrangement for determining the pre-set values independent of the low-altitude bombing attachment to the Mark 15.

FIG. 17. An experimental pre-set computer for use with the Mark 15 bomb sight.

In 1944 NDRC assigned the Institute certain projects concerning the development of experimental sights for use with such radio-controlled (guided) missiles as AZON, RAZON, and ROC bombs. The available space facilities in the Institute's building were already overtaxed and the staff, increased to approximately 100 people, was overloaded. Additional experienced research engineers and scientists were not available, and it was necessary to obtain and train individuals for this special type of work. The success of the training program was most encouraging.

After many months of studying and analyzing the problem of sighting and guiding a bomb release from an aircraft to a ground target, a sales engineer-scientist, H. A. Van Dyke, conceived a simple device to perform the job. This device, referred to as CRAB, was used by the Army Air Forces and tested by the Navy Department.

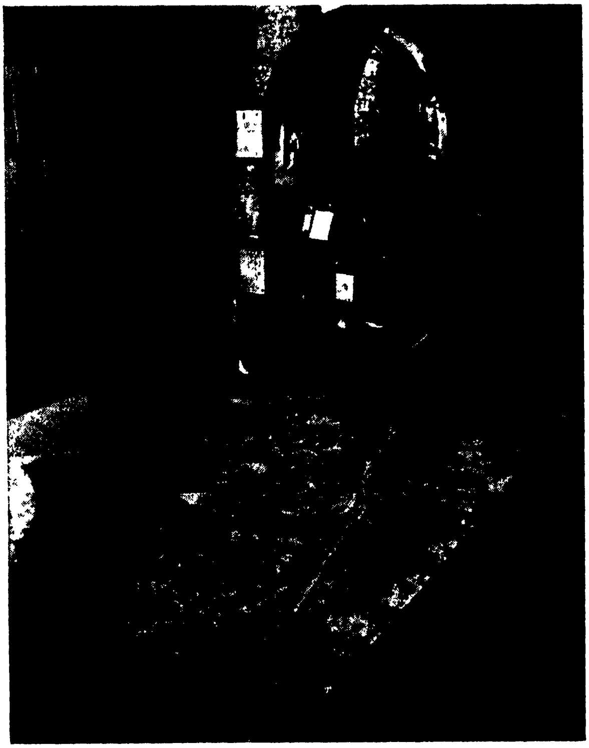


FIG. 18. Bombing trainer A-5 adopted for Azon and Razon bombing training.

In addition to conceiving, designing, and developing the CRAB sight, the Institute, in cooperation with Columbia University, adapted a Navy A-5 bombing trainer with simulated control equipment and a sighting system for training bombers. Figure 18 illustrates "Miss Guided Missile" in operation. For practice a map of the New York, New Jersey, and Philadelphia area was used, and members of the staff developed a pretty knack for guiding missiles to such targets as the

George Washington Bridge, entrances to the Holland Tunnel, the Pulaski Skyway, Cloverleafs in New Jersey, the Delaware River Bridge, and The Franklin Institute.

ROCKET SIGHTS.

In November 1943 another special group was organized to investigate and develop rocket gunnery sights for airborne use. This work led to the first automatic computing sight for directing the firing of 3½ and 5-in. high-velocity aircraft rockets, the RASP III rocket sight. Except for the manual insertion of target or wind speed and temperature of the rocket values, the RASP sight is fully automatic. The sight, with its predecessor, Model I, was tested at the Naval Air Station, Quonset, R. I., with U. C. S. Dilks, W. Sheppard and E. C. Lewis in charge of the equipment. More than 700 rockets were fired during the tests of principle and evaluation.

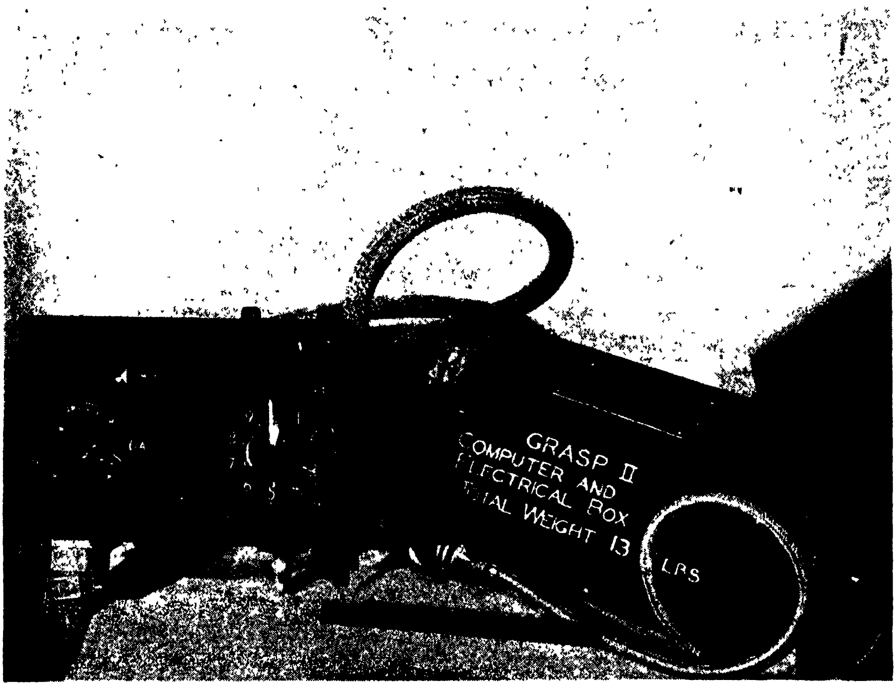


FIG. 19. GRASP Model II, Computer and electric box.

The combination rocket and gun sight shown in Fig. 19 and referred to as GRASP II, was developed by Dr. E. P. Cooper for use with the Mark 23 gun sight to provide a combination sighting system for directing the fire of both guns and forward-firing aircraft rockets. The unit was tested at the Naval Ordnance Testing Station, Inyokern, Calif., during July and August 1945.

PILOT'S UNIVERSAL SIGHTING SYSTEM.

From December 1944 until October 1945 the Institute was commissioned to capitalize upon its work with torpedo directors, bomb sights, gun sight, and rocket sights by investigating and developing a universal sighting system for fighter aircraft. During the war shipborne fighter aircraft, as well as land-based fighters, were performing triple duty; that is, during the invasion of some strategic island group, fighter aircraft would function first as fighters to clear the sky of enemy aircraft, then load with bombs and perform as light bombers, and finally take off with a supply of rockets, dive on enemy gun installations, knock them out, and ease the landing of ground troops. Consequently there was an urgent need for a sighting system which would enable the pilot to direct ammunition in accordance with his particular mission. Work proceeded on the development of the universal sighting system until shortly after V-J day.

The Laboratories of The Franklin Institute, as they exist today, are an outgrowth of the war work. The state of the world at the conclusion of hostilities was such that the Government felt research on all varieties of fire control projects and other military development should be continued in the interests of the national defense. As a consequence, The Franklin Institute established a formal structure for its laboratories in 1946 and placed these laboratories, to an extent, at the disposal of the branches of the armed services. Fire control work continued and numerous other electronic instrument and aircraft problems were undertaken. Moreover it was appreciated that the state of industry in respect to research and development was approximately the same as the state of the Government. Enormous amounts of data had been accumulated in almost all technical fields during the war. For business to operate to the best interests of the public welfare subsequent to the war required an application of the accumulated technical data to improve industrial products and methods. Consequently the Laboratories have acquired a wide variety of facilities for doing research for industrial concerns. A number of contracts have been taken with industrial concerns, and the progress of laboratory work for industry is proceeding quite firmly. We are continuing work for the armed services, in the way that the Institute has always served the country in times of stress. Thus, 124 years after its founding, The Franklin Institute is still broadening its scope of service to the nation and to the public.

Bacteria and Vibrations.—It's doubtful that "looks can kill" but University of Wisconsin scientists know that certain sounds have interesting, often lethal effects—at least on bacteria and protozoa, very small one-celled plants and animals, some of which cause disease.

Sounds or "audible vibrations," to be specific, are being studied by Eugene Ackerman, research assistant to H. B. Wahlin, Professor of Physics, in an attempt to learn whether they can be useful in research on the chemistry and physics of tissue cells and bacteria.

Vibrations, Ackerman says, have one great advantage for biologic extraction purposes; they will break tissue up into its chemical "parts" without the use of heat which often destroys the very substances biochemists want to study.

They may, in addition, be used to break up cells of micro-organisms to release heat-sensitive cellular enzymes, and he hopes that they may even induce changes in heredity in certain forms of life by changing chromosome structure. Yeast and liver extracts can be manufactured without the use of heat.

The vibrations are produced by three devices designed by Ackerman. Two of the vibrators, which differ from one another only in size, are based on the same principle as a telephone receiver, each having a magnetic coil which causes vibration in a thin metal disk. The third vibrator has a plunger which can be lowered into a container of liquid.

When in operation they sound like a radio with a bad tube—but each can kill protozoa in small quantities of liquid within a matter of minutes. Bacteria are affected just as lethally, but it sometimes takes slightly longer.

The smallest vibrator will hold about a teaspoon of bacteria, mold spores, virus, liver, or yeast, in solution, and the larger will hold about a tablespoonful. The third device will induce vibrations in a test-tube full of liquid.

One practical use for the vibrators has been in the field of virus research. Scientists in the college of agriculture believe they may be helpful in learning the nature of two viruses with which they are working.

These viruses cause a disease in chicken embryos, and appear to be a mixture of two different kinds. It is hoped the selective action of the vibrators will destroy one virus to let scientists know which of the two causes disease, or if both play a part. The viruses can be seen only under the electron microscope.

Although vibrations have been used previously in some biologic research, Ackerman will study the effect on tissue and bacteria of vibrations over the whole audible range—from basso profundo to the highest soprano, to use an operatic analogy.

"No one to our knowledge has studied more than one or two frequencies," Ackerman said. "I've been working on the development of magnetic vibrators with an adjustable pitch."

It is possible, scientists say, for particular frequencies to be more efficient in breaking up tissue, just as certain X-rays and ultraviolet light frequencies have advantages others do not.

Work has been done at other laboratories with super-sonic vibrations, which are above the frequencies audible by the human ear, but these have attendant heating capacities which often destroy their usefulness for biologic extraction purposes.

THE EFFECTIVE PATH LENGTH THROUGH A VERTICAL COSMIC-RAY TELESCOPE.*

BY

ENOS E. WITMER [1] AND MARTIN A. POMERANTZ.[2]

I. INTRODUCTION.

One of the most widely used methods for investigating the properties of the cosmic radiation involves measurement of their absorption in matter. By means of coincidence-counter telescopes (see Fig. 1) which record an event only when every member of a train of Geiger-Mueller counters is discharged simultaneously by an ionizing particle, it is possible to select rays which have followed paths lying within specified limits. Thus, a path may be defined such that all of the rays actuating the apparatus must pass through an absorber interposed between successive counter trays. From observations of the counting-rate of the telescope as a function of the thickness and nature of the interposed absorber, the desired information is obtained.

In practice, owing to the finite aperture of the counter train, it is evident that the cosmic-ray particles detected in a particular absorption experiment may traverse total paths varying in length from L, the perpendicular separation between extreme trays, to $(L^2 + l^2 + w^2)^{\frac{1}{2}}$, the diagonal distance between opposite corners. For an accurate analysis of the results, it is thus necessary to determine the average path length through the telescope with respect to all of the radiation concerned, in order to obtain a factor for converting the measured thickness of the interposed material to effective thickness as regards absorption properties. The necessity for applying this correction has only rarely been appreciated (**1**),[3] and, as the following results indicate, the contribution is not always negligible. The error may become especially serious in the comparison of results obtained with different counter trains.

The effective path length must of course be calculated with respect to a particular dependence upon zenith angle θ of the unidirectional cosmic-ray intensity, $I(\theta)$, defined as the number of ionizing particles per square centimeter per unit solid angle per second arriving in a given direction. It has been established for some time that, at sea level, the variation is in good accord with the equation (**2,3,4,5**)

$$I(\theta) = I(0) \cos^2 \theta. \tag{1}$$

---

* The investigations described herewith were supported in part by the Office of Naval Research.

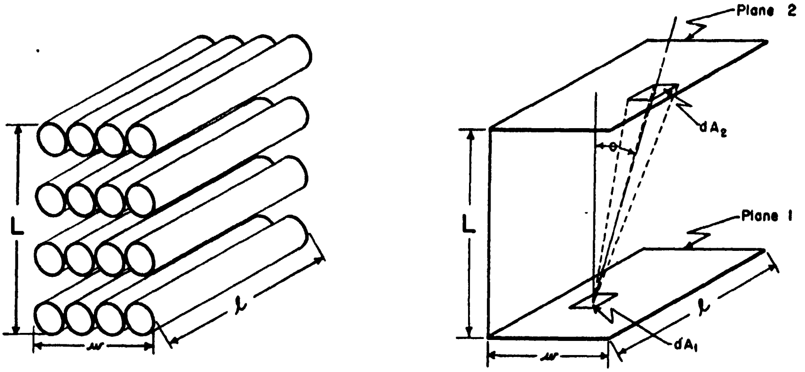
[1] Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Penna.

[2] Bartol Research Foundation of The Franklin Institute, Swarthmore, Penna.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

At an altitude of 4300 m. approximately the same law (1) applies to high energy mesons having ranges exceeding 15 cm. of Pb, whereas for the soft component the exponent may be 3. A departure from this relationship apparently occurs at higher altitudes, as was shown by Swann, Locher and Danforth (6), who obtained a value of 0.21 for the ratio $I\ (90°)/I(0°)$.

In order to compare absorption measurements made in the stratosphere with those obtained at low altitudes, it is evidently important to ascertain, for the particular cosmic-ray telescopes utilized in obtaining the data, the magnitude of the dependence of the effective path length upon the nature of the angular distribution of cosmic-ray intensity. For this purpose, calculations have been based upon several different laws in addition to (1), representing alternative possible modes for the variation of intensity with zenith angle. At extremely high altitudes, such as have been attained by apparatus carried aloft in free balloons during investigations conducted by one of the authors (7), the correct law probably lies within the range considered. Formulas have have been obtained, and families of curves have been computed showing the average path length through a cosmic-ray telescope as a function of all of the dimensions of the apparatus (8). These results may be utilized for obtaining corrections to be applied to the previously reported data of numerous investigators, or for designing systems for experiments at high altitudes in a manner such as to minimize the effects of changes in the directional characteristics of the cosmic radiation.



(a) (b)

FIG. 1.

II. MATHEMATICAL DERIVATIONS.

A. Theory.

The trays in this cosmic-ray telescope are assumed to be rectangular and of length l and width w. The vertical distance between the uppermost and lowest trays is L. As shown in Fig. 1 the x axis is taken to coincide with one of the long edges of the lowest tray, and the z axis is vertical. The lowest tray is designated plane 1 and the cartesian coordinates of a point in it are $(x_1, y_1, 0)$. The uppermost tray is designated plane 2 and the coordinates of a point in it (x_2, y_2, L).

Consider two elements of area, dA_1 in the lowest tray and dA_2 in the uppermost tray. The cosmic rays which pass through both of these elements of area make an angle θ with the vertical. The number of cosmic rays which pass through both dA_1 and dA_2 per unit time is given by dN, where

$$dN = I(\theta) d\omega\, dA_1 \cos\theta, \tag{2}$$

where $d\omega$ is the solid angle which dA_2 subtends at dA_1. Obviously

$$d\omega = \frac{dA_2 \cos\theta}{\lambda^2}, \tag{3}$$

where λ is the distance from dA_2 to dA_1.

$I(\theta)$ is a constant for any given direction, but will depend upon the zenith angle θ, if the cosmic-ray intensity varies with θ. Substituting Eq. 3 in Eq. 2,

$$dN = \frac{I(\theta) dA_1 dA_2 \cos^2\theta}{\lambda^2}. \tag{4}$$

The most common formula given for the dependence of cosmic-ray intensity on zenith angle is represented by Eq. 1.

We will assume

$$I(\theta) = I(0) \cos^p \theta. \tag{1a}$$

Then

$$dN = \frac{I(0) dA_1 dA_2 \cos^{p+2}\theta}{\lambda^2}. \tag{5}$$

But from Fig. 1

$$\cos\theta = \frac{L}{\lambda}. \tag{6}$$

Hence,

$$dN = \frac{I(0) L^{p+2} dA_1 dA_2}{\lambda^{p+4}}. \tag{7}$$

Also

$$\lambda^2 = L^2 + (y_2 - y_1)^2 + (x_2 - x_1)^2 \tag{8}$$

and

$$\left.\begin{aligned} dA_1 &= dx_1 dy_1 \\ dA_2 &= dx_2 dy_2. \end{aligned}\right\} \tag{9}$$

If $\bar{\lambda}$ is the average path length through the cosmic-ray telescope, obviously

$$\bar{\lambda} = \frac{\int \lambda dN}{\int dN}. \tag{10}$$

This gives

$$\bar{\lambda} = \frac{\int_0^w dy_1 \int_0^w dy_2 \int_0^l dx_1 \int_0^l \frac{dx_2}{\lambda^{p+3}}}{\int_0^w dy_1 \int_0^w dy_2 \int_0^l dx_1 \int_0^l \frac{dx_2}{\lambda^{p+4}}}. \tag{11}$$

For any value of p the problem consists in evaluating integrals of the type

$$C_n \equiv \int_0^w dy_1 \int_0^w dy_2 \int_0^l dx_1 \int_0^l \frac{dx_2}{\lambda^n}. \tag{12}$$

We consider the cases when

$p = 2, 1, 0,$ and -1, so that
$n = 2, 3, 4, 5,$ and 6.

B. Two-Dimensional Problem ($w \ll 1$).

In the special case when w is so small that variation in y can be neglected

$$\bar{\lambda} = \frac{A_{p+3}}{A_{p+4}}, \tag{13}$$

where

$$A_n \equiv \int_0^l dx_1 \int_0^l \frac{dx_2}{[L^2 + (x_2 - x_1)^2]^{n/2}}. \tag{14}$$

Even when dealing with the general case where the variation in y cannot be neglected, it is worth while evaluating the A_n first, because by merely substituting $\sqrt{L^2 + (y_2 - y_1)^2}$ for L in these results one has the result of carrying out the two integrations in x_1 and x_2 in Eq. 12. Furthermore, the work of evaluating A_n is so easy for all n considered here that it is not necessary to give the details of the process of integration but merely the results as follows:

$$A_2 = \frac{2l}{L} \tan^{-1} \frac{l}{L} - \log \frac{L^2 + l^2}{L^2}, \tag{15}$$

$$A_3 = \frac{2}{L^2} \sqrt{L^2 + l^2} - \frac{2}{L}, \tag{16}$$

$$A_4 = \frac{l}{L^3} \tan^{-1} \frac{l}{L}, \tag{17}$$

$$A_5 = \frac{4}{3L^4}\sqrt{L^2 + l^2} - \frac{2}{3L^3} - \frac{2}{3L^2\sqrt{L^2 + l^2}}, \tag{18}$$

$$A_6 = \frac{1}{4L^4} - \frac{1}{4L^2(L^2 + l^2)} + \frac{3l}{4L^5}\tan^{-1}\frac{l}{L}. \tag{19}$$

It is desirable for both theoretical and practical reasons to write the formulas for $\bar{\lambda}$ in dimensionless form. For this reason we introduce $B_n(u)$, where

$$B_n(u) \equiv L^{n-2}A_n \tag{20}$$

and

$$u \equiv \frac{l}{L}. \tag{21}$$

It follows that

$$B_2(u) = 2u\tan^{-1}u - \log(1 + u^2), \tag{22}$$

$$B_3(u) = 2\sqrt{1 + u^2} - 2, \tag{23}$$

$$B_4(u) = u\tan^{-1}u, \tag{24}$$

$$B_5(u) = \frac{4}{3}\sqrt{1 + u^2} - \frac{2}{3(1 + u^2)} - \frac{2}{3}, \tag{25}$$

$$B_6(u) = \frac{3}{4}u\tan^{-1}u - \frac{1}{4(1 + u^2)} + \frac{1}{4}. \tag{26}$$

Substituting Eq. 20 in Eq. 13,

$$\frac{\bar{\lambda}}{L} = \frac{B_{p+3}(u)}{B_{p+4}(u)}, \tag{27}$$

where, as indicated in Eq. 1a, p determines the dependence of the intensity on zenith angle.

1. Series Approximations when $u < 1$.

$p = 2$ ($\cos^2\theta$ distribution):

$$\frac{\bar{\lambda}}{L} = 1 + \frac{1}{12}u^2 - \frac{1}{15}u^4 + \frac{377}{6720}u^6 - \frac{29}{600}u^8 + \cdots. \tag{28}$$

$p = 1$ ($\cos\theta$ distribution):

$$\frac{\bar{\lambda}}{L} = 1 + \frac{1}{12}u^2 - \frac{41}{720}u^4 + \frac{263}{6048}u^6 - \cdots. \tag{29}$$

$p = 0$ (isotropic distribution):

$$\frac{\bar{\lambda}}{L} = 1 + \frac{1}{12}u^2 - \frac{17}{360}u^4 + \frac{391}{12\,096}u^6 - \frac{44\,089}{1\,814\,400}u^8 + \cdots. \tag{30}$$

$p = -1$ (*sec. θ distribution*):

$$\frac{\bar{\lambda}}{L} = 1 + \frac{1}{12}u^2 - \frac{3}{80}u^4 + \frac{19}{840}u^6 - \cdots. \tag{31}$$

All these series formulas are approximations valid only when $u < 1$.

It is interesting to note that for all the values of p considered:

$$\frac{\bar{\lambda}}{L} = 1 + \frac{1}{12}u^2 + \cdots \tag{32}$$

to that degree of approximation.

A study of the coefficients in formulas 28 to 31 shows that the numerical values obtained with the different distribution laws differ only slightly except when u is large. This is in accordance with the results of a qualitative approach to this problem.

C. *Three-Dimensional Problem.*

1. *Evaluation of C_n.*

We will now evaluate the integrals C_n defined in Eq. 12.

$n = 2$:

From Eqs. 12 and 15

$$\begin{aligned} C_2 = 2l \int_0^w dy_1 \int_0^w \frac{1}{\sqrt{L^2 + (y_2 - y_1)^2}} \tan^{-1} \frac{l}{\sqrt{L^2 + (y_2 - y_1)^2}} dy_2 \\ - \int_0^w dy_1 \int_0^w \log [L^2 + l^2 + (y_2 - y_1)^2] dy_2 \\ + \int_0^w dy_1 \int_0^w \log [L^2 + (y_2 - y_1)^2] dy_2. \end{aligned} \tag{33}$$

The only one of the three integrals on the right that offers any difficulty is the first. It is evaluated by making the substitution:

$$\tan^{-1} \frac{l}{\sqrt{L^2 + (y_2 - y_1)^2}} = \sin^{-1} \frac{l}{\sqrt{L^2 + l^2 + (y_2 - y_1)^2}}. \tag{34}$$

Then

$$\begin{aligned} &\int_0^w dy_1 \int_0^w \frac{1}{\sqrt{L^2 + (y_2 - y_1)^2}} \tan^{-1} \frac{l}{\sqrt{L^2 + (y_2 - y_1)^2}} dy_2 \\ &= \int_0^w dy_1 \int_0^w \frac{1}{\sqrt{L^2 + l^2 + (y_2 - y_1)^2}} \\ &\times \left[1 - \frac{l^2}{L^2 + l^2 + (y_2 - y_1)^2} \right]^{-\frac{1}{2}} \sin^{-1} \frac{l}{\sqrt{L^2 + l^2 + (y_2 - y_1)^2}} dy_2 \\ &= \sum_{r=0}^{\infty} \left\{ \sum_{j=0}^{r} \frac{(2r - 2j)!}{[(r - j)!]^2} \cdot \frac{(2j)!}{(j!)^2} \cdot \frac{1}{(2j + 1)} \right\} \frac{l^{2r+1}}{2^{2r}} G_r', \end{aligned} \tag{35}$$

where

$$G_r' \equiv \int_0^w dy_1 \int_0^w \frac{dy_2}{[L^2 + l^2 + (y_2 - y_1)^2]^{r+1}}. \tag{36}$$

This result is obtained by expanding the $\sin^{-1}$ and the square bracket in the integrand. The integration (9) of Eq. 36 yields:

$$G_0' = \frac{2w}{\sqrt{L^2 + l^2}} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}} - \log \frac{L^2 + l^2 + w^2}{L^2 + l^2}, \tag{37}$$

$$G_1' = \frac{w}{(L^2 + l^2)^{\frac{1}{2}}} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}} \tag{37a}$$

and for $r \geqslant 2$

$$\begin{aligned} G_r' \equiv {} & \frac{1}{(L^2 + l^2)^r} \sum_{\sigma=1}^{r-1} \frac{(2r)!}{(2r - 2\sigma + 1)!} \frac{[(r - \sigma)!]^2}{2^{2\sigma}(r!)^2} \cdot \frac{1}{(r - \sigma)} \\ & - \sum_{\sigma=1}^{r-1} \frac{(2r)!}{(2r - 2\sigma + 1)!} \frac{[(r - \sigma)!]^2}{2^{2\sigma}(r!)^2} \cdot \frac{1}{(r - \sigma)} \frac{1}{(L^2 + l^2)^\sigma} \\ & \times \frac{1}{(L^2 + l^2 + w^2)^{r-\sigma}} + \frac{2(2r)!}{2^{2r}(r!)^2} \cdot \frac{w}{(L^2 + l^2)^{r+\frac{1}{2}}} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}}. \end{aligned} \tag{37b}$$

Then

$$\begin{aligned} C_2 = {} & (L^2 + l^2 - w^2) \log (L^2 + l^2 + w^2) - (L^2 - w^2) \log (L^2 + w^2) \\ & - (L^2 + l^2) \log (L^2 + l^2) + L^2 \log L^2 + 4wL \tan^{-1} \frac{w}{L} \\ & - 4w\sqrt{L^2 + l^2} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}} \\ & + \sum_{r=0}^{\infty} \left\{ \sum_{j=0}^{r} \frac{(2r - 2j)!}{[(r - j)!]^2} \cdot \frac{(2j)!}{(j!)^2} \cdot \frac{1}{(2y + 1)} \right\} \frac{l^{2r+2}}{2^{2r-1}} G_r'. \end{aligned} \tag{38}$$

n = 3:

From Eqs. 12 and 16:

$$\begin{aligned} C_3 = {} & 2 \int_0^w dy_1 \int_0^w \frac{[L^2 + l^2 + (y_2 - y_1)^2]^{\frac{1}{2}}}{[L^2 + (y_2 - y_1)^2]} dy_2 \\ & - 2 \int_0^w dy_1 \int_0^w \frac{1}{[L^2 + (y_2 - y_1)^2]} dy_2. \end{aligned} \tag{39}$$

The first integral is evaluated by expanding the numerator of the integrand by the binomial theorem taking $(L^2 + l^2)$ as the larger term.

This gives

$$2\int_0^w dy_1\int_0^w \frac{[L^2+l^2+(y_2-y_1)^2]^{\frac{1}{2}}}{[L^2+(y_2-y_1)^2]}dy_2$$

$$= 2\sqrt{L^2+l^2}\int_0^w dy_1\int_0^w \frac{dy_2}{[L^2+(y_2-y_1)^2]}$$

$$+ 4\sqrt{L^2+l^2}\sum_{r=1}^{\infty}\frac{(-1)^{r+1}}{2^{2r-1}}\cdot\frac{(2r-2)!}{(r-1)!r!}\cdot\frac{H_r'}{(L^2+l^2)^r}. \qquad (40)$$

Here

$$2H_r' \equiv \int_0^w dy_1\int_0^w \frac{(y_2-y_1)^{2r}dy_2}{[L^2+(y_2-y_1)^2]}. \qquad (41)$$

This integral can easily be evaluated by dividing the denominator into the numerator; there is a remainder term. This gives

$$H_r' = \sum_{\sigma=0}^{r-1}\frac{(-1)^{\sigma}L^{2\sigma}w^{2r-2\sigma}}{(2r-2\sigma)(2r-2\sigma-1)}$$

$$+ (-1)^r L^{2r-1}w\tan^{-1}\frac{w}{L} - (-1)^r\tfrac{1}{2}L^{2r}\log\frac{L^2+w^2}{L^2}. \qquad (42)$$

The final result for C_3 is

$$C_3 = 4\sqrt{L^2+w^2} - 4L - 2w\log\frac{\sqrt{L^2+w^2}+w}{\sqrt{L^2+w^2}-w}$$

$$- 2\sqrt{L^2+l^2}\log\frac{L^2+w^2}{L^2} + \frac{4w\sqrt{L^2+l^2}}{L}\tan^{-1}\frac{w}{L}$$

$$+ 4\sqrt{L^2+l^2}\sum_{r=1}^{\infty}\frac{(-1)^{r+1}}{2^{2r-1}}\cdot\frac{(2r-2)!}{(r-1)!r!}\cdot\frac{H_r'}{(L^2+l^2)^r}. \qquad (43)$$

$n = 4$:

Combining Eq. 12 and 17

$$C_4 \equiv l\int_0^w dy_1\int_0^w \frac{1}{[L^2+(y_2-y_1)^2]^{\frac{1}{2}}}\tan^{-1}\frac{l}{[L^2+(y_2-y_1)^2]}dy_2. \qquad (44)$$

In all the integrations for C_n the independent variable for the first integration should be $(y_2 - y_1)$, with y_1 constant, of course. In this case it is advantageous to use partial integration first in such a way as to get rid of the $\tan^{-1}$ factor by differentiation in the double integral. The first integration in the resulting double integral can be done by separating into partial fractions. *All* the integrands for the integration with respect to y_1 involve a $\tan^{-1}$ term. Once more partial integration can be used in each integral to eliminate the $\tan^{-1}$ term in the resulting integral.

In this way one obtains

$$C_4 = \frac{2l\sqrt{L^2 + w^2}}{L^2} \tan^{-1} \frac{l}{\sqrt{L^2 + w^2}} + \frac{2w\sqrt{L^2 + l^2}}{L^2} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}} - \frac{2l}{L} \tan^{-1} \frac{l}{L} - \frac{2w}{L} \tan^{-1} \frac{w}{L} - \log \frac{L^2(L^2 + l^2 + w^2)}{(L^2 + l^2)(L^2 + w^2)}. \quad (45)$$

n = 5:

Using Eqs. 12 and 18

$$C_5 = \frac{4}{3} \int_0^w dy_1 \int_0^w \frac{[L^2 + l^2 + (y_2 - y_1)^2]^{\frac{1}{2}}}{[L^2 + (y_2 - y_1)^2]} dy_2 - \frac{2}{3} \int_0^w dy_1 \int_0^w \frac{dy_2}{[L^2 + (y_2 - y_1)^2][L^2 + l^2 + (y_2 - y_1)^2]^{\frac{1}{2}}} - \frac{2}{3} \int_0^w dy_1 \int_0^w \frac{dy_2}{[L^2 + (y_2 - y_1)^2]^{\frac{3}{2}}}. \quad (46)$$

In the case of the first two integrals the factors in the integrands with the exponent ½ are expanded by means of the binomial theorem in such a way that the positive powers of $(y_2 - y_1)$ appear in all terms except the first. It is exactly the same procedure as that used in evaluating C_3. Therefore the details will not be given. The result is

$$C_5 = \frac{4}{3L} - \frac{4\sqrt{L^2 + w^2}}{3L^2} + \frac{4w\sqrt{L^2 + l^2}}{3L^3} \tan^{-1} \frac{w}{L} + \frac{2}{3} \sum_{r=1}^{\infty} \frac{(-1)^r}{2^{2r-2}} \cdot \frac{(2r - 2)!}{(r - 1)!r!} (L^2 + l^2)^{-r+\frac{1}{2}} J_r'. \quad (47)$$

Here

$$J_r' = \sum_{\sigma=1}^{r-1} (-1)^{\sigma-1} \frac{w^{2r-2\sigma} L^{2\sigma-2}}{(2r - 2\sigma - 1)} + (-1)^{r-1} L^{2r-3} w \tan^{-1} \frac{w}{L}. \quad (48)$$

n = 6:

From Eqs. 12 and 19

$$C_6 = \frac{1}{4} \int_0^w dy_1 \int_0^w \frac{dy_2}{[L^2 + (y_2 - y_1)^2]^2} - \frac{1}{4} \int_0^w dy_1 \int_0^w \frac{dy_2}{[L^2 + l^2 + (y_2 - y_1)^2][L^2 + (y_2 - y_1)^2]} + \frac{3l}{4} \int_0^w dy_1 \int_0^w \frac{1}{[L^2 + (y_2 - y_1)^2]^{5/2}} \tan^{-1} \frac{l}{\sqrt{L^2 + (y_2 - y_1)^2}} dy_2. \quad (49)$$

The second and third integrals are evaluated by the same methods as in the case of C_4, namely by partial integration on integrands involving $\tan^{-1}$ so as to get rid of $\tan^{-1}$ in the resulting *integral* term, and separation in partial fractions whenever integrands such as those in the second integral appear.

The result is

$$C_6 = \frac{(2l^2 + L^2)w}{2L^4\sqrt{L^2 + l^2}} \tan^{-1} \frac{w}{\sqrt{L^2 + l^2}} + \frac{(2w^2 + L^2)l}{2L^4\sqrt{L^2 + w^2}} \tan^{-1} \frac{l}{\sqrt{L^2 + w^2}} - \frac{w}{2L^3} \tan^{-1} \frac{w}{L} - \frac{l}{2L^3} \tan^{-1} \frac{l}{L}. \tag{50}$$

The infinite series in C_2 and C_3 are convergent for all values of the parameters L, l, and w; and the infinite series in C_5 is convergent provided $w \leqslant l$, a condition which can be satisfied in all cases.

2. Reduction to Dimensionless Form.

Equation 11 may be rewritten as

$$\bar{\lambda} = \frac{C_{p+3}}{C_{p+4}}. \tag{51}$$

For both theoretical and practical reasons it is desirable to write this equation in dimensionless form.

Let

$$D_n(u, v) = L^{n-4}C_n, \tag{52}$$

$$u \equiv \frac{l}{L}, \tag{21}$$

$$v \equiv \frac{w}{L}. \tag{53}$$

Then D_n is dimensionless.

From Eqs. 51 and 52

$$\frac{\bar{\lambda}}{L} = \frac{D_{p+3}(u, v)}{D_{p+4}(u, v)}. \tag{54}$$

This dimensionless equation gives $\bar{\lambda}/L$ for the cosmic-ray distribution determined by the value of p.

Below, the formulas for $D_n(u, v)$ are given:

$n = 2$:

$$D_2(u, v) = \log \frac{1 + u^2 + v^2}{(1 + u^2)(1 + v^2)} + u^2 \log \frac{1 + u^2 + v^2}{1 + u^2} + v^2 \log \frac{1 + v^2}{1 + u^2 + v^2} + 4v \tan^{-1} v - 4v\sqrt{1 + u^2} \tan^{-1} \frac{v}{\sqrt{1 + u^2}} + \sum_{r=0}^{\infty} g_r G_r u^{2r+2}, \tag{55}$$

where

$$g_r = \frac{1}{2^{2r-1}} \sum_{j=0}^{r} \frac{(2r-2j)!}{[(r-j)!]^2} \frac{(2j)!}{[j!]^2} \cdot \frac{1}{(2j+1)} . \tag{56}$$

and

$$G_0 = \frac{2v}{\sqrt{1+u^2}} \tan^{-1} \frac{v}{\sqrt{1+u^2}} - \log \frac{1+u^2+v^2}{1+u^2} , \tag{57}$$

$$G_1 = \frac{v}{(1+u^2)^{3/2}} \tan^{-1} \frac{v}{\sqrt{1+u^2}} , \tag{57a}$$

and for $r \geqslant 2$

$$\begin{aligned} G_r = {} & \frac{1}{(1+u^2)^r} \sum_{\sigma=1}^{r-1} \frac{(2r)![(r-\sigma)!]^2}{2^{2\sigma}(r!)^2(2r-2\sigma+1)!} \cdot \frac{1}{(r-\sigma)} \\ & - \sum_{\sigma=1}^{r-1} \frac{(2r)![(r-\sigma)!]^2}{2^{2\sigma}(r!)^2(2r-2\sigma+1)!} \cdot \frac{1}{(r-\sigma)} \frac{1}{(1+u^2)^{\sigma}(1+u^2+v^2)^{r-\sigma}} \\ & + \frac{(2r)!}{2^{2r}(r!)^2} \cdot \frac{v}{(1+u^2)^{r+\frac{1}{2}}} \tan^{-1} \frac{v}{\sqrt{1+u^2}} . \end{aligned} \tag{57b}$$

n = 3:

$$\begin{aligned} D_3(u, v) = {} & 4\sqrt{1+v^2} - 4 - 2v \log \frac{\sqrt{1+v^2}+v}{\sqrt{1+v^2}-v} \\ & - 2\sqrt{1+u^2} \log(1+v^2) + 4v\sqrt{1+u^2} \tan^{-1} v \\ & - 8\sqrt{1+u^2} \sum_{r=1}^{\infty} \frac{(-1)^r(2r-2)!}{2^{2r}(r-1)!r!} (1+u^2)^{-r} H_r , \end{aligned} \tag{58}$$

where

$$\begin{aligned} H_r = {} & \sum_{\sigma=0}^{r-1} \frac{(-1)^{\sigma} v^{2r-2\sigma}}{(2r-2\sigma)(2r-2\sigma-1)} \\ & + (-1)^r v \tan^{-1} v - (-1)^r \tfrac{1}{2} \log(1+v^2) . \end{aligned} \tag{59}$$

n = 4:

$$\begin{aligned} D_4(u, v) = {} & 2u\sqrt{1+v^2} \tan^{-1} \frac{u}{\sqrt{1+v^2}} \\ & + 2v\sqrt{1+u^2} \tan^{-1} \frac{v}{\sqrt{1+u^2}} - 2u \tan^{-1} u \\ & - 2v \tan^{-1} v - \log \frac{1+u^2+v^2}{(1+u^2)(1+v^2)} . \end{aligned} \tag{60}$$

n = 5:

$$D_5(u, v) = \frac{4}{3} - \frac{4}{3}\sqrt{1 + v^2} + \frac{4}{3}v\sqrt{1 + u^2}\tan^{-1} v$$
$$+ \frac{8}{3}\sum_{r=1}^{\infty}\frac{(-1)^r(2r-2)!}{2^{2r}(r-1)!r!}(1 + u^2)^{-r+\frac{1}{2}}J_r, \quad (61)$$

where

$$J_r = \sum_{\sigma=1}^{r-1}(-1)^{\sigma-1}\frac{v^{2r-2\sigma}}{2r - 2\sigma - 1} + (-1)^{r-1}v\tan^{-1} v. \quad (62)$$

n = 6:

$$D_6(u, v) = \frac{(2u^2 + 1)v}{2\sqrt{1 + u^2}}\tan^{-1}\frac{v}{\sqrt{1 + u^2}}$$
$$+ \frac{(2v^2 + 1)u}{2\sqrt{1 + v^2}}\tan^{-1}\frac{u}{\sqrt{1 + v^2}}$$
$$- \tfrac{1}{2}v\tan^{-1} v - \tfrac{1}{2}u\tan^{-1} u. \quad (63)$$

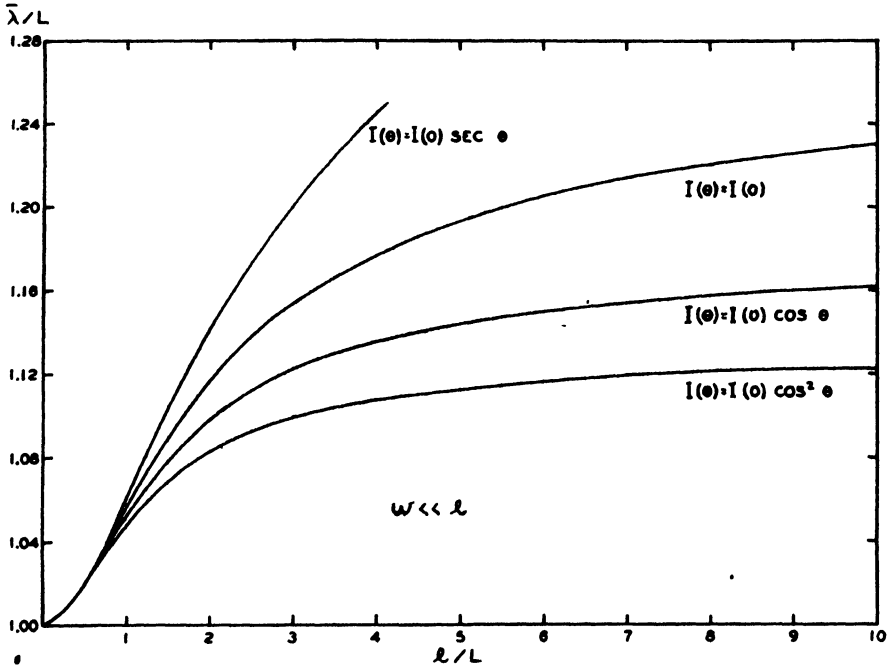


FIG. 2.

III. RESULTS.

The results obtained from numerical computations utilizing the formulas developed in Section II are best presented in a series of graphs.

Figure 2 shows $\bar{\lambda}/L$ (the ratio of the average path length to the normal path length) plotted as a function of l/L (the ratio of counter

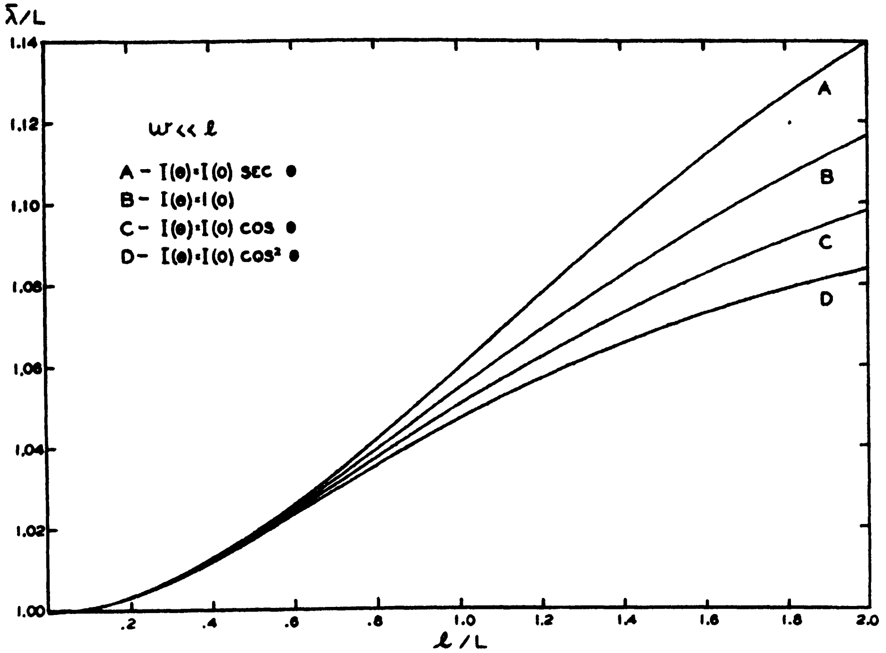

λ̄/L
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
w << ℓ
A - I(θ)=I(0) sec θ
B - I(θ)=I(0)
C - I(θ)=I(0) cos θ
D - I(θ)=I(0) cos² θ
A
B
C
D
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
ℓ/L

FIG. 3.

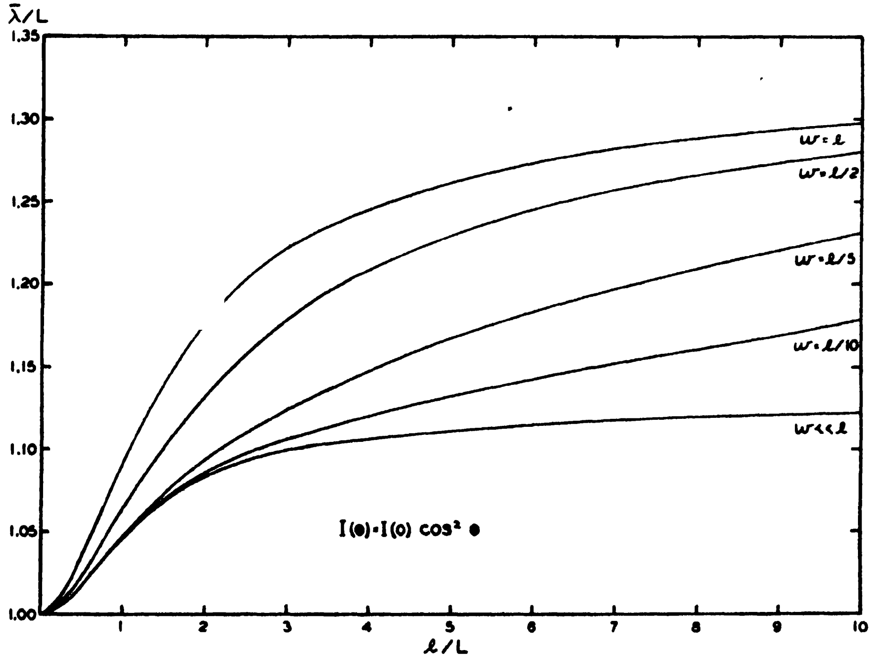

λ̄/L
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
w=ℓ
w=ℓ/2
w=ℓ/5
w=ℓ/10
w<<ℓ
I(θ)=I(0) cos² θ
1
2
3
4
5
6
7
8
9
10
ℓ/L

FIG. 4.

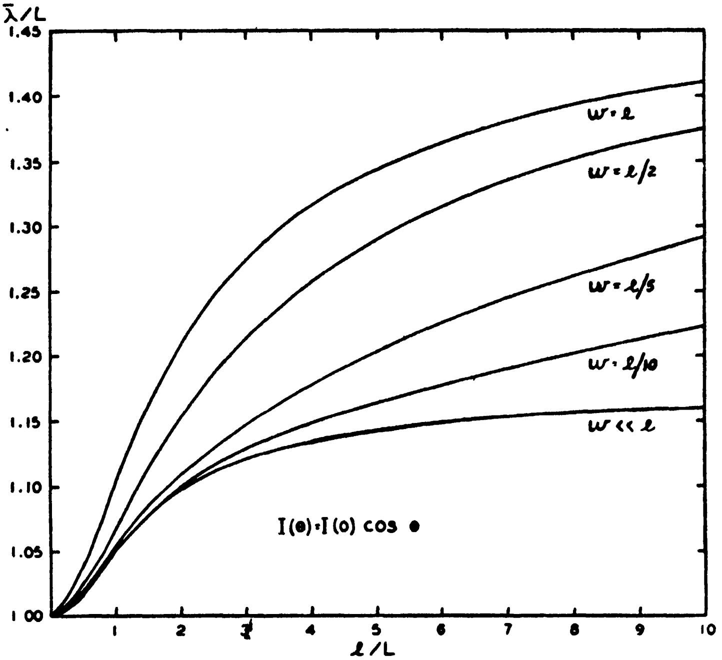

λ̄/L
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
1
2
3
4
5
6
7
8
9
10
ℓ/L
w=ℓ
w=ℓ/2
w=ℓ/5
w=ℓ/10
w<<ℓ
I(θ)=I(0) cos θ

FIG. 5.

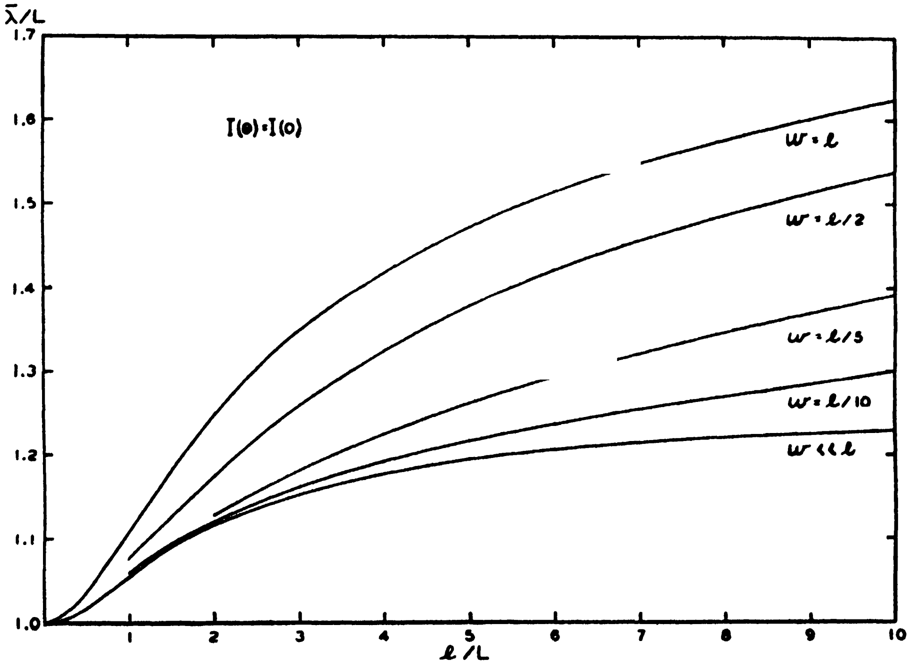

λ̄/L
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
1
2
3
4
5
6
7
8
9
10
ℓ/L
I(θ)=I(0)
w=ℓ
w=ℓ/2
w=ℓ/5
w=ℓ/10
w<<ℓ

FIG. 6.

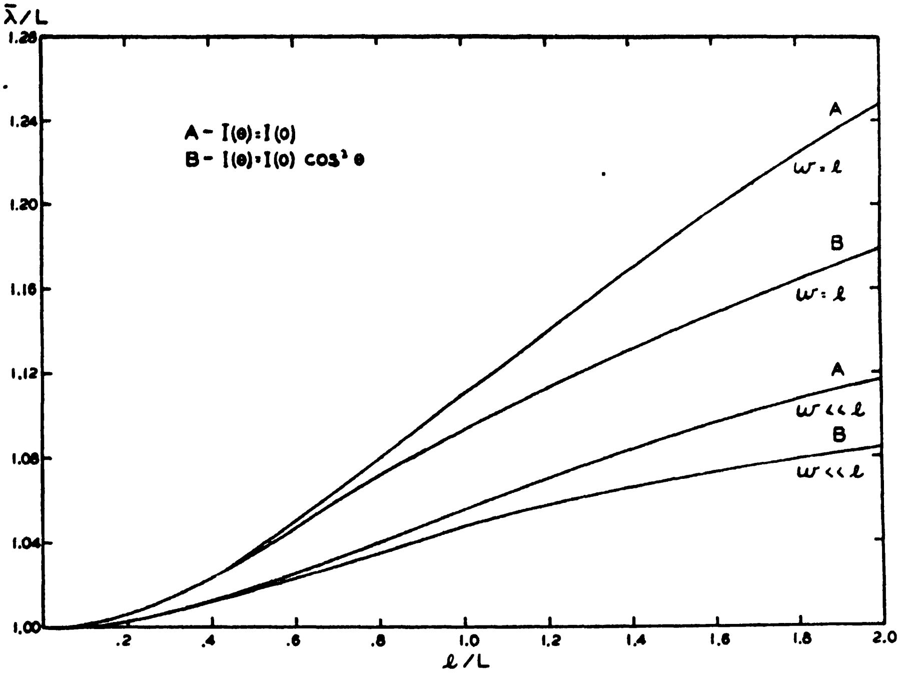

$\bar{\lambda}/L$
1.28
1.24
1.20
1.16
1.12
1.08
1.04
1.00
A- I(θ)=I(0)
B- I(θ)=I(0) cos² θ
A
w = l
B
w = l
A
w << l
B
w << l
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
l/L

FIG. 7.

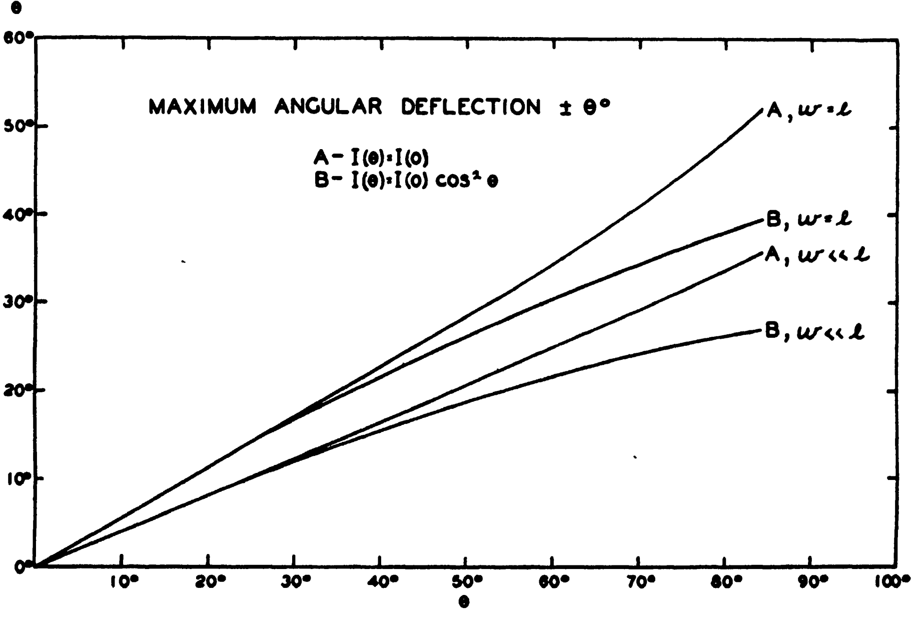

$\bar{\theta}$
60°
50°
40°
30°
20°
10°
0°
MAXIMUM ANGULAR DEFLECTION ± θ°
A- I(θ)=I(0)
B- I(θ)=I(0) cos² θ
A, w = l
B, w = l
A, w << l
B, w << l
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
θ

FIG. 8.

length to the vertical separation between extreme counters) for various indicated laws of cosmic-ray intensity *versus* zenith angle. These curves, as well as those plotted in Fig. 3 on an expanded scale encompassing a range of dimensions more commonly encountered in the usual experimental arrangements, apply when the counter diameter is negligible compared with its length.

Figure 4 contains a family of curves relating the same quantities for a $\cos^2 \theta$ distribution of intensity. Here, the width of the tray is taken into account. The magnitude of w expressed in terms of counter length appears as the parameter indicated in Figs. 5 and 6 which represent $\cos \theta$ and isotropic distributions in a similar manner. The extreme cases of $w = l$ and $w \ll l$ for both the isotropic and $\cos^2 \theta$ distributions are shown on an expanded scale in Fig. 7.

It is of interest to plot the results in terms of angles, as has been done in Fig. 8. The so-called angular resolution of a vertical cosmic-ray telescope is defined as the maximum allowed zenith angle for a ray which can produce a coincidence. This angle $\theta_R = \tan^{-1} l/L$. The angle between the vertical and the average path through the telescope is $\bar{\theta} = \sec^{-1} \bar{\lambda}/L$. This greatly simplifies the shape of the curves, as may be seen in Fig. 8. For small angles, the curves are practically straight lines. Thus, the results are conveniently summarized by the following simple expression

$$\bar{\theta} = k_1 \theta_R. \tag{64}$$

Values of k_1 and the condition under which they pertain, are tabulated in Table I.

TABLE I.

Zenith Angle Distribution Law	k_1	Range of θ_R	w
$I(\theta) = I(0) \cos^2 \theta$	0.40	0°–25°	$\ll l$
	0.36	25°–40°	
	0.33	40°–55°	
	0.55	0°–30°	$= l$
	0.47	30°–60°	
	0.37	60°–80°	
$I(\theta) = I(0)$	0.41	0°–45°	$\ll l$
	0.43	45°–80°	
	0.55	0°–30°	$= l$
	0.60	30°–60°	
	0.68	60°–80°	

IV. CONCLUSIONS.

These derivations and calculations have been performed only for arrangements in which the dimensions of the uppermost tray are the same as those for the lowest tray. Other cases concerning trays of different sizes could of course be treated in a manner similar to that used above. If the dimensions of extreme trays are not very different, the use of arithmetical means of the corresponding dimensions should yield results which are sufficiently accurate for most practical purposes. The most striking aspects of the results presented quantitatively in the various figures may be summarized qualitatively as follows: (1) the average path length through a vertical coincidence counter train is not greatly affected by a change in the zenith angle distribution law, provided the separation between extreme trays is roughly comparable with the dimensions of the trays; (2) even for geometrical arrangements which intuitively appear to be unsatisfactory as regards angular definition and the possible complicating effects arising from the distribution of allowed path lengths, the departure of the average path length from the vertical path length may be rather small.

V. ACKNOWLEDGMENTS.

The numerical computations have been performed by Stephen H. Forbes, with the assistance of Henrietta Fricke. It is a genuine pleasure to acknowledge their active participation in this investigation and the invaluable assistance which they have rendered thereby.

REFERENCES.

(1) K. GREISEN, *Phys. Rev.*, Vol. 61, p. 212 (1942).
(2) T. H. JOHNSON, *Phys. Rev.*, Vol. 43, p. 307 (1933).
(3) G. BERNARDINI, *Nature*, Vol. 129, p. 578 (1932).
(4) G. MEDICUS, *Zeits. f. Phys.*, Vol. 74, p. 350 (1932).
(5) L. TUWIM, *Berliner Berichte*, Vol. 19, pp. 91, 360 (1931).
(6) W. F. G. SWANN AND G. L. LOCHER, *Nat. Geog. Soc. Cont. Tech. Papers Stratosphere Series No. 1*, p. 714 (1935). W. F. G. SWANN, G. L. LOCHER and W. E. DANFORTH, *ibid.*, No. 2, p. 13 (1936). W. F. G. SWANN, JOURNAL FRANKLIN INST., Vol. 222, p. 45 (1936).
(7) MARTIN A. POMERANTZ, *Phys. Rev.*, Vol. 73, p. 651 (1948). *Ibid.*, p. 1253 (1948).
(8) ENOS E. WITMER AND MARTIN A. POMERANTZ, *Phys. Rev.*, Vol. 73, p. 651 (1948). *Ibid.*, p. 1274 (1948). A preliminary report of the results herein described in detail has been given in abstract form.
(9) H. B. DWIGHT, "Tables of Integrals and Other Mathematical Data," The Macmillan Co., New York, 1934. See Eq. 120.9.

Eye-Dropper Technique Proves Effective Method of Testing Steel.—Science has removed the eye-dropper from the family medicine cabinet and put it to work sampling molten steel at 2700° F.

The "eye-dropper," as now used industrially in the G. E. Schenectady Works Laboratory, is a heat-resistant glass tube, about the diameter of a lead pencil and 18 in. long, with a rubber bulb at one end. Liquid steel fresh from the furnace is drawn up into the tube by squeezing the rubber bulb, just as an eye-dropper draws up medicine. The steel hardens into a smooth, homogeneous rod within 5 min., the glass is cracked from it, and the rod is ready to be checked for quality.

Eye-dropper samples may be prepared for analysis in a few minutes. Older techniques required long hours and involved such laborious operations as cutting a block from a hardened steel mass, machining it into a rod, and buffing it to a high polish.

Three men, wearing gloves and goggles, form a highly co-ordinated testing team. In a sequence that must take no more than 60 seconds, the first man removes a puddle from the heart of the furnace with a ladle on the end of a 10-ft. pole. A second worker skims away the slag forming on the surface as the metal approaches the solidifying point. The third man then steps forward with the "eye-dropper" to suck up a glowing sample.

Cut in two and fitted into sockets before a spectrograph, the sample forms two electrodes of a high-voltage arc. An electric spark jumps between the two, giving off a light which varies with the makeup of the electrodes. By breaking up this light into its component colors with a prism, trained observers are able to judge the quality of any given batch of steel sampled by the "eye-dropper."

R. H. O.

Results of Conservation.—Conservation endeavors in the United States have already meant much to the Nation and to the world. When World War II boiled up in Europe and the plea for American food was raised again, the farmers of the United States did a far better job of production than in World War I—and they did it with far less devastation of the soil. Just one comparison illustrates the difference. During and immediately after World War I, more than 30 million acres of grass and pasture were plowed up. In World War II, less than 3 million acres unsuited for cultivation felt the plow.

What's New in Plastics (*Plastics*, Vol. 8, No. 7).—A radio active isotope, byproduct of the nation's atomic energy plants, is the heart of a new gage for measuring the thickness of Pliofilm and other thin films on continuous production equipment. Developed by W. E. Morris, of the research laboratory, the Goodyear Tire and Rubber Co., the device is capable of measuring material as thin as one-thousandth of an inch.

A combination edging and folding machine is now available through the Taber Instrument Corporation, North Tonawanda, New York. It will simultaneously bead or fold one or both parallel edges of sheet plastic in like or different beads or folds. The machine can bead or fold 500 to 1000 in. of sheet plastic per minute ranging in gage from 0.005 to 0.020 in. in thickness and from 2 to 20 in. in width.

ELECTRON LENSES OF HYPERBOLIC FIELD STRUCTURE.*

BY

REINHOLD RÜDENBERG.[1]

Part I.

SUMMARY.

From the conditions of focus formation and Laplace's equation, a hyperbolic field structure is derived which permits a rigorous mathematical description and an explicit determination of electron trajectories of any amplitude. Various possible electron paths are formulated and charts are given for the most outstanding solutions, namely for symmetrical electron lenses and mirrors.

The focal properties of such field formations are derived and the results condensed to closed formulations and plots easy to survey. Uniform rays of parallel incidence do not suffer any aberration within the volume of this field. The performance of the focal length, the principal planes and the back focus are discussed in detail, the latter being of great significance with respect to perfect image formation at high magnification. For thin lenses an approximation leads to a concise formulation which is in accord with published experiments.

1. INTRODUCTION.

Many of the field lenses applied in electronic or ionic instruments make use of the focusing properties of cylindrical gaps or circular apertures or their equivalents arranged between magnetic poles or in electrostatic electrodes. The mathematical description of the electric or magnetic fields of such devices is possible, but cumbersome. Moreover, the solutions do not allow for an explicit determination of the actual electron trajectories. The electronic properties of the lenses, therefore, either must be derived by graphical or experimental methods, both of moderate accuracy and applicable to lenses of given parameters only, or they must be deduced from a consideration of the paraxial rays, necessitating methods of successive approximation. Other restrictions frequently applied for making a mathematical treatment possible are that the lenses be thin or weak, both excluding the most interesting case of small focal length.

In this paper we shall investigate the electronic properties of a field structure which will be developed directly from the postulate of electron focusing. Its field lines and equipotential surfaces are easy to survey and can be produced by appropriate mechanical and electrical means. The field is elementary in structure, and thus leads easily to an analytical determination of the electron trajectories. Therefrom the electron-optical performance can be derived, resulting in rigorous expressions for all the principal properties of the lens field. No aberrations of uniform electron rays occur within the formative volume of this field; such aperture defects are confined rather to the boundaries of the lens.

---

* Paper read at the Philadelphia meeting of the Electron Microscope Society of America on December 11, 1947. The investigation was originally made for the Farrand Optical Co., Inc., New York.

[1] Gordon McKay Professor of Electrical Engineering, Graduate School of Engineering, Harvard University, Cambridge, Mass.

This analysis gives a direct insight into the details of ray-focusing and image-formation. It allows for correlation of the final results to the individual electron paths traversing the various parts of the field. Thus it permits us to visualize the performance of the lens better than do integral methods frequently used for such purposes.

By emphasizing the dependence of the field upon the radius as of the same importance as its dependence along the axis, we consider every element of the active space as equally significant. This method allows the development of a closed formulation for the cardinal points and for the lens aberrations, thus leading the way to a scientific design of electron lenses in contrast to the trial and error method often used heretofore.

Although such fields can be produced either by use of electrostatic electrodes or of magnetic poles, only the electrostatic case shall be considered here in detail. We limit ourselves in this paper further to the consideration of symmetrical converging fields. Diverging and accelerating fields may be treated in a following paper. No restrictions will be made as to paraxial rays or to thin or weak lenses. However, we shall exclude the consideration of space charge and of relativity effects, both being well-known by themselves for many types of electron beams.

2. FIELD STRUTURE.

Let us start with a short survey of the mathematical requirements for the electron lens fields. The analytical relations between field strength, potential and stream-function in a field of circular symmetry were developed by G. G. Stokes (1)[2] in 1842, and were first applied to hydrodynamic problems. In cylindrical coordinates r, z, the radial and axial components of the field strength $\mathcal{E}$ can be derived from a potential-function Φ by

$$\mathcal{E}_r = -\frac{\partial \Phi}{\partial r}; \qquad \mathcal{E}_z = -\frac{\partial \Phi}{\partial z}. \tag{1}$$

The same components may be derived also from a stream-function Ψ by

$$\mathcal{E}_r = \frac{1}{r}\frac{\partial \Psi}{\partial z}; \qquad \mathcal{E}_z = -\frac{1}{r}\frac{\partial \Psi}{\partial r}. \tag{2}$$

Potential and stream-function thus are interrelated by

$$\frac{\partial \Psi}{\partial r} = r\frac{\partial \Phi}{\partial z}; \qquad \frac{\partial \Psi}{\partial z} = -r\frac{\partial \Phi}{\partial r}. \tag{3}$$

---

[2] The boldface numbers in parentheses refer to the references appended to this paper, see Part II.

The field strengths of Eqs. 1 satisfy the condition of irrotationality

$$\frac{\partial \mathcal{E}_r}{\partial z} - \frac{\partial \mathcal{E}_z}{\partial r} = 0, \tag{4}$$

while the field strengths of Eqs. 2 satisfy the condition of continuity

$$\frac{\partial (r\mathcal{E}_r)}{\partial r} + \frac{\partial (r\mathcal{E}_z)}{\partial z} = 0. \tag{5}$$

On the other hand, Eqs. 1 substituted into Eq. 5 yield the differential equation which the potential-function must satisfy

$$\left.\begin{aligned} &\frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) = 0; \\ \text{or}\quad &\frac{\partial^2 \Phi}{\partial z^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial r^2} = 0, \end{aligned}\right\} \tag{6}$$

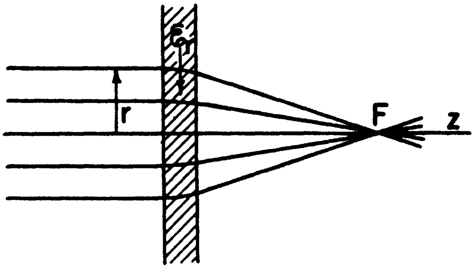


FIG. 1. Condition for focus formation.

and Eqs. 2 substituted into Eq. 4 yield the differential equation for the stream-function

$$\left.\begin{aligned} &\frac{\partial^2 \Psi}{\partial z^2} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi}{\partial r}\right) = 0; \\ \text{or}\quad &\frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial r^2} = 0. \end{aligned}\right\} \tag{7}$$

Equations 6 and 7 are by no means of identical form as they would be with two-dimensional problems.

If by action of a field lens the rays of a parallel electron beam of homogeneous velocity are to be deflected to converge toward or diverge from a point-focus on the axis of the beam, as shown in Fig. 1, the necessary condition is that the radial field strength, deflecting the electrons, is proportional to the radius

$$\mathcal{E}_r = p \cdot r. \tag{8}$$

In general, p might be a function of the axial coordinate z, but here we will consider p as a constant, characteristic for the strength of the lens-forming field. We will denote p as the power of the field.

Hence, the potential must contain a component as integrated from Eq. 1

$$\Phi' = -\int \mathcal{E}_r dr = -p\int r dr = -p\frac{r^2}{2} \tag{9}$$

to which an arbitrary constant or a function of z may be added. The actual dependence of the potential upon z can be determined by Eq. 6, as

$$\frac{\partial^2\Phi}{\partial z^2} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(-p\frac{r^2}{2}\right)\right) = 2p. \tag{10}$$

Two-fold integration with respect to z gives a second component of the potential

$$\Phi'' = \int dz \int 2p dz = z^2 + az + b. \tag{11}$$

The two last terms on the right-hand side are trivial, indicating merely a constant potential and a homogeneous field. We will not consider them further since they express only a displacement of the entire field in the direction of the z-axis.

The potential with which we will deal thus is the sum of the main terms in Eqs. 9 and 11, namely

$$\Phi = p\left(z^2 - \frac{r^2}{2}\right). \tag{12}$$

The factor p determines the power, the function of z and r the structure of the electromagnetic field, which shall be used here for the purpose of forming an electron lens.

The stream-function can be derived by integration of one of the Eqs. 3, yielding generally

$$\Psi = \int r\frac{\partial(\Phi' + \Phi'')}{\partial z} dr = \int r(2pz + a)dr = pzr^2 + a\frac{r^2}{2}. \tag{13}$$

With exclusion of the trivial last term, indicating merely a parallel flow, the stream-function is

$$\Psi = pzr^2. \tag{14}$$

Equations 12 and 14 indicate that in a meridian plane of z and r the stream lines, Ψ = constant, are cubic hyperbolas, while the equipotential lines, Φ = constant, are ordinary hyperbolas, having a saddle-point at the origin $r = 0$, $z = 0$. The two families of curves are shown in Figs. 2 and 3. They are symmetrical to the origin. In the three-dimensional field of rotational symmetry around the z-axis, these

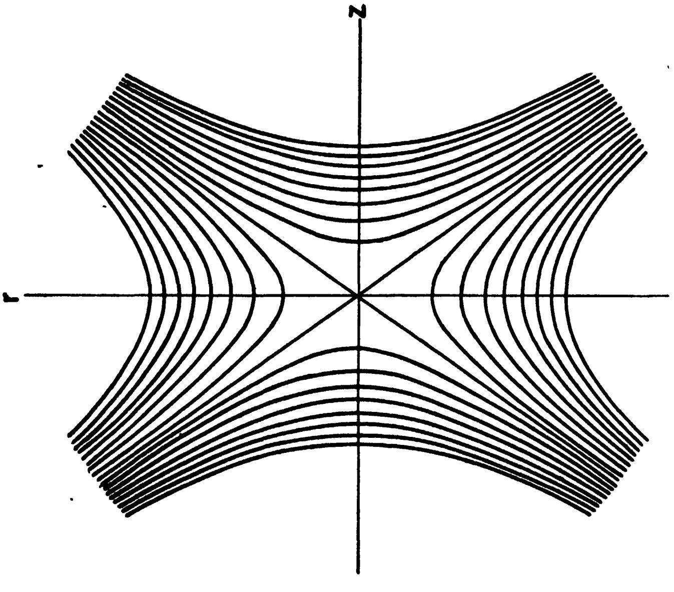


FIG. 2. Equipotential hyperboloids, Φ = constant, of lens field.

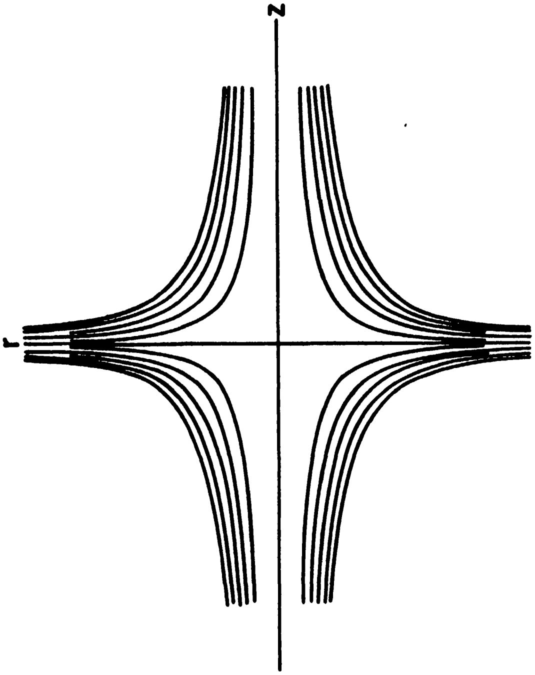


FIG. 3. Field lines, Ψ = constant, in hyperbolic lens.

curves are the generatrices of the equipotential surfaces and of the stream tubes, both constituting hyperboloids in space of the order just mentioned.

The equipotential lines have rectilinear asymptotes given by the zero potential, $\Phi = 0$. Their slope is by Eq. 12

$$\frac{r}{z} = \sqrt{2}. \tag{15}$$

Thus the apex angle of the cone formed by this linear generatrix is

$$\alpha = \tan^{-1} \sqrt{2} = 54^\circ\, 44'. \tag{16}$$

According to Fig. 2 the double cone separates the lateral hyperboloids within the cones from the annular hyperboloids in the space between the cones.

If a perfect focusing action is expected for a wide electron beam as in Fig. 1, the asymptotic lines of the field should be absolutely straight as far as the focusing effect extends. Any departure of the zero potential lines from linearity is an indication that the field deviates from the hyperbolic structure given by Eqs. 12 and 14. Then the radial field strength is no more proportional to the radius. Hence, the straightness of the equipotential lines through the saddlepoint is a criterion of the quality of the field structure.

Whereas in the case of light-optics, spherical or elliptical lenses can focus sharply a light beam of considerable radial extension, it appears that a uniform electron beam of substantial radial extension will be sharply focused by a field of hyperbolic structure.

The radial and axial field strengths, as derived by Eq. 1 from the potential of Eq. 12, are

$$\mathcal{E}_r = pr; \qquad \mathcal{E}_z = -\,2pz. \tag{17}$$

Thus, the radial field strength depends only on the radius, the axial field strength only on the axial distance, each being proportional to its respective coordinate and being independent of the other coordinate. With separation from the origin, the axial field strength increases at twice the rate of the radial field strength.

The curvature of the equipotential hyperbolas in Fig. 2 is a maximum at the axes. For constant potential the equation of the hyperbolas is

$$r = \sqrt{2\left(z^2 - \frac{\Phi}{p}\right)}; \qquad \text{or} \qquad z = \sqrt{\frac{\Phi}{p} + \frac{r^2}{2}}. \tag{18}$$

The second derivatives are

$$\frac{d^2r}{dz^2} = -\,2\,\frac{2z^2 - r^2}{r^3}\,; \qquad \frac{d^2z}{dr^2} = \frac{2z^2 - r^2}{4z^3} \tag{19}$$

and, therefore, the radii of curvature on the r-axis and on the z-axis are

$$\rho_r = \frac{r}{2}; \qquad \rho_z = 2z, \tag{20}$$

as represented in Fig. 4. Thus, for equal distance from the origin the annular hyperboloids are curved four times as much as the lateral hyperboloids.

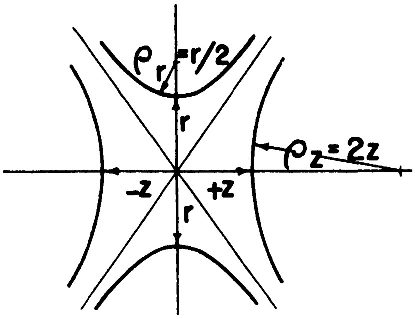


FIG. 4. Radii of curvature of equipotential hyperboloids.

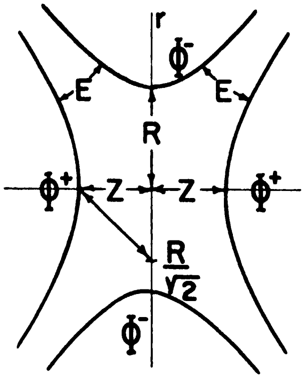


FIG. 5. Electrical and geometrical parameters of hyperbolic field.

The total field strength at any point of the field is

$$\mathcal{E} = \sqrt{e_r^2 + e_z^2} = p\sqrt{r^2 + (2z)^2}. \tag{21}$$

Along the asymptotes, as defined by Eq. 15, the field strength is

$$\mathcal{E}_a = \sqrt{3}pr = \sqrt{6}pz = \sqrt{2}pa, \tag{22}$$

increasing linearly with distance a from the origin. Thus the curves of constant field strength, as derived from Eq. 21, are

$$r^2 + (2z)^2 = \left(\frac{e}{p}\right)^2. \tag{23}$$

In the meridian plane they are ellipses of an axis-ratio 2:1.

If the power p of the field is chosen positive, the radial field strength from Eq. 17, is positive throughout the field. The axial field strength, however, is negative for positive z and positive for negative z. Thus electrons of negative charge moving through the field are throughout radially accelerated towards the axis, while axially they are retarded on the entrance side and accelerated on the exit side of the field. This performance produces an excellent lens effect, converging the electrons to a focus from whichever side they may come. A negative constant p, however, would diverge a stream of electrons away from the axis.

For positive p, the lateral hyperboloids, intersecting the z-axis at the distance Z for $r = 0$, as indicated in Fig. 5, have positive potential Φ^+ on both sides of the field. The annular hyperboloids, on the other hand, intersecting the r-axis at R for $z = 0$ have negative potential Φ^- throughout. Thus, the potentials are given by Eq. 12 as

$$\Phi^+ = pZ^2; \qquad \Phi^- = -p\frac{R^2}{2}. \tag{24}$$

A voltage applied between such hyperboloidal electrodes, as indicated by Fig. 5, is therefore

$$E = \Phi^+ - \Phi^- = p\left(Z^2 + \frac{R^2}{2}\right). \tag{25}$$

Hence, the power p of the field structure is determined by the voltage between any lateral and annular hyperboloids and by the apex distances Z and R of these hyperboloids on the axes as

$$p = \frac{E}{Z^2 + R^2/2}. \tag{26}$$

The denominator can be represented by a line from the end of Z to $R/\sqrt{2}$, as shown in Fig. 5.

The power p of the field is expressed by a magnitude of the dimension "volts per square centimeter," as distinguished from the dimension "volts per centimeter" of the field strength $\mathcal{E}$. Substitution of Eq. 26 into Eq. 21 or 22 permits the determination of the field strength at any place for given geometry and voltage between the electrodes.

3. ELECTROSTATIC TRAJECTORIES.

Since a field of hyperbolic structure, as described by Eqs. 12 and 14, develops field strengths, the components of which depend only on their own coordinates, see Eq. 17, the equations of motion of electrons through the field are extremely simple.

We consider converging lens fields of positive constant p. For electrons of mass m and charge $-q$, Newton's law is

$$m\frac{dv}{dt} = -q\mathcal{E}. \tag{27}$$

This yields, with the axial and radial components of velocity

$$v_r = \frac{dr}{dt}; \qquad v_z = \frac{dz}{dt} \tag{28}$$

and with use of Eq. 17,

$$\frac{d^2r}{dt^2} + \frac{q}{m}pr = 0; \qquad \frac{d^2z}{dt^2} - 2\frac{q}{m}pz = 0. \tag{29}$$

These are ordinary linear differential equations, separated in r and z. Their solution for the radial position of the electron is

$$r = A\cos(\nu t - \varphi); \qquad \nu = \sqrt{\frac{q}{m}p} \tag{30}$$

and for the axial position

$$z = B\sinh\mu t + C\cosh\mu t; \qquad \mu = \sqrt{2\frac{q}{m}p}. \tag{31}$$

A, B, C and φ are constants of integration which must be determined by the boundary conditions.

We realize that radially the electrons perform ordinary sinusoidal oscillations about the z-axis with frequency ν, while axially the movement will be unilaterally retarded or accelerated on either side of the center plane. The time factor μ of the latter motion is related to the radial frequency ν of the former motion by the simple ratio

$$\frac{\mu}{\nu} = \sqrt{2}, \tag{32}$$

both parameters being determined solely by the power p of the field and by the ratio of charge to mass of the electrons. Hence, the motions of the electrons will be isochronous.

Let us first consider the sinh-solution of the electron rays with $C = 0$ in Eq. 31. In order to obtain the orbital equation, we eliminate the time from Eqs. 30 and 31. This yields with use of Eq. 32,

$$\left.\begin{aligned} \frac{r}{A} &= \cos\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{z}{B}\right) - \varphi\right]; \\ \text{or} \qquad & \\ \frac{z}{B} &= \sinh\left[\sqrt{2}\left(\cos^{-1}\left(\frac{r}{A}\right) + \varphi\right)\right]. \end{aligned}\right\} \tag{33}$$

Both these equal expressions for the electron trajectories show that the radial amplitude A about the axis does not play any role for the shape of the curve. Only the ratio of radial distance r to amplitude A enters the equations. Therefore, the trajectories of different amplitudes, but of the same phase angle φ, are all of identical character. This indicates that all such trajectories intersect the z-axis at exactly the same points. These points are focal points of the electron rays. Hence this hyperbolic lens field does not produce any aberration with respect to the amplitude of the individual rays.

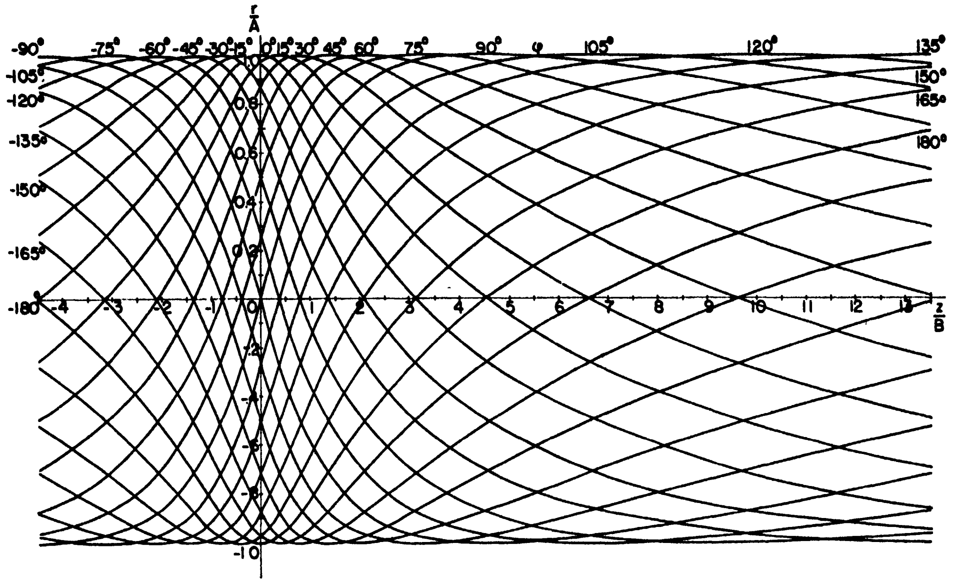


FIG. 6. Electron trajectories in hyperbolic lens field.

In Fig. 6 the entire family of trajectories for various phase angles φ is plotted, the variables being taken as r/A and z/B. The chart is extended on the right-hand side to larger values of z/B. If we follow from a point, as marked on the left-hand side of Fig. 7, the course of two of the trajectories taken from Fig. 6 and intersecting at this point, we see that they will intersect again on the right-hand side at a point

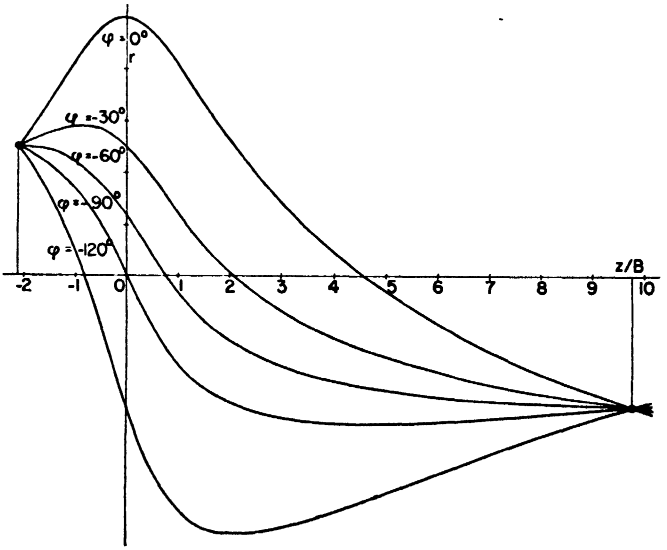


FIG. 7. Rays between conjugate image points.

the same distance from the axis as the point of origin. The same holds true for any two corresponding intersections of Fig. 6.

The two extreme traces in Fig. 7 belong to $\varphi = 0$ and $\varphi = -120°$. Three other trajectories are shown, belonging to $\varphi = -30°$, $-60°$ and $-90°$. These are reduced from Fig. 6 in amplitude in order to originate from the same point as marked on the left-hand side. They all join at the same intersection on the right-hand side, thus showing graphically that electron rays emerging from any point on the left-hand side unite at a common point on the right-hand side of the field, these conjugate points having the same distance from the axis. Hence, any pattern in a plane on the left-hand side of the origin can be imaged free of aberration and to scale on the right-hand side, and vice versa.

The value of the phase angle φ determines the position of the maxima and of the zero passages of the trajectories, as seen by Fig. 6. $\varphi = 0$ gives the ray of symmetry about the center plane, cutting the axis at the points

$$\pm \frac{z}{B} = \sinh\left(\sqrt{2}\,\frac{\pi}{2}\right) = 4.55. \tag{34}$$

$\varphi = \pm 90°$ gives the central rays traversing the axis at the center plane and reaching their amplitudes at the axial distances of Eq. 34.

We see that this family of trajectories constitutes an ideal field of rays, securing a perfect imaging of one radial plane to another, without, however, producing either magnification or reduction of the scale. The fact that the trajectories are compressed near the center plane and are expanding with separation from this plane is due to the increase of the axial field strength with axial distance, retarding or accelerating the electrons more and more, as determined by Eq. 31.

An electron beam consisting of parallel rays of homogeneous velocity entering the field structure at any radial plane, will be continued within the field structure by trajectories of various amplitudes, all having the same phase angle φ as determined by the plane of incidence. All these rays will be focused at a point on the axis which may lie on the left-hand side, in the center plane, or on the right-hand side, depending on the value of the phase angle φ at the plane of incidence.

The trajectories have points of inflection at regions where the axial field strength increases more than the radial field strength decreases. In order to obtain a relatively short focal distance of the trajectories, the field should not extend axially too far from the center plane since for large z the rays are converging very slowly.

If we express the trajectories, Eqs. 33, in the form

$$\varphi = \frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{z}{B}\right) - \cos^{-1}\left(\frac{r}{A}\right) \tag{35}$$

and note that the inverse cosine is double-valued, we realize that emerging from any point given by r/A and z/B there will develop two trajectories with different angles φ. These curves will proceed independently with changing z/B but will reach uni-valued points again for r/A equal in magnitude to that of the original point, but alternating in sign. By this mechanism of the trajectories, as illustrated by Figs. 6 and 7, electrons leaving any point within the field will meet again at an infinite number of points which are alternately above and below the z-axis. Hence there exist an infinite number of conjugate points and planes to the left and to the right, each forming a real image of all the others at a magnification ratio ±1:1. If z extends to great distances, or if B is small, the electrons oscillate many times during their transmission through the field. This is indicated in Fig. 8, where one of the trajectories is traced up to greater values of z/B.

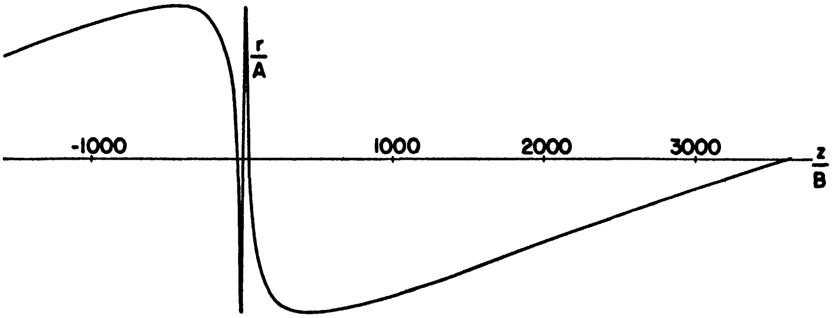


FIG. 8. Multiple oscillations of trajectory.

The velocity of the electrons in the radial direction is, by Eqs. 28 and 30,

$$v_r = -\nu A \sin(\nu t - \varphi) \tag{36}$$

and in the axial direction, by Eqs. 28 and 31,

$$v_z = \mu B \cosh \mu t. \tag{37}$$

Eliminating t from Eqs. 30 and 36, the radial velocity becomes

$$\frac{v_r}{\nu A} = -\sin\left(\cos^{-1}\left(\frac{r}{A}\right)\right) = \sqrt{1 - \left(\frac{r}{A}\right)^2}. \tag{38}$$

This is mainly dependent on the radius r and graphically would form a circle diagram. The axial velocity, by elimination of t from Eqs. 31 and 37, is

$$\frac{v_z}{\mu B} = \cosh\left(\sinh^{-1}\left(\frac{z}{B}\right)\right) = \sqrt{1 + \left(\frac{z}{B}\right)^2}. \tag{39}$$

This is dependent essentially on the axial distance z, and is given by a hyperbola, as indicated in Fig. 9.

Hence, on the left-hand side of the center plane the entering electrons are retarded to a minimum velocity which at $z = 0$ is

$$v_{z\,\mathrm{min}} = \mu B, \tag{40}$$

while on the right-hand side the electrons are accelerated to high velocities again. At considerable distances off the center plane the velocity changes nearly linearly with z, corresponding to the asymptotes of the hyperbolic velocity diagram in Fig. 9.

The radial and axial velocity components on the electron paths are independent of each other, see Eqs. 38 and 39, each being determined merely by its own coordinate, r or z. This independence is due to the fact that the potential Φ of the field does not contain any product of r and z, or product of functions of r and z. On the other hand, in an electromagnetic field having such product-structure, the differential equations for the coordinates of the electron paths, and still more the velocity components, would not be independent of each other. Therefore the formulation for the trajectories and for the velocities would be much more involved, rendering extremely difficult a rigorous analytical determination of the electronic qualities of the lens field.

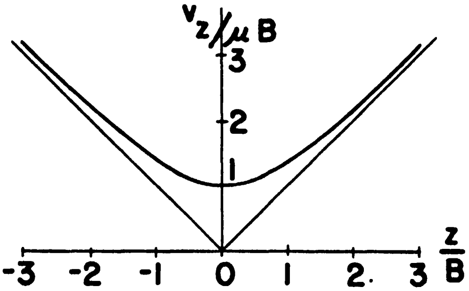


FIG. 9. Axial velocity distribution in lens field.

4. TRANSMISSION CONSTANT.

In Fig. 10(a) is shown a beam of parallel electrons leaving an emitting cathode, which may have any negative potential, and entering a hyperbolic field at a distance $-Z$ from the origin. The electrons leave the field at a distance $+Z$ from the center, the field thus forming a symmetrical electron lens. While passing through the field, the electrons will follow the trajectories of Fig. 6, and therefore will converge towards a focus which may lie within or without the range of the lens field. Figure 10(b) shows the change of potential along the z-axis. The potential increases between the emitting cathode and the left-hand anode of the lens by a voltage E_V, constituting the accelerating voltage of the electrons, or the velocity-voltage of the electrons entering the field. Within the lens space the potential changes parabolically, reaching a minimum 0 at the center. On the right-hand side externally to the lens field, the potential may be constant. Outside of the axis, the distribution of potential departs from that on the axis, as given by Eq. 12. The lower curve in Fig. 10(*b*) shows the distribution at the radius $r = R$. The voltage between the lateral equipotential hyperboloids, passing through axial points $\pm Z$, and the annular equipotential hyperboloid, passing through radial points R, again is denoted by E.

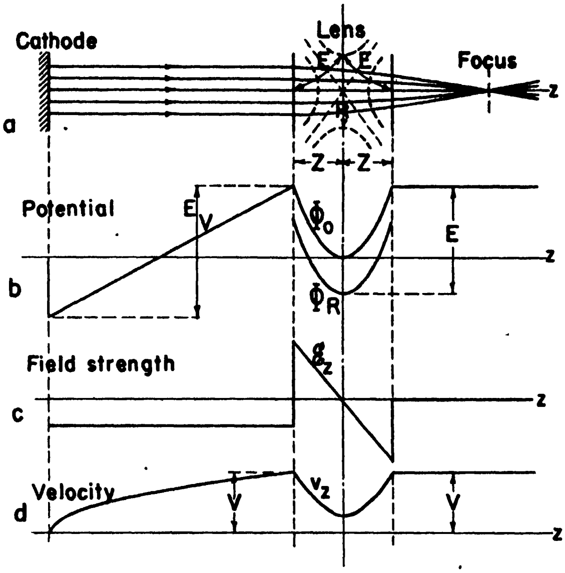


FIG. 10. Boundary conditions of a symmetrical lens.

Figure 10(*c*) shows the distribution of the axial field strength $\mathcal{E}_z$ along the axis, changing linearly within the lens space and, in our scheme, jumping at the boundaries to the field strength just outside of the lens space. Figure 10(*d*) shows the distribution of the axial electron velocity, having a value V at the entrance into the lens field. Within the lens the velocity varies hyperbolically with distance, attaining a minimum value at the center plane and again reaching the original velocity V on exit from the lens field.

In terms of the velocity voltage E_V of any incident electron considered, the entrance and exit velocity is given by

$$V = \sqrt{2\frac{q}{m}E_V}. \tag{41}$$

Within the lens, the axial velocity at the boundaries $\pm Z$ is given by Eq. 39 as

$$v_z = \mu\sqrt{B^2 + Z^2}. \tag{42}$$

Equating the last two expressions and substituting μ from Eq. 31, gives

$$E_V = p(B^2 + Z^2) \tag{43}$$

and therefrom the integration constant of the ray can be determined as

$$B = \sqrt{\frac{E_V}{p} - Z^2} = \sqrt{\left(\frac{V}{\mu}\right)^2 - Z^2}. \tag{44}$$

The last expression contains the entrance velocity V rather than the incident velocity voltage E_V.

If in the lens space the initial distribution of the axial velocity over the cross section of the beam is uniform, it remains uniform throughout the lens field. We will consider here this condition as being fulfilled, although it may require special arrangements to secure this performance of the electron beam when entering the lens field. A later section of this paper will deal with the deviations if this condition is not satisfied.

We denote B as the transmission constant of the lens-ray arrangement because its value determines the axial development of the trajectories as seen by Eq. 33 and Fig. 6. The smaller B is, the more the trajectories are refracted over a certain axial distance z. Substituting the power p of the field by Eq. 26 into Eq. 44 the transmission constant becomes

$$B = \sqrt{\frac{E_V}{E}\left(Z^2 + \frac{R^2}{2}\right) - Z^2} = \sqrt{\left(\frac{E_V}{E} - 1\right)Z^2 + \frac{1}{2}\frac{E_V}{E}R^2}. \tag{45}$$

The dimension of this constant is centimeters.

We see from the right-hand side of Eq. 45 that the transmission constant depends on three parameters only, namely: the half-thickness Z of the lens, the radius R of the central aperture, and the ratio of the velocity voltage E_V of the incident electrons to the lens voltage E between the hyperboloids through Z and R.

With increasing lens voltage E, the transmission constant B decreases, becomes zero and finally imaginary. In these latter cases the lens loses its ability to transmit electrons. Hence, the condition for transmission of an electron beam is, that the lens voltage with respect to the incident electron voltage must be smaller than

$$\left(\frac{E}{E_V}\right)_0 = 1 + \frac{1}{2}\left(\frac{R}{Z}\right)^2, \tag{46}$$

a ratio which is plotted in Fig. 11 as depending on the relative lens thickness Z. For larger lens voltage or smaller electron voltage than is determined by this condition, the potential in the center of the field would be lower than that of the cathode, and therefore no electrons could pass through the lens. Thus, Eq. 46 determines the blocking voltage of the lens.

When approaching this blocking condition, the transmission constant B becomes very small. Then the electrons will perform an increasing number of radial oscillations within the lens volume, as shown by Fig. 8.

If the annular electrode of the lens is directly connected to the emitting cathode, the lens voltage is $E = E_V$, and the transmission constant becomes by Eq. 45

$$B = \frac{R}{\sqrt{2}}. \tag{47}$$

Hence, for a unipotential lens the transmission constant is independent of the thickness Z of the lens and solely determined by R.

For short lenses Z is relatively small and for weak lenses E_V/E is relatively large, both cases leading to a similar simplification of Eq. 45.

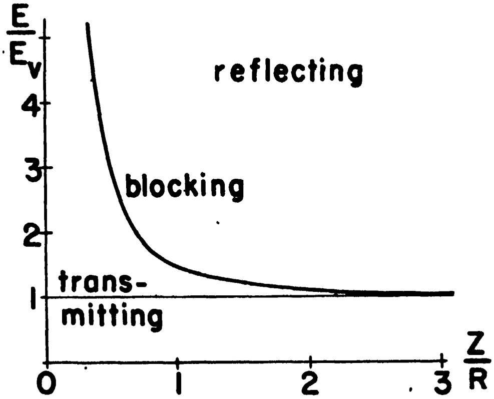


FIG. 11. Blocking voltage *versus* lens thickness.

The use of these approximations, however, would not simplify the development of the further analysis in this paper.

In all cases where E is equal to or smaller than E_V, the lens will always transmit the electron beam, see Eq. 46. If, however, the lens voltage is higher than the incident electron voltage, the ratio Z/R determines whether the beam will be transmitted or blocked.

Figure 11 shows that with large half-width Z the blocking condition is reached at lower lens voltages than with large aperture-radius R. For $Z = R$ the blocking voltage is 1.5 times the incident velocity voltage of the electrons. In order to avoid a relatively unstable performance near the blocking condition, it is not advisable to operate at too large a ratio Z/R for unipotential lenses.

The minimum axial velocity, Eq. 40, to which the electron rays are reduced at the lens center, can be expressed by substitution of μ from

Eq. 31, of B from Eq. 44, and of p from Eq. 26 as

$$v_{z\,\mathrm{min}} = V\sqrt{1 - \frac{E/E_V}{1 + R^2/2Z^2}}. \tag{48}$$

This is plotted in Fig. 12 and shows that near the blocking condition of Eq. 46 the axial velocity in the center plane of the lens rapidly approaches zero.

In Fig. 6 the electron trajectories have been plotted as depending on an axial coordinate z/B. For actual examples it is convenient to convert this coordinate to a ratio z/Z which has a direct bearing on the physical width Z of the lens. This can be done by multiplying the variable z/B by a factor B/Z, the value of which, by Eq. 45, is

$$\frac{B}{Z} = \sqrt{\frac{E_V}{E}\left[1 + \frac{1}{2}\left(\frac{R}{Z}\right)^2\right] - 1}. \tag{49}$$

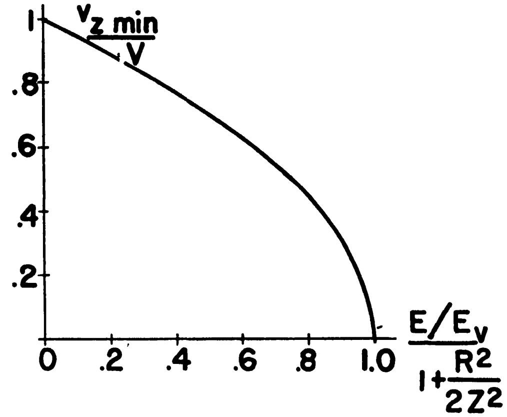


FIG. 12. Change of minimum velocity in lens center.

Table I gives some values of this ratio for a number of lens voltages and central lens apertures. We see that for such relations B is of the order of magnitude of the half-thickness of the lens. In the blank spaces the lens is blocking the beam.

TABLE I.—*Ratio B/Z of Transmission Constant to Half-Width of the Lens.*

$E/E_V =$	$\frac{1}{2}$	1	2
$\frac{R}{Z} = \frac{1}{2}$	$\frac{5}{4} = 1.12$	$\frac{1}{\sqrt{8}} = 0.354$	
$\frac{R}{Z} = 1$	$\sqrt{2} = 1.41$	$\frac{1}{\sqrt{2}} = 0.707$	
$\frac{R}{Z} = 2$	$\sqrt{5} = 2.24$	$\sqrt{2} = 1.41$	$\frac{1}{\sqrt{2}} = 0.707$

Now we can determine the characteristic frequencies ν and μ of the electron motion as in Eqs. 30 and 31. Substituting p from Eq. 26, and expressing q/m from Eq. 41, the frequency of the radial oscillations of the electrons is

$$\nu = \frac{V}{\sqrt{2Z^2 + R^2}} \sqrt{\frac{E}{E_V}}. \tag{50}$$

For a unipotential lens with $E = E_V$, an incident electron velocity of half the velocity of light, $V = 1.5 \cdot 10^{10}$ cm. per second corresponding to $E_V = 64$ kv., and dimensions $R = Z = 1$ cm., the frequency is

$$\frac{\nu}{2\pi} = \frac{1.5 \cdot 10^{10}}{2\pi\sqrt{2 \cdot 1^2 + 1^2}} \sqrt{1} = 1.38 \cdot 10^9 \text{ cps.}$$

Thus, for such an example, the frequency of ν, and also that of μ, will be of the order of thousands of megacycles, and the corresponding electromagnetic wave length of the order of centimeters.

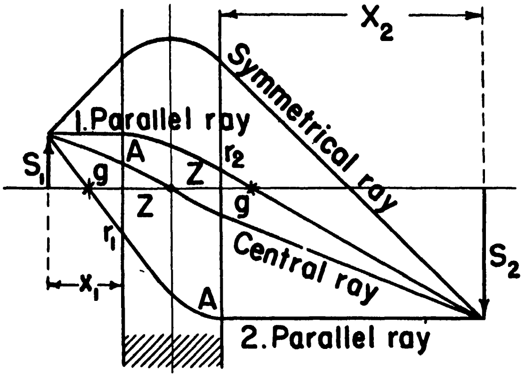


FIG. 13. Image formation by four principal rays.

Beyond the blocking condition of Eq. 46 the transmission constant B becomes imaginary and therefore the second solution of Eq. 31, with integration constant C, should be used. Since cosh has a minimum value of unity and has two branches of the same sign, the electrons now cannot reach $z = 0$, but will return on the second branch. Hence, whereas the first term of Eq. 31 represents a lens action, the second term indicates the action of a mirror.

5. CARDINAL POINTS.

We will consider four characteristic rays as indicated in Fig. 13, namely incident and emerging parallel rays, central rays, and symmetrical rays. The investigation of these significant rays transmitted through a hyperbolic field may stand for a more general analysis. We suppose the lens field to be axially symmetrical, and to be limited by two radial planes at the distance $\pm Z$ from the center of the field.

Incident or emerging parallel rays in the external space are continued through the boundaries of the lens by rays within the field which have at $z = \pm Z$ the value $r/A = \pm 1$, as seen immediately from Fig. 6. Inserted into Eq. 33 this yields

$$\pm \frac{Z}{B} = \sinh\left[\sqrt{2}(0 + \varphi)\right]; \qquad \varphi = \pm \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{Z}{B}\right), \tag{51}$$

thus determining the phase angles φ of the two parallel rays of Fig. 13.

The equations of these rays within the volume of the lens are given by inserting φ from Eq. 51 into the first Eq. 33

$$\frac{r}{A} = \cos\left[\frac{1}{\sqrt{2}}\left(\sinh^{-1}\left(\frac{z}{B}\right) - \frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right)\right]. \tag{52}$$

For both of the parallel rays the abscissa z at the converging end of the ray in the lens is opposite in sign to the abscissa of the parallel end of the ray, $z = -Z$. Both sinh's in Eq. 52 then have the same magnitude and sign, yielding the radii at the converging ends of the rays as

$$\frac{r}{A} = \cos\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]. \tag{53}$$

We may ask for the condition that parallel incident rays will directly focus at the opposite boundary of the field. Then we have in Eq. 53, $r/A = 0$, giving with use of Eq. 51

$$\pm \sinh^{-1}\left(\frac{Z}{B}\right) = \frac{\pi}{2\sqrt{2}}; \qquad \frac{Z}{B} = 1.35; \qquad \varphi = \pm\frac{\pi}{4}. \tag{54}$$

It may be verified by Fig. 6 that parallel rays entering a symmetrical lens at $\varphi = -45°$, focus just at the other end at $\varphi = +45°$. In this case the thickness of the lens would be 90°, measured in units of φ.

Outside of the lens field the rays propagate linearly, with constant slope. Quite generally, the slope of the rays within the lens field, as given by differentiation of Eq. 33, is

$$\frac{dr}{dz} = -\frac{A}{B}\,\frac{\sin\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{z}{B}\right) - \varphi\right]}{\sqrt{2\left(1 + \left(\frac{z}{B}\right)^2\right)}}. \tag{55}$$

Introducing herein the value of the phase angle φ from Eq. 51 for parallel entrance of the ray at $-Z$, and the abscissa $z = +Z$ for the converging exit, the slope of the ray leaving the lens is

$$\frac{dr}{dz} = -\frac{A}{B}\,\frac{\sin\left[\sqrt{2}\,\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}}. \tag{56}$$

This ray intersects the axis at a distance g from the surface of the lens field, which length usually is denoted as back-focus and is given in Fig. 14 by the subtangent

$$g = \frac{r}{dr/dz}. \tag{57}$$

Substituting the numerator by Eq. 53 and the denominator by Eq. 56, the back-focus is

$$g = \frac{\sqrt{2}\,B\sqrt{1+\left(\frac{Z}{B}\right)^2}}{\tan\left[\sqrt{2}\,\sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{58}$$

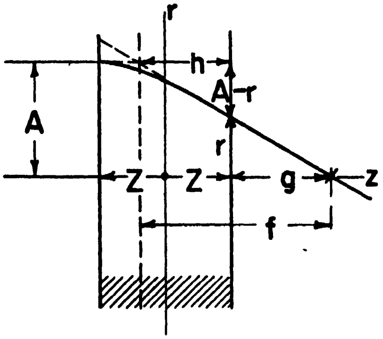


FIG. 14. Parallel incident ray and cardinal lens points.

This value is independent of the initial amplitude A, indicating that bundles of uniform parallel rays are refracted rigorously toward a point-focus, independent of the width of the incident beam. This shows again that the hyberbolic structure of the field is entirely free from aperture defects for uniform parallel beams.

Due to our supposition of a symmetrical field, the back-focus is the same for both sides of the lens. Its magnitude depends only on the transmission constant B and on the ratio of half-thickness Z to this constant B. Figure 15 shows the change of the back-focus with the lens thickness Z, both values taken as ratio to the transmission constant B. If the relative lens thickness Z/B increases from 0 to 4.55, the back-focus changes from $+\infty$ through 0 to $-\infty$.

If we express the main factors in the numerator of Eq. 58 in terms of the original lens parameters, we obtain with use of Eq. 45

$$B\sqrt{1+\left(\frac{Z}{B}\right)^2} = \sqrt{B^2+Z^2} = \sqrt{\frac{E_V}{E}\left(Z^2+\frac{R^2}{2}\right)}. \qquad (59)$$

This is a characteristic length which will appear in many of the following equations, and which is given solely by the square root of the voltage ratio and the main dimensions of the hyperbolic lens.

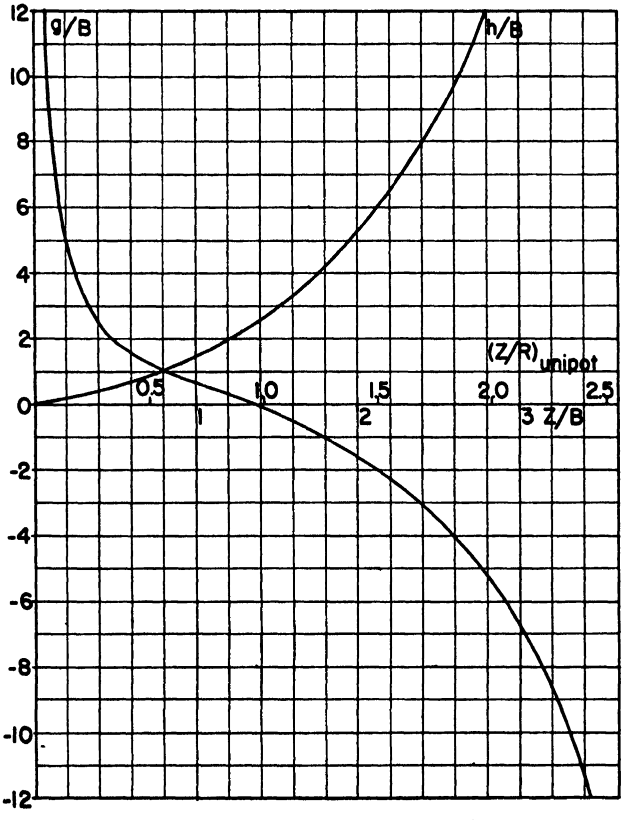


FIG. 15. Change of back-focus and principal plane with lens thickness.

The position of the principal plane, as shown in Fig. 14, is determined by the intersection of the entrance and exit tangents of the parallel ray. The distance of this point from the end of the lens is

$$h = \frac{A-r}{dr/dz} = B\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}\,\frac{1-\cos\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sin\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} \qquad (60)$$

which transforms by a well-known trigonometric relation to

$$h = \sqrt{2}\, B \sqrt{1 + \left(\frac{Z}{B}\right)^2} \tan\left[\frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{Z}{B}\right)\right]. \tag{61}$$

The second curve in Fig. 15 shows the change of this value with lens width. We see that h is larger than Z, showing that for the emerging ray the principal plane in such electron lenses always is situated on the entrance side of the rays. This is in contrast to simple light optical lenses, where this principal plane for the most part is on the exit side of the rays. This difference effects a performance of the principal electron rays transmitted through the electron lens as in Fig. 16(a), while light rays in glass lenses ordinarily behave as in Fig. 16(b).

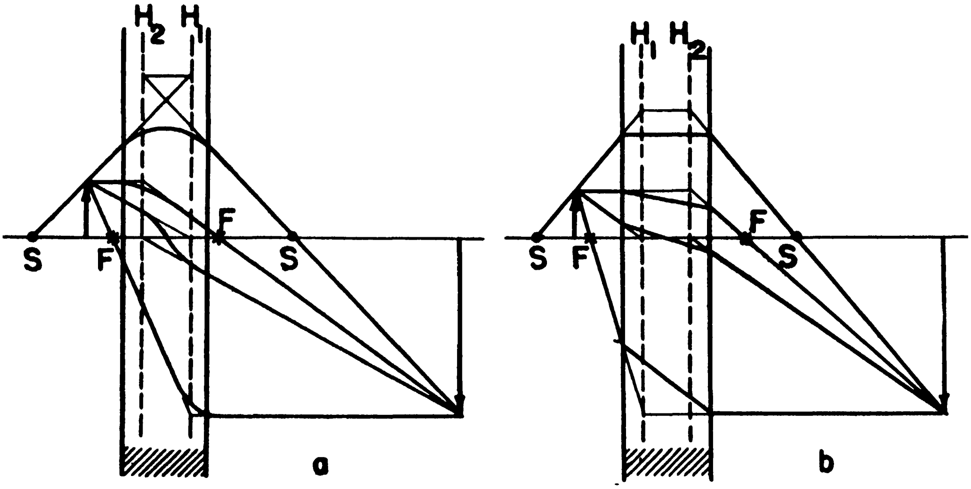


FIG. 16. Ray propagation, (a) in electron lens, (b) in glass lens.

The focal length of the lens is given by the separation of the focus from the principal plane. According to Fig. 14, this is

$$f = \frac{A}{dr/dz} = \frac{\sqrt{2}\, B \sqrt{1 + \left(\frac{Z}{B}\right)^2}}{\sin\left[\sqrt{2} \sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{62}$$

The ratio f/B is plotted in Fig. 17, giving large values for both thin and thick lenses. The focal-length ratio shows a marked minimum at the point

$$\frac{Z}{B} = 0.89; \qquad \frac{f}{B} = 2.08, \tag{63}$$

leading to $f = 2.35\, Z$ as optimum focal length with respect to the ratio Z/B of the half-width. However, the transmission constant B, to which Z and f here are related, is in general dependent on Z, as given by Eq. 45.

Only for unipotential lenses is the constant B independent of Z as shown by Eq. 47. For such lenses the optimum length is

$$Z_{\text{opt}} = 0.63R \tag{64}$$

and the minimum focal length will be

$$f_{\text{min}} = 1.48R. \tag{65}$$

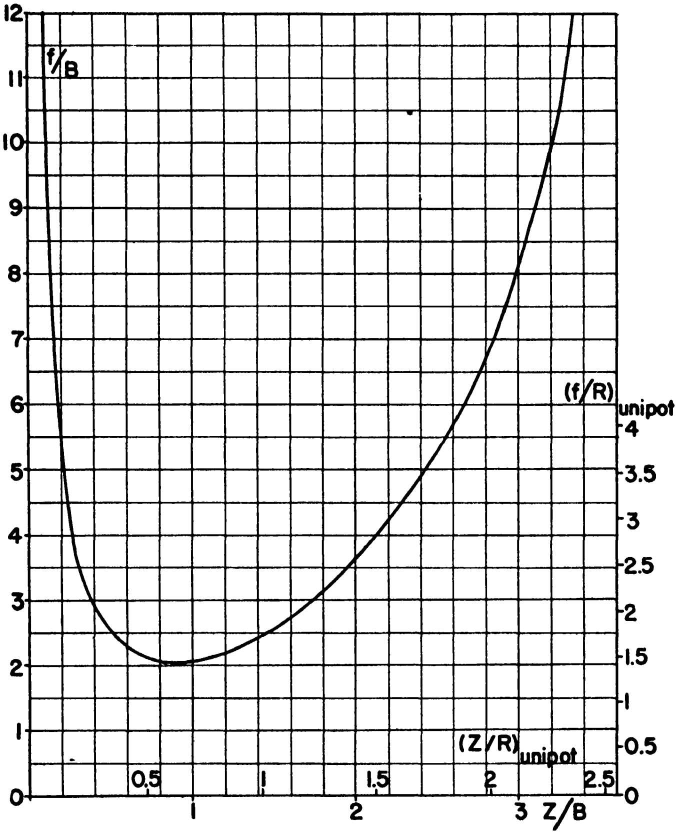


FIG. 17. Change of focal length with lens thickness.

In Fig. 17 the values are indicated which the coordinates assume in f/R and Z/R for a unipotential lens. For any other lens voltage the actual position and magnitude of the minimum of f are depending also on E/E_V and can be determined by supplementary differentiation of B from Eq. 45.

We see from Fig. 17 that axially thin lenses have a considerable focal length. This can be diminished either by greater thickness of the lens field, as given by Z, or by smaller transmission constant B, which is further determined by lens voltage E and radius R, see Eqs. 45 or 47.

Both changes lead to a greater effect of the field on the electrons. However, extending this effect beyond a certain limit will be of no avail since the focal length after passing through a minimum will rise again due to the decreasing slope of the trajectories in Fig. 6 with increasing Z/B.

By use of Eq. 62 and Fig. 17, such lenses can be readily designed with respect to voltage, width and aperture in order to obtain a desired focal length.

From the curve in Fig. 15 we see that for the minimum focal length, as in Eq. 63, the back-focus has a value $g/B = 0.86$, which is still positive. Thus, in this case the focus is outside the surface of the lens at a distance almost equal to the half-width, see Eq. 63. For thicker lenses the focus moves into the lens field, the limiting value being given by Eq. 54 as $Z/B = 1.35$. At $g = -Z$, the focus passes through the center of the lens field, which occurs according to Fig. 15 for a half-width $Z/B = 2.16$. This is different from the $\varphi = -90°$ trajectory of Fig. 6 starting parallel at $Z/B = -4.55$, since inside the lens Eq. 58 indicates the virtual position of the focus of the emerging rectilinear rays rather than the actual intersection of the trajectories.

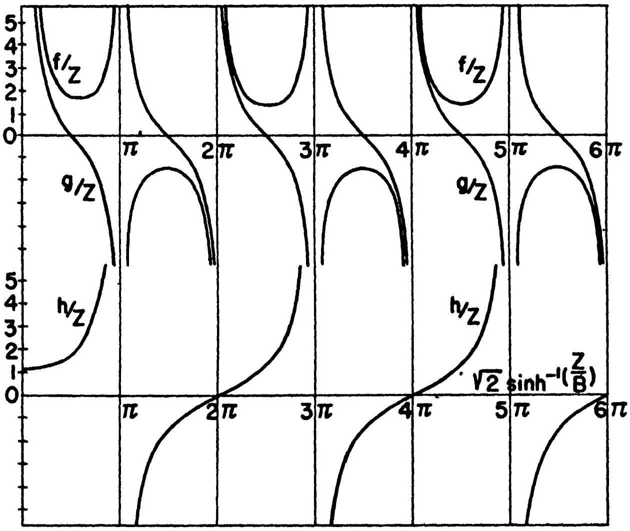


FIG. 18. Cardinal values for very thick lenses.

For still thicker lenses, the position of the virtual focus, given by g, moves rapidly toward $-\infty$ on the incident side of the lens. The principal plane, however, determined by h, moves much faster to the same side, see Fig. 15. Thus, the focal length f, as separation of focus and principal point, finally is entirely to the far left of the lens field.

If we increase the lens width beyond the value $Z/B = 4.55$, the back-focus g, as determined by a trigonometric tangent, repeats moving similar to Fig. 15. However the focal length f, as determined by a sine, now becomes negative and will change its sign repeatedly with further increase of Z. This indicates the formation of a diverging lens in the zones of negative sign. Figure 18 shows the performance of f, g, and h with greatly increasing ratio Z/B. The cardinal values are related here to Z rather than to B in order to yield more uniform curves; and as abscissa there is plotted the inverse sinh rather than its argument Z/B for the same reason. We see from Eq. 49 in combination with Eq. 46, that we are now close to the blocking condition and thus, such lenses will be somewhat sensitive.

It is interesting to see that, in principle, diverging lenses can be produced by the same technical means as converging lenses. From the trajectories of Fig. 6 it is clear that for formation of a diverging effect, the electrons must stay within the lens for more than one half a radial oscillation, thus converting the condensing action of the field into a diverging effect. For unipotential lenses, we derive from Eq. 47 that the width of the field must be $Z > 3.21R$. However, the minimum focal length will not occur before $Z = 10R$ and then has a value of $f = 1.42Z$. Hence, strongly diverging lenses must have a relatively small central aperture.

For lenses of greater width Fig. 18 indicates that alternately converging and diverging effects are produced. This behavior soon becomes periodic with respect to the abscissa shown.

For thin lenses we can develop Eq. 62 into a power series, the first two terms of which, with use of Eq. 45, are

$$f \cong \frac{B^2}{Z} + Z = \frac{E_V}{E}\left(Z + \frac{1}{2}\frac{R^2}{Z}\right). \tag{66}$$

Therefore, the focal length of a thin lens is parabolically dependent on the aperture and hyperbolically dependent on the thickness. For a thin unipotential lens with $E = E_V$, the focal length is always greater than the half-width, and there is a very simple relation with respect to the dimensions R and Z. This formulation may be used as an approximation also for other types of electron lenses and is in good accord with some experimental results (6) for three-plate-aperture lenses.

The last approximate formula (66) for the focal length, or even the former rigorous equations for all of the cardinal points, may be used to advantage if lenses of arbitrary field distribution are to be analysed or synthesized. The actual variation of the potential, paraxial and extra-axial, may be approximated by a number of potential fields as in Eq. 12 each forming an elemental lens of hyperbolic structure.

6. IMAGE FORMATION.

Let us now consider the formation of the image and the magnification of the object. If S_1 and S_2 are the magnitudes of object and image, we see from Fig. 19 that the following geometrical relations hold between the distances x from the lens surfaces and the back-focus g: For the object space

$$\frac{x_1}{g} = \frac{S_1 + r_1}{r_1} = \frac{S_1}{S_2 \cos\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} + 1 \tag{67}$$

and for the image space

$$\frac{x_2}{g} = \frac{S_2 + r_2}{r_2} = \frac{S_2}{S_1 \cos\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} + 1. \tag{68}$$

On the right-hand sides of both equations we have substituted Eq. 53 for the ratios of exit radius and entrance amplitude of a parallel incident ray, the radii being r_1 and r_2 and the amplitudes being S_2 and S_1.

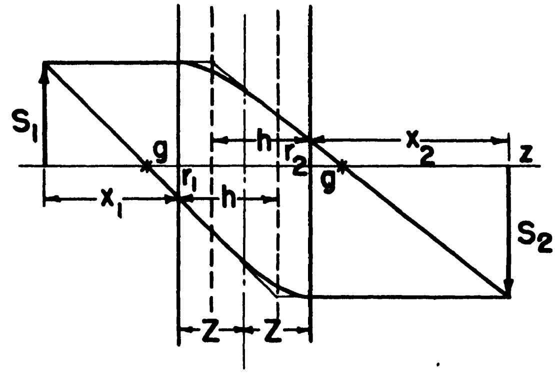


FIG. 19. Image formation by parallel rays.

By multiplying the first terms of the right-hand sides of Eqs. 67 and 68, the S-ratios cancel and there remains

$$\frac{g^2}{\cos^2\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} = (x_1 - g)\cdot(x_2 - g). \tag{69}$$

Substituting now g from Eq. 58 and using f from Eq. 62 yields

$$f^2 = (x_1 - g)(x_2 - g). \tag{70}$$

This is Newton's formula, correlating the focal length and the distances of object and image from the respective focal points, see Fig. 19. By purely geometrical means this formulation can be transformed into

$$\frac{1}{f} = \frac{1}{x_1 + h} + \frac{1}{x_2 + h}, \tag{71}$$

where h is the principal distance, according to Eq. 61. Thus, a hyperbolic electron lens behaves, with respect to the distances of object and image, like a perfect light-optical lens.

By dividing the first terms of the right-hand sides of Eqs. 67 and 68, the cosines cancel and there results

$$\left(\frac{S_2}{S_1}\right)^2 = \frac{x_2 - g}{x_1 - g}, \tag{72}$$

yielding the magnification ratio, also in Newton's form. This can be transformed by purely geometrical manipulation into the conventional form

$$\frac{S_2}{S_1} = \frac{x_2 + h}{x_1 + h}. \tag{73}$$

Thus, a hyperbolic electron lens behaves also with respect to the magnification like a perfect light-optical lens. For many practical applications of electron lenses, Eqs. 70 and 72 are more convenient to use than the conventional Eqs. 71 and 73. They show that, particularly for high magnification, the back-focus g rather than the focal length f plays the dominant role in precise image formation.

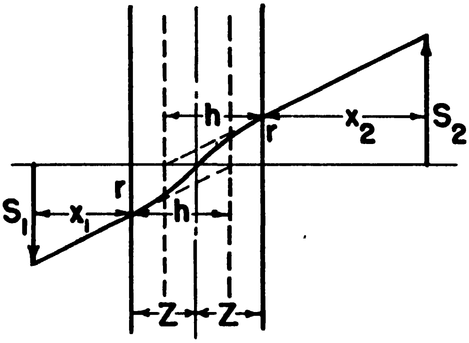


FIG. 20. Image formation by central ray.

So far we have derived the performance of the lens from the two parallel rays transmitted through the field. It is interesting to see that use of the central ray, as shown in Fig. 20, will lead to identical results. For this ray the phase angle is $\varphi = \pm 90°$, as in Fig. 6, and thus the trajectories are given by the first Eq. 33 as

$$\frac{r}{A} = \pm \sin\left[\frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{z}{B}\right)\right]. \tag{74}$$

The slope at the entrance and exit of the lens field is, from Eq. 55,

$$\frac{dr}{dz} = \frac{A}{B}\,\frac{\cos\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}}, \tag{75}$$

having the same sign for positive and negative Z.

The magnitudes of object and image are given, as in Fig. 20, by the sums of the entrance or exit radius r and the elevation above the distance x from the lens surface

$$S_1 = r + x_1\left(\frac{dr}{dz}\right); \qquad S_2 = r + x_2\left(\frac{dr}{dz}\right). \tag{76}$$

Thus, the magnification is

$$\frac{S_2}{S_1} = \frac{x_2 + \dfrac{r}{dr/dz}}{x_1 + \dfrac{r}{dr/dz}} = \frac{x_2 + h}{x_1 + h}, \tag{77}$$

wherein h stands for the quotient of Eqs. 74 and 75. This, however, is identical with Eq. 61 for the principal distance, and thus we are led

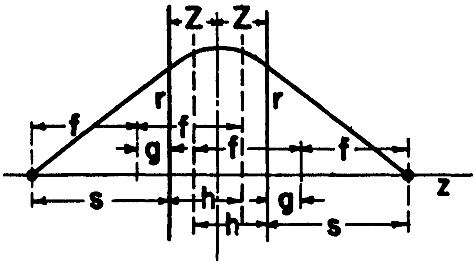


FIG. 21. Image formation by symmetrical ray.

to the conventional value of the magnification. We see further from Eq. 77 and Fig. 20 that the tangents to the central ray intersect the axis at the principal points given by the distance h from the lens surface. Hence, the central ray participates correctly in the image formation of the object.

The same can be shown for the symmetrical ray, as indicated in Fig. 16, or for any other ray. It may be noted, however, that the straight line diagram for the fictitious path of the symmetrical electron ray, as in Fig. 16(*a*), deviates considerably from the corresponding symmetrical path ordinarily assumed for light rays as in Fig. 16(*b*).

Instead of repeating the pattern of the last derivation, let us consider the intersection of the symmetrical ray with the axis as shown in

Fig. 21. At the lens surface the radius, with $\varphi = 0$ according to Fig. 6 and Eq, 33, is given by

$$\frac{r}{A} = \cos\left[\frac{1}{\surd 2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]. \tag{78}$$

The slope of the ray at the surfaces, by Eq. 55, is

$$\frac{dr}{dz} = \pm\frac{A}{B}\,\frac{\sin\left[\frac{1}{\surd 2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}}. \tag{79}$$

The ray intersects the axis at distances from the surfaces on both sides of the lens

$$s = \frac{r}{dr/dz} = B\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}\cot\left[\frac{1}{\surd 2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]. \tag{80}$$

If we add to this focal distance s the depth h of the principal plane beyond the surface, see Eq. 61, we obtain after the common factor $B\sqrt{2\left(1+\left(\frac{Z}{B}\right)^2\right)}$ a sum which can be condensed to

$$\cot\left[\frac{1}{\surd 2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]+\tan\left[\frac{1}{\surd 2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]=\frac{2}{\sin\left[\surd 2\sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{81}$$

Thus, the entire distance from principal point to focus is

$$s + h = 2\,\frac{\surd 2\,B\sqrt{1+\left(\frac{Z}{B}\right)^2}}{\sin\left[\surd 2\sinh^{-1}\left(\frac{Z}{B}\right)\right]} = 2f, \tag{82}$$

the quotient in the middle term being identical with the focal length of Eq. 62. Hence, every symmetrical ray, whatever its amplitude, cuts the axis on both sides at a distance of twice the focal length from the principal planes, or at a distance of focal length from the focus.

(*To be continued.*)

America's Most Powerful Aircraft Jet Engine in Production.—America's most powerful aircraft jet engine now in production, General Electric's torpedo-shaped TG-190, which is capable of producing approximately 10,000 hp. at the top speed of the new planes being built for its application, has been announced by the U. S. Air Force and the company's Aircraft Gas Turbine Divisions.

Known as the Air Force J-47A, the new engine is a "greatly advanced" development of the company's axial-flow TG-180 (J-35) jet engine which is the power plant of 10 different types of military jet-propelled fighters and bombers.

Although having approximately the same frame size as the TG-180, the TG-190 has many design improvements, a greater thrust rating and is substantially the same weight as the former unit. Compared with the slightly more than 4000-lb. thrust developed by the TG-180, the new engine has 25 per cent more power, according to the Air Force.

Future development of the TG-190 jet, according to engineers, promised to increase its rating to 6000-lb. thrust.

With a rating of approximately 5000-lb. thrust, which is equivalent to about twice as many horsepower under certain conditions, the TG-190 has been in production at the company's Lynn River, Mass., plant since the U.S.A.F. officially accepted the prototype in the fall of 1947. Engineers explained that "thrust" is a force, and the propulsive horsepower being developed depends upon the speed of the airplane. At an airplane speed of 375 miles an hour, one pound of thrust is equal to one horsepower of propulsive effort.

First units of the new engine have been undergoing extensive ground tests at the plant. Following completion of the military-required type-testing in the factory, the TG-190 now has been installed as the powerplant of the North American F-86A, an Air Force fighter recently revealed as having a speed of "over 650 miles an hour." The North American XF-86 now uses the General Electric-designed TG-180 jet engine.

Altitude and other tests with the new engine also have been underway aboard the B-29 flying laboratory operated by G.E. at its Flight Test Center near Schenectady, N. Y. As in the case of the TG-180, the TG-190 has been installed in the bomb bay of the Boeing Superfortress for flight testing under altitude conditions. The flying laboratory, operated by G-E engineers and manned by a Pan American crew, also is an Air Force project.

R. H. O.

Induction Heating Equipment Available Without Capital Investment.—For the first time, THER–MONIC Induction and Dielectric Heating equipment is made available to industry under a Rental-Purchase Plan. Under this plan, recently established by the Induction Heating Corporation, Brooklyn, N. Y., new or used (latter, when available) Induction or Dielectric Generators may be rented on a monthly basis with the option to buy at any time. A portion of rental payments forms an equity towards outright purchase.

This Rental-Purchase Plan is unique in the heavy equipment field in that it benefits the following—the manufacturer who insists upon "proof prior to purchase," the manufacturer who for some reason or other would rather not make an immediate capital investment and the manufacturer whose set-up is such as to prefer a continuing "pay-as-you-go" arrangement.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

THE EFFECT OF SCREENS IN WIND-TUNNEL WIDE-ANGLE DIFFUSERS.

Extensive experimental research under the direction of G. B. Schubauer and W. G. Spangenberg of the National Bureau of Standards has led to a more complete understanding of the effect of damping screens in wind-tunnel wide-angle diffusers.[1] Results have shown that the usual undesirable turbulence in such rapidly expanding ducts can be greatly reduced by suitable location of fine wire screens. Thus, wide-angle diffusers may now be utilized for increasing the cross section of an air stream within a very short length of travel.

A wind tunnel is designed to simulate, as nearly as possible, the motion of a body through still air by producing a smooth, uniform flow of air past the body. The air stream, drawn through a circuit of ducts by a propeller, is accelerated into the test section—the part of smallest cross-sectional area—through a nozzle or entrance cone. The greater the contraction ratio (ratio of the area of the larger end of the entrance cone to that of the smaller end) the less turbulent is the flow through the test section.

Recent wind-tunnel investigations at the Bureau have shown that damping screens [2] placed across the section upstream of the entrance cone are a very effective means for reducing turbulence in the test section of a tunnel. Whether screens are used or not, however, it is always desirable to have the contraction ratio as large as possible, and this has usually been accomplished in complete-circuit tunnels by making the return circuit of long, gradually expanding ducts known as narrow-angle diffusers.

A few years ago, it was observed that the cross section of an air stream could be increased in an unusually short length of travel through a rapidly expanding duct if the larger end of the duct was covered by a screen. This observation by members of the staff of the National Advisory Committee for Aeronautics was very significant, as years of experience had apparently shown that the only way an enclosed stream could be induced to increase its cross section uniformly was by means of a long, gradually expanding duct.

The conventional narrow-angle diffuser is a very long funnel-shaped tube with an included wall angle of about 8 deg. The term "wide-angle diffuser" is applied to ducts with walls diverging at much greater angles —for example, 30 deg., or perhaps even 90 deg. in extreme cases. If the transition from a smaller to a larger duct is made through a wide-angle diffuser, flow from the smaller to the larger duct ordinarily becomes very nonuniform because the air stream will not follow the walls of the diffuser. The flow may also whip from side to side and pulsate violently. These effects are summed up by saying that the diffuser fails to fill.

---

* Communicated by the Director.

[1] Complete details of this work are described in NACA Technical Note No. 1610 by G. B. Schubauer and W. G. Spangenberg.

[2] Damping wind-tunnel turbulence, *NBS Technical News Bulletin*, 31, 1 (1947).

Because of the importance in engineering applications of this little-understood phenomenon, an investigation of the mechanics of the process was undertaken by the National Bureau of Standards with the cooperation of the National Advisory Committee for Aeronautics. A system was set up in which several different wide-angle diffusers could be used to connect a duct 18 in. in diameter to a 36-in. duct, and air was drawn through the system by a centrifugal fan at speeds of about 100 ft. per second. By probing along diameters with pitot-static tubes, studies were made of the effects on the flow of fine wire screens (sometimes called wire cloth) placed at various cross sections of the diffusers. While the screens were all of smaller mesh than ordinary insect screen, the porosity to flow was about the same because of the finer wires.

From detailed analysis of data obtained in a 28-deg. diffuser, it became clear that the characteristic action of a screen is to spread a stream and in this way prevent separation of flow from the diffuser walls. However, the screen must be located sufficiently near to the natural separation point to be fully effective. When it is placed at the large end of the diffuser, it generally does not prevent separation and thus does not produce complete filling. Nevertheless, complete filling may always be attained by the use of several screens at properly chosen cross sections of the diffuser. By giving the diffuser walls a special shape, filling may be obtained with a single screen, but this method offers no advantage over the use of several screens in a duct of simple conical form.

Mathematical relations for efficiency—that is, the ratio of available energy leaving the diffuser to that entering—were found for diffusers containing screens. In general, efficiency is low compared to that of an 8-deg. diffuser without screens. Thus, while screens make it possible to spread a stream over a large cross section in a short distance, they do not permit retention of much of its initial kinetic energy.

Flow measurements were also made on multiple-screen arrangements in a 28-deg. and a 90-deg. diffuser to investigate a possible application of diffuser-screen combinations to the general problem of reduction of turbulence in wind tunnels. When damping screens are used upstream of the entrance cone to decrease turbulence, there is ordinarily an appreciable pressure drop across them because of their resistance to flow. However, if the duct area is increased by a wide-angle diffuser, the pressure drop through the screen may be reduced to negligible proportions, with a significant saving in the power required to drive the tunnel.

FRICTIONAL AND THERMAL BEHAVIOR IN JOURNAL-BEARING LUBRICATION.

The operation of a journal bearing under a steady state of temperature distribution with forced-feed lubrication is dependent upon a number of factors related both to the conditions of operation and to the geometry of the bearing. The relation of these factors to the performance of the bearing is very complex, involving both the rate at which the heat is generated in the bearing and the rate at which it is carried away. In an extensive investigation of these factors at the National Bureau of Standards [1] using a four-bearing friction machine, a method was developed for studying bearing performance through measurements of the combined frictional and thermal behavior in journal-bearing lubrication. These investigations also led to a practical graphical procedure for estimating safe bearing loads.

The work involved operation of bearings of two diameters, in two length-diameter ratios and three clearance-diameter ratios, and at two journal speeds. Three oils, of different viscosity, were used at three oil-inlet temperatures. Particular attention was given to the resultant effects of the combined hydrodynamic and thermodynamic actions involved in bearing operation with forced-feed lubrication.

The four-bearing friction machine, developed and used in former investigations at the National Bureau of Standards, consists essentially of four test bearings on a common shaft. The bearings are mounted in self-aligning ball-bearing swivels which are prevented from rotating by flat springs. The two outer bearings are fastened to the housing while the two inner ones are located in plates that slide in guides. The complete housing floats on the horizontal shaft and acts as a cradle dynamometer. Load is applied by hydraulic jacks under the two inner bearings.

Oil is fed to the bearings through the drilled shaft which has two oil holes at the longitudinal center of each bearing. A motor-driven pump and a relief valve maintain the oil-feed pressure. Bearing temperatures are measured by thermocouples imbedded in the metal at the loaded sides of the bearings.

A study of the frictional-torque data for operation under load indicates that when a bearing is operating at a constant speed, using a given oil at constant oil-inlet temperature and oil-feed pressure, an

---

NOTE: For further technical details of this work, see "Measurements of Combined Frictional and Thermal Behavior in Journal-Bearing Lubrication," by S. A. McKee, H. S. White, and J. F. Swindells, *ASME*, 70, 409 (1948).

[1] These tests were part of a research program carried out with the cooperation and financial assistance of the National Advisory Committee for Aeronautics.

increase in load produces an approximately proportional increase in frictional-torque. The increase in torque with unit increase in load is influenced chiefly by the viscosity of the oil, the oil-inlet temperature, the oil-feed pressure, the clearance-diameter ratio, and the length-diameter ratio.

The results also indicate that, under these operating conditions, an increase in load causes a proportional increase in the fluidity of the oil in the bearing. For a given oil, oil-inlet temperature, and oil-feed pressure, this increase in fluidity (reciprocal of viscosity) is a function of the speed of the journal, the diameter of the journal, the length-diameter ratio, and the clearance-diameter ratio. Empirical equations have been developed to define this relationship.

The straight-line relationship between fluidity and pressure provides a graphical means for estimating safe loads. The limit for safe operation is based upon a minimum value of a generalized operating variable, ZN/P, where Z is the absolute viscosity of the lubricant, N is the speed of the journal, and P is the pressure on the bearing. The intersection of a line representing a limiting value of this variable with the straight line drawn through the fluidity-pressure data for a given set of conditions gives directly the safe bearing pressure for these conditions.

In analyzing these results it is emphasized that the thermal behavior of a journal bearing operating with forced-feed lubrication is dependent upon a complex relation that involves heat losses by radiation, conduction, and convection, as well as the heat carried away by the oil leaving the bearing. Therefore, the numerical values obtained in any series of tests are dependent upon the heat-dissipation characteristics of the specific apparatus used. In the course of the bearing-research program, however, the operating conditions were such that it was possible to differentiate between the heat carried away by the oil and the heat losses through the apparatus itself.

These data show that under conditions typical of normal high-speed, high-temperature operation practically all of the heat generated in the bearing is carried away by the oil. Accordingly, the straight-line relationship between fluidity and pressure should hold reasonably well in most bearing applications. In these cases considerable information on bearing performance may be obtained by making a few observations of bearing temperature for various known operating conditions. This is of considerable significance since in many bearing installations friction measurements are not practicable.

STRESS-CORROSION TESTS OF HIGH-STRENGTH ALUMINUM ALLOYS.

The current demand for larger, heavier, and faster aircraft has resulted in the development of a number of new high-strength aluminum alloys. As these materials have become available, their resistance to corrosion has been determined by the National Bureau of Standards, usually at the request of the Armed Services. In connection with the program, several aluminum alloys introduced during the war—namely, the 75S-T, R301-T, R303-T, and the artificially aged 24S-T—were studied to determine their ability to withstand the highly corrosive conditions to which military planes are subjected in tropical areas.

In the course of this investigation, methods and apparatus for accelerated laboratory tests of stressed samples in corrosive solutions were developed and applied [1] by Hugh L. Logan and Harold Hessing of the Bureau staff. Marine-atmosphere exposure tests of the same materials under similar experimental conditions indicate a high degree of correlation between results of the laboratory tests and what may be expected in actual service.

The duralumin type of aluminum-copper-magnesium alloy known as 24S-T was introduced in 1932 and within a decade had largely replaced other aluminum alloys as sheet material for aircraft construction. Although its resistance to corrosion was generally satisfactory, under some conditions it was appreciably attacked upon exposure to a marine atmosphere or sea water. To increase corrosion resistance, it has been used in the form of a "clad" alloy, in which a duralumin sheet is sandwiched between, and integrally bonded to, two thin protective layers of commercially pure aluminum. This, however, results in some sacrifice of tensile strength. As this material has been found to be adequately resistant to corrosion in service, it was used as a standard of comparison in the marine-atmosphere testing of the new alloys.

With the outbreak of the war in 1939, even stronger aluminum alloys became desirable for aircraft use. Efforts were therefore made to develop alloys of greater tensile strength with adequate corrosion resistance. In the R301-T alloy, this result was sought by covering a duralumin-type alloy with an alloy-cladding layer of higher strength than commercially pure aluminum. In the 75S-T and R303-T alloys, tensile properties were improved by the addition of appreciable amounts of zinc. Use was also made of the discovery that elevated-temperature aging of the commercial flat 24S-T alloy results in a marked increase in yield strength.

In the Bureau's investigation of the stress-corrosion resistance of these new materials, standard ASTM flat tensile specimens with ½-in. reduced sections were tested both in the laboratory and in a marine atmosphere under a stress equal to three-fourths of the yield strength. In the laboratory, stressed samples of all alloys were continuously immersed in a sodium chloride-hydrogen peroxide solution (NaCl, 57 g.; 30-per cent H_2O_2, 10 ml.; H_2O, 990 ml.); those containing zinc as an alloying element (R303-T and 75S-T) were also exposed in a boiling

---

[1] For further technical details see "Stress-Corrosion Tests on High-Strength Aluminum Alloy Sheet," by Hugh L. Logan and Harold Hessing, *J. Research NBS*, 41, 69 (1948), RP1905.

6-per cent solution of sodium chloride. Unstressed specimens were subjected to the same corrosive conditions in order that the effect of stress in increasing corrosion damage could be evaluated. All clad materials were tested with the cladding intact since the purpose of the tests was to determine the resistance of the commercial alloy as actually used rather than that of the core material itself. Losses in ultimate tensile strength and per cent elongation were taken as criteria of corrosion damage.

Specimens supported vertically and stressed by means of weighted levers were tested in the sodium chloride-hydrogen peroxide solution in cells made of 60-mm. Pyrex cylindrical tubes fitted at each end into slotted bakelite disks. Rubber gaskets placed between the bakelite and the glass, and rubber stoppers molded with rectangular slots slightly smaller than the grip ends of the specimens, completed the cell assemblies. Samples up to 0.064 in. in thickness were kept in the sodium chloride-hydrogen peroxide solution for 24 hours. One-eighth inch specimens were immersed for 72 hr., the solution being renewed at the end of each 24-hr. period. Breaking of a specimen under stress automatically opened a knife switch in the circuit of a solenoid counter actuated once every 6 min. by a clock. The time required for failure of the specimen was thus recorded in units of 1/10 hour. After removal from the solution and cleaning, the specimens were broken in a hydraulic-type tensile testing machine to determine the tensile properties of the corroded material. Metallographic examinations were also made to determine the types of corrosion that had developed. For marine-atmosphere exposure tests, the specimens were supported and stressed in a similar fashion but were left exposed to the atmosphere.

Specimens immersed in boiling sodium chloride solution were stressed by bowing. This was accomplished by a device consisting of a threaded monel rod, insulating washers slotted to hold the ends of the specimen, and two nuts adjacent to the washers. Tension on the bowed specimens was controlled by adjustment of the nuts. The specimens were then placed in wide-mouthed flasks connected to reflux condensers and containing the sodium chloride solution. They remained in the boiling solution for 14 days unless earlier failures occurred. At the end of this time the specimens were removed, cleaned, and broken in tensile tests.

One of the difficulties in stressing specimens by bowing has been the fact that extremely small changes in the length of the chord connecting the ends of the specimen will make appreciable changes in the stress in the outer fiber. An instrument was therefore constructed by means of which the distance from the chord to the outer fiber could be measured to 0.0001 in. and the stress computed much more precisely than by earlier methods. Essentially, this device consists of two fixed pins and, between them, a third removable pin, which is attached to the plunger of a dial gage. Stresses computed from dial-gage readings differed from stresses obtained from wire strain gages (attached to the concave and convex surfaces of the specimen) by less than 2 per cent at three-fourths of the yield strength.

The results of the investigation indicate that flat, bare 24S-T aluminum alloy sheet aged four hours or longer at 375° F. is not susceptible to stress-corrosion cracking in either the laboratory or marine atmosphere tests, and is at least as resistant to the combined action of stress and corrosion as the commercially heat-treated but unaged material. It was found that aging of a sample of this alloy for three hours at 385° F. produced an increase in yield strength of about 25 per cent above an initial value of approximately 50,000 psi., an increase in tensile strength of about 3 per cent above an initial value of approximately 70,000 psi., and a decrease of about two-thirds in the initial elongation of 17 to 18 per cent. Similar results were obtained when the material was aged for 20 hr. at 350° F., 5 hr. at 375° F., or $1\frac{1}{2}$ hr. at 400° F.

The samples of the other alloys that were tested, with the exception of R301-T, were found adequately resistant to stress-corrosion cracking. The failure of the R301-T samples was the result of the penetration of stress-corrosion into the core material at the machined edges of the relatively narrow ($\frac{1}{2}$ in. wide) specimens that were tested. Such damage would probably not be significant in wide sheets, particularly those cut by shearing.

In general, it was apparent that the short-time laboratory tests developed by the National Bureau of Standards give a good indication of the corrosion resistance to be expected of the unclad alloys in a marine atmosphere. However, results of continuous immersion tests on clad material in the laboratory do not necessarily agree with those obtained by exposure in a marine atmosphere.

THE FRANKLIN INSTITUTE.

MUSEUM NOTES.

ATOMIC ENERGY.

The fears of many people who imagine that, by developing nuclear fission for the purpose of making destructive atomic bombs, the United States had seized a bear by the tail, will be set at rest by the exhibition planned for display in the Museum during the months of November, December, and January.

The exhibit, entitled "Atomic Energy is Here for Good," is sponsored by the Brookhaven National Laboratory and Associated Universities, Inc. Scientists have designed the exhibit to help make atomic energy, as Chairman David E. Lilienthal of the U. S. Atomic Energy Commission terms it, "everybody's business." A "world" that is our greatest frontier of science—the nucleus, or the heart of the atom—is brought into focus for the general public in demonstrations, models, displays, panel illustrations, and photomurals.

Featured in the exhibit is a model atomic pile which splits single uranium atoms before the eyes of the spectators. Energy released by this fission process is amplified into visual and sound effects. A tiny capsule of a mixture of radium and beryllium is used in the model to produce the neutrons that pierce the nuclei of uranium atoms in the fission chamber.

The power possibilities of nuclear energy are demonstrated in a model power plant showing how a nuclear reactor, or atomic pile, may some day be employed to generate electric power by means of the great heat release resulting from nuclear fission—heat that may be transformed into steam to drive electric generating machinery.

Other demonstrations and models depict other beneficial uses, stressing the great value of nuclear energy in scientific research and processes relating to the fields of medicine and biology, chemistry, agriculture, metallurgy, and numerous others. A striking feature of this section will be the radioactive flower, growing in soil fertilized by a phosphate containing radioactive phosphorus. A Geiger counter is used to demonstrate the course of "tracer" atoms into the roots, stem, leaves, and flowers of the plant.

A miniature Van de Graaf electrostatic generator, a machine that builds up high charges of static electricity, is one of several scientific instruments demonstrated. When visitors touch the machine when it is accumulating charges up to 125,000 volts, it will literally "make their hair stand on end."

A Wilson cloud chamber shows the paths made by atomic particles in a pressure chamber containing an atmosphere of fog.

To enable the general public to visualize the construction of the atom the exhibit will include a section devoted to basic physics. This portion will analyze the simplest atoms and go on to explain radioactivity, and why the nuclei of the heavy elements, such as uranium, are used in fission processes. Nor is the historical element of the subject neglected.

No pains have been spared to assist visitors in forming a decision of what is to be gained from nuclear energy.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays, and Saturdays from 9 A.M. until 5 P.M., and Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AGRICULTURE.

NELSON, JOHN A. AND TROUT, G. MALCOLM. Judging Dairy Products. Second Edition. 1948.

ASTRONOMY.

BERNHARD, HUBERT J.; BENNETT, DOROTHY A. AND RICE, HUGH S. New Handbook of the Heavens. Revised Edition. 1948.

BOTANY.

CROCKER, WILLIAM. Growth of Plants. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

FRANCIS, WILFRED. Boiler House and Power Station Chemistry. Second Edition. 1947.
STEINER, LUKE EBY. Introduction to Chemical Thermodynamics. Second Edition. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

HOLM, RAGNAR. Electric Contacts. 1946.
REED, MYRIE B. Alternating Current Circuit Theory. 1948.
WALKER, R. C. Photoelectric Cells in Industry. 1948.
WILSON, W. The Cathode Ray Oscillograph in Industry. Third Edition. 1948.

ELECTRONICS.

COSSLETT, V. E. The Electron Microscope. 1947.
HATSCHEK, PAUL. Electron-optics. Second Edition. 1948.

ENGINEERING.

EVANS, ULICK R. An Introduction to Metallic Corrosion. 1948.
FINCH, JAMES KIP. Trends in Engineering Education. 1948.

FIRE PROTECTION.

NATIONAL FIRE PROTECTION ASSOCIATION. Crosby-Fiske-Forster Handbook of Fire Protection. Tenth Edition. 1948.

HISTORY.

STEVENS, SYLVESTER K. Pennsylvania; Titan of Industry. 1948.

MANUFACTURE.

ANGUS-BUTTERWORTH, L. M. The Manufacture of Glass. 1948.
PHILLIPS, C. J. Glass, the Miracle Maker. Second Edition. 1948.

MATHEMATICS.

LASSER, J. K. Handbook of Accounting Methods. 1943.
RELTON, F. E. Applied Bessel Functions. 1946.

MECHANICAL ENGINEERING.

OLDENBOURG, RUFUS AND SARTORIUS, H. The Dynamics of Automatic Controls. 1948.

PUTNAM, PALMER COSSLETT. Power from the Wind. 1948.

MEDICINE.

DORLAND, W. A. NEWMAN. American Pocket Medical Dictionary. Seventeenth Edition. 1943.

METALLURGY.

AMERICAN SOCIETY FOR METALS. Metals Handbook. 1948.

PHOTOGRAPHY.

COX, ARTHUR. Optics; the Technique of Definition. Seventh Edition. 1947.
HENNEY, KEITH. Color Photography for the Amateur. Revised Edition. 1948.
MORTENSEN, WILLIAM. Flash in Modern Photography. Second Edition. 1947.

PHYSICS.

COLE, ROBERT H. Underwater Explosions. 1948.
GRIFFITHS, EZER. Methods of Measuring Temperature. Third Edition. 1947.
INGERSOLL, LEONARD ROSE; ZOBEL, OTTO J. AND INGERSOLL, ALFRED C. Heat Conduction. 1948.
MARTIN, L. C. Technical Optics. Volume 1. 1948.
UBBELOHDE, A. R. Time and Thermodynamics. 1947.

PLASTICS.

SOCIETY OF THE PLASTICS INDUSTRY. SPI Handbook. 1947.

RADIO.

SANDEMAN, E. K. Radio Engineering. Volume 1. 1948.
STURLEY, K. R. Radio Receiver Design. Volume 2. 1947.
TERMAN, FREDERICK EMMONS. Radio Engineers Handbook. First Edition. 1943.
TIBBS, CHRISTOPHER S. Frequency Modulation Engineering. 1947.
WILLIAMS, EMRYS. Thermionic Valve Circuits. 1947.

SCIENCE.

CHWISTEK, LEON. The Limits of Science. 1948.
HASLETT, A. W. Science in Transition. 1947.
HOLMSTROM, J. EDWIN. Records and Research in Engineering and Industrial Science. Second Edition. 1947.
STEELMAN, JOHN. Science and Public Policy; a Report to the President. 1947.
WOODBURY, DAVID O. Battlefronts of Industry; Westinghouse in World War II. 1948.

TEXTILES.

DISERENS, LOUIS. Neueste Fortschritte und Verfahren in der Chemischen Technologie der Textilfasern. Two volumes. 1946–1948.
MARSH, J. T. Textile Science. 1948.

PUBLICATIONS RECEIVED.

Introduction to Physics, by Harley Howe. Second ed., 599 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

Cathode Ray Tube Displays, edited by Theodore Soller & Others. 746 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $10.00.

The Nature of Patentable Invention, by John E. R. Hayes. Second ed., 187 pages, 15 × 23 cm. Cambridge, Mass., Addison-Wesley Press, Inc., 1948. Price, $5.00.

Design of Machine Elements, by M. F. Spotts. 402 pages, 16 × 23 cm., tables and drawings. New York, Prentice-Hall, Inc., 1948. Price, $6.65.

A CHECK LIST OF FRANKLIN IMPRINTS IN THE LIBRARY OF THE FRANKLIN INSTITUTE.

Compiled by

GEORGE E. PETTENGILL.[1]

The Franklin Institute has recently become indebted to Samuel H. McVitty of Salem, Virginia for a collection of books printed or written by Benjamin Franklin. This generous gift offers the Institute an opportunity to present a list of its Franklin imprints, the number of which has now been brought to sixty. While the collection is not large in proportion to the more than eight hundred titles listed by Campbell,[2] it may be said to be generally representative. There are examples from both the early and late years of Franklin's active printing career, with various highspots in the intervening period, as well as many of the later Franklin and Hall imprints. The most significant lack is any specimen from Franklin's Passy press in France. This is understandable, however, when one considers their comparative scarcity.

Benjamin Franklin began his printing career in Boston, apprenticed to his brother James in 1718. Among the books which issued from the latter's shop during the period of young Benjamin's apprenticeship are to be found the first examples of his work as a printer. The Institute now has several of these James Franklin imprints including Hodder's *Arithmetick* (X9).[3] This is "the first separate English textbook on arithmetic to be printed in America," according to Louis C. Karpinski in his *Bibliography of Mathematical Works Printed in America through 1850.*

Another book of considerable interest is the 1725 edition of William Wollaston's *The Religion of Nature Delineated* (X67), printed by S. Palmer in London. Franklin tells in his *Autobiography* of having worked on this book and of its having influenced him to write and print *A Dissertation on Liberty and Necessity, Pleasure and Pain* (X66 facs.). Only four copies of this pamphlet are known to exist as he destroyed almost all of them because of his subsequent dissatisfaction with the ideas expressed. A facsimile reprint is in the Institute's collection.

The next item of interest is Sewell's *History of the Rise, Increase and Progress of the Christian People called Quakers* (1) published by Samuel Keimer in 1728. Since this book was begun earlier it is probable that Franklin worked on it while he was still associated with Keimer. After he set up his own business with Hugh Meredith, he finished printing approximately the last third of the book for Keimer. Thus this is the first known book on which Franklin worked in his own shop.

Several of the Poor Richard almanacs are included, notably the one for 1758 (607) which was the last written by Franklin and which contains the first printing of "The Speech of Father Abraham." Better known perhaps as "The Way to Wealth," this was a collection of the many proverbs which had appeared in the previous almanacs. Its popularity is attested to by the fact that Paul Leicester Ford in his *Franklin Bibliography* in 1889 listed nearly one hundred and fifty editions. There is also a copy of the last Poor Richard almanac to bear Franklin's name as printer, that for 1766 (723).

Cicero's *Cato Major* of 1744 (290) in the first issue is important because it has been considered by some as the handsomest product of Franklin's press. An apparently unique item is David Evans's *A Short, Plain Help for Parents and Heads of Families,* 2nd edition, Philadelphia, 1740 (145.5). This is not listed in Campbell nor does any other copy appear in the Union Catalog at the Library of Congress. The last year of Franklin's association with the printing business he established, 1766, is represented by Letchworth's *A Morning and Evening's Meditation* (728).

---

[1] Assistant Librarian, The Franklin Institute Library, Philadelphia, Pa.

[2] Campbell, William J. "The Collection of Franklin Imprints in the Museum of The Curtis Publishing Company, with a Short-Title Check List of All the Books, Pamphlets, Broadsides, &c., known to have been printed by Benjamin Franklin." Philadelphia, Curtis Publishing Co., 1918.

The Curtis Collection is now in the University of Pennsylvania library. Its catalog will be subsequently referred to as Curtis, and the check list will be referred to as Campbell.

[3] Numbers appearing after the titles mentioned in this introduction are those used in the accompanying check list.

To facilitate comparison with Campbell's catalog of the Curtis collection and his check list, the various titles have been listed under the numbers assigned them by Campbell. The Evans item has been placed in its proper order and assigned the previous number with .5 added. In giving descriptions, a short-title entry following Campbell's check list has been used for those items which are fully described in Curtis. For those items not so described, twelve in number, a full collation comparable to that used in the Curtis catalog has been given. Notes have been added of interesting provenance or incomplete copies. All titles are from the McVitty gift unless otherwise indicated.

This is in no sense a supplement to Campbell's work, although it contains supplementary information. It purports to be merely a check list of the holdings of Franklin imprints in the Library of The Franklin Institute.

X2b

PEMBERTON, EBENEZER. A Discourse / Had / By the late Reverend and Learned / Mr. Ebenezer Pemberton, / Previous to the Ordination / Of the Reverend / Mr. Joseph Sewall, / At Boston, September 16. 1713. / Affirming and proving the Validity / of / Presbyterial Ordination. / [Ornament] / Boston: / Printed by J. Franklin, for S. Gerrish, / and Sold at his Shop near the Old Meeting- / House. 1718.

Title, 1 leaf; preface, (2); text, 15. 18 cm.

Generally issued with and as part of the following:

Prince, Thomas. A Sermon Delivered . . . on Wednesday October 1, 1718. At his ordination to the pastoral charge of the South Church in Boston, N.E. . . . To which is added A discourse of the validity of ordination by the hands of Presbyters, previous to Mr. Sewall's on September 16, 1713. By the late Reverend and Learned Mr. Ebenezer Pemberton. . . . Boston: Printed by J. Franklin for S. Gerrish . . . 1718. (8), 76, (4), 15

Presented by Hamilton Vaughan Bail.

X9

HODDER, JAMES. Hodder's / Arithmetick: / Or, That / Necessary Art / Made most Easy. / Being explained in a way familiar to / the Capacity of any that desire to / learn it in a little Time. / By James Hodder, Writing-Master. / The Five and Twentieth Edition, / Revised, Augmented, and above a Thou- / sand Faults Amended, by Henry / Mose, late Servant and Successor / to the Author. / Boston: Printed by J. Franklin, for S. Phillips, / N. Buttolph, B. Elliot, D. Henchman, G. Phil- / lips, J. Elliot, and E. Negus, Booksellers / in Boston, and Sold at their Shops. 1719.

Title, 1 leaf; dedication, (3); to the reader, (4–5); (6–7); contents, (8–10); text, 1–216. 15 cm.

Lacks portrait. Fragment missing from title page affecting border but not text.

X18

CARE, HENRY. English Liberties, / Or The / Free-born Subject's Inheritance; / Containing / Magna Charta, Charta de Foresta, / The Statute De Tallagio non concedendo, / The Habeas Corpus Act, and several o- / ther Statutes; with Comments on each / of them. / Likewise / The Proceedings in Appeals of Murder: Of / Ship-Money; Of Tonnage and Poundage. Of Par- / liaments, and the Qualification and Choice of / Members: Of the Three Estates, and of the Set- / tlement of the Crown by Parliament. / Together with / A short History of the Succession, not by any Here- / ditary Right: Also a Declaration of the Liberties of the / Subject: And of the Oath of Allegiance and Supremacy. / The Petition of Right; with a short but impartial Re- / lation of the Difference between K. Charles I. and the / Long Parliament, concerning the Prerogative of the King, / the Liberties of the Subject, and the Rise of the Civil / Wars. Of Trials by Juries, and of the Qualifications / of Jurors; their Punishment for Misbehaviour, and of / Challenges to them. / Lastly, / Of Justices of the Peace, Coroners, Constables, / Church-wardens, Overseers of the Poor, Surveyors of / the High-ways, &c. With many Law-Cases through- / out the Whole. / Compiled first by Henry Care, and contiued, [sic] with / large

Additions, by W. N. of the Middle-Temple, Esq. / The Fifth Edition. / Boston: Printed by J. Franklin, for N. Buttolph, B. / Eliot, and D. Henchman, and Sold at their Shops. 1721.

Title, 1 leaf; to the reader, (3–4); contents, (5–12); text, 1–288. 17 cm.

'A few leaves slightly torn and defective.

X41

WALTER, THOMAS. The sweet Psalmist of Israel. / A / Sermon / Preach'd at the Lecture held in Boston, / by the Society for promoting / Regular & Good Singing, / And for Reforming the / Depravations and Debasements / Our / Psalmody labours under, / In order to introduce the proper and true / Old Way of Singing. / Now published at the Desire of several Ministers / that heard it, and at the Request of the Society / aforesaid. / By Thomas Walter, M.A. / Minister of a Church in Roxbury. / [Three lines of quotation] / Boston: Printed by J. Franklin, for T. Fleet, / in Savage's Court on Cornhill, 1722.

(8), 28 pp. 17 cm. Collation according to Evans 2403. The first and last leaves are missing, possibly a half-title as found in the edition "for S. Gerrish" noted in Curtis, and pp. 27–28.

X66 facs.

[FRANKLIN, BENJAMIN.] A Dissertation on Liberty and Necessity, Pleasure and Pain. London: Printed in the Year 1725.

This is the facsimile reprint by Charles Whittingham, London, listed by Curtis.

X67

[WOLLASTON, WILLIAM.] The Religion of Nature Delineated. London: Printed by S. Palmer, 1725. Third edition.

The collation of this copy varies from that in Curtis, having 219, (11), the additional pages constituting an index.

1

SEWELL, WILLIAM. The History of the Rise, Increase, and Progress of the Quakers. Third edition, corrected. Philadelphia: Samuel Keimer, 1728.

35

(a) ARSCOT, ALEXANDER. Some Considerations Relating to the Present State of the Christian Religion. B. Franklin, 1732.

(b) ——— Some Considerations Relating to the Present State of the Christian Religion. Part II. B. Franklin, 1732.

42 facs.

SAUNDERS, RICHARD. Poor Richard, 1733. B. Franklin [1732]

Facsimile, published by George S. Appleton, Philadelphia [1849]

122

SAUNDERS, RICHARD. Poor Richard, 1739. An Almanack. B. Franklin [1738]

145.5

EVANS, DAVID. A Short, Plain / Help / For / Parents / And / Heads of Families, / To feed their Babes with the / sincere Milk of God's Word. / Being a Short, Plain / Catechism, / Grounded upon God's Word, and agree- / able to the Westminster Assembly's excellent Catechisms. / By David Evans, / A poor Labourer in the Gospel at Tredyffryn in / Pensilvania. / [Four lines of quotations.] / The Second Edition. / Philadelphia: / Printed by B. Franklin, 1740.

Title, 1 leaf; iii–viii; 9–46+. 14 cm.

There is at least 1 leaf missing after p. 46.

Unique copy. Not in Campbell, Evans, or Union Catalog at Library of Congress.

175

(a) WHITEFIELD, GEORGE. A / Continuation / Of the Reverend / Mr. Whitefield's / Journal / During the Time he was detained / in England, by the Embargo. / Vol. II. / [Ornament] / Philadelphia: / Printed and Sold by B. Franklin, in / Market-street, 1740.

Title, 1 leaf; poem, iii–iv; text, 5–63. 14 cm.

(b) ——— A / Continuation / Of the Reverend / Mr. Whitefield's / Journal / From / His Embarking after the Embargo. / To / His Arrival at Savannah in Georgia. / [Ornament] / Philadelphia: / Printed and Sold by B. Franklin, in / Market-street, 1740.

Title, 1 leaf; text, 67–205; Books sold by B. Franklin, (3) pp. 14 cm.

Some pages lack corners affecting the text in a few cases. See note to 176.

176

WHITEFIELD, GEORGE. A / Continuation / Of the Reverend / Mr. Whitefield's / Journal / From / His Embarking after the Embargo. / To / His Arrival at Savannah in Georgia. / [Ornament] / Philadelphia: / Printed and Sold by B. Franklin, in / Market-street, 1740.

Title, 1 leaf; text, 3–145; Books sold by B. Franklin, (3) pp. 14 cm.

Apparently both 175b and 176 lack pp. [3–4] of the ads, for other copies have five pages.

233e

[GRUBER, JOHANN ADAM.] Gründliche An- und aufforderung. B. Franklin, 1742.

235

BECHTEL, JOHANNES. Kurzer Catechismus. B. Franklin, 1742.

237

(a) PENNSYLVANIA. The Charters of the Province of Pensilvania and City of Philadelphia. B. Franklin, 1742.

(b) ——— A Collection of all the Laws of the Province of Pennsylvania Now in Force. B. Franklin, 1742.

(c) ——— An Appendix; Containing a Summary of Such Acts of Assembly as have been formerly in force. B. Franklin, 1742.

Two copies. Copy 1 was presented by Mrs. Arthur Biddle.

256

ZINZENDORFF, NIKOLAUS LUDWIG. B. Ludewigs / Wahrer / Bericht / De dato Germantown den 20sten Febr. 174½, / An seine liebe Teutsche, / Und / Wem es sonst nützlich zu wissen ist, / Wegen Sein und seiner Brüder / Zusammenhanges / Mit Pennsylvania, / Zu Prüfung der Zeit und Umstände ausgefertiget; / Nebst einem P.S. de dato Philadelphia den 5ten Martii; / Und einigen / Unsre Lehre überhaupt und dieses Schriftgen insonderheit / Erläuternden Beylagen. / Philadelphia, / Gedruckt bey Benjamin Franklin.

Title, 1 leaf; 3–26. 23 cm.

258

ZINZENDORFF, NIKOLAUS LUDWIG. Etliche Zu dieser Zeit nicht unnütze Fragen über Einige Schrift-Stellen, Welche . . . B. Franklin, [1742]

289 facs.

[FRANKLIN, BENJAMIN.] A Catalogue of Choice and Valuable Books, Consisting of near 600 Volumes. [B. Franklin, 1744]

"A facsimile of the unique copy in the Curtis Collection of Franklin Imprints, in the University of Pennsylvania Library, with a note by Carl Van Doren. Printed in an edition of 250 copies on the occasion of the meeting of the Bibliographical Society of America in Philadelphia, June 5th, 1948."

Catalogue, p. 1–16; note, 4 pages; colophon, 1 leaf; 1 blank leaf.

Presented by the University of Pennsylvania Library, through Dr. Charles W. David.

290

CICERO, MARCUS TULLIUS. M. T. Cicero's Cato Major. B. Franklin, 1744.

The first issue. In this copy "the upper corner of the title-page is restored and the four last letters of Cicero are pen and ink facsimile."

294

ESTAUGH, JOHN. A Call to the Unfaithful Professors of Truth. B. Franklin, 1744.

"Presentation copy from the author's wife to John Fothergill, the noted English Quaker physician, who acted as an intermediary between Franklin and the Penns in the Pennsylvania troubles. With the following autograph inscription on a front flyleaf: 'Jno Fothergil from E: E: 1744'."

295

[FRANKLIN, BENJAMIN]. An / Account / Of the New Invented / Pennsylvanian / Fire-Places: / Wherein / Their Construction and Manner of / Operation is particularly explained; / Their Advantages above every other / Method of Warming Rooms de- / monstrated; / And all Objections that have been raised against / the Use of Them, answered and obviated. / With Directions for putting them up, and for Using / them to the best Advantage. And a Copper-Plate, / in which the several Parts of the Machine / are exactly laid down, from a Scale of equal Parts. / Philadelphia: / Printed and Sold by B. Franklin. 1744.

Title, 1 page; (1); 1–37; (1) 22 cm.

Folded plate is lacking in this copy.

Signed "T. Godfrey 1744" on the title-page.

An early acquisition of the Library.

312

ARMSTRONG, JOHN. The Art of Preserving Health. B. Franklin, 1745.

P. 87–88 is torn and text damaged.

314

CADWALADER, THOMAS. An Essay on the West India Dry-Gripes. B. Franklin, 1745.

317 a–k

THE CONFESSION of Faith, The Larger and Shorter Catechisms. B. Franklin, 1745.

363

(a) MORGAN, ABEL. Anti-Paedo-Rantism. B. Franklin, 1747.

(b) AN APPENDIX to the Foregoing Work, Written by Another Hand. B. Franklin, 1747.

378

CURRIE, WILLIAM. A Treatise on the Lawfulness of Defensive War. B. Franklin and D Hall, 1748.

381

FISHER, GEORGE. The American Instructor. Ninth edition. B. Franklin and D. Hall, 1748.

A worn copy affecting text in spots. Three plates wanting.

411 a, b, c,

CHALKLEY, THOMAS. A Collection of the Works of Thomas Chalkley. B. Franklin and D. Hall, 1749.

Paging as given in Curtis is incorrect. Pages ccclxii–ccclxiv are in Roman rather than in Arabic numerals.

Two copies. One contains autograph signature of Chalkley tipped in.

417 facs.

[FRANKLIN, BENJAMIN.] Proposals Relating to the Education of Youth in Pensilvania. Philadelphia: Printed [by B. Franklin and D. Hall] in the year 1749.

"Facsimile reprint, with an introduction by William Pepper. Philadelphia, University of Pennsylvania Press, 1931."

Purchased through the Bloomfield H. Moore Memorial Fund.

447

MORGAN, ABEL. Anti-Paedo-Rantism Defended. B. Franklin and D. Hall, 1750.

458

PENNSYLVANIA. Anno Regni Georgii II. Vigesimo Quarto. General Assembly holden at Philadelphia, Oct. 14, 1750. B. Franklin, 1751.

Lacks p. 151.

460

Arndt, Johann. Des Hocherleuchteten Theologi. B. Fräncklin und J. Böhm, 1751.
Lacks one preliminary leaf and the last leaf, i.e. pp. 1355–1356. Various other pages are frayed and torn.

475

Saunders, Richard. A Pocket Almanac for 1752. B. Franklin and D. Hall [1751]
Interleaved.

478

Short, Thomas. Medicina Britannica: Or A Treatise On Such Physical Plants, As Are Generally to be found in the Fields or Gardens in Great Britain. B. Franklin and D. Hall, 1751.

486

Delaware. Laws of the Government of New-Castle, Kent and Sussex, upon Delaware. B. Franklin and D. Hall, 1752.

489

Jerman, John. The American / Almanack / For the Year of Christian Account / 1753, / Wherein is contained, / The Lunations, Eclipses, Planets Motions and Aspects, / Judgment of the Weather, the Time of the Sun's / and Moon's Rising and Setting, Seven-stars Rising, / Southing and Setting, High-water, Spring tides, / Fairs, Courts, Meetings, and other observable / Days. / Fitted to the Latitude of Forty Degrees North, and a / Meridian of Five Hours West from London, but may / without much Error serve from Newfoundland to South- / Carolina. / By John Jerman, Philom. / [Ten lines of quotation.] / Philadelphia: / Printed by B. Franklin and D. Hall. [1752]
Title, 1 leaf; text, (22) pp. 17 cm.
This is undoubtedly the item listed by Campbell as his 489 although the title is slightly different. Campbell gives Jerman, J. An Almanac for 1753. B. Franklin and D. Hall. [1752] and cites Hildeburn 1261, who locates no copy. The only other copy located is an imperfect one in the Library of Congress.

490

(a) **Johnson, Samuel.** Elementa Philosophica. B. Franklin and D. Hall, 1752.
(b) ——— Noetica: or the First Principles of Human Knowledge. B. Franklin and D. Hall, 1752.
(c) ——— Ethica: or the First Principles of Moral Philosophy. Second edition. B. Franklin and D. Hall, 1752.

497

Saunders, Richard. Poor Richard improved, for 1753. B. Franklin and D. Hall [1752]

499

(a) **Pennsylvania.** Votes and Proceedings of the House of Representatives of Pennsylvania, beginning Dec. 4, 1682. Volume the First. In two parts. B. Franklin and D. Hall, 1752.
(b) ——— Votes and Proceedings of the House of Representatives of Pennsylvania. Part the Second. B. Franklin and D. Hall, 1752.
Two copies. See also nos. 515 and 530.

502

Fisher, George. The American / Instructor: / Or, / Young Man's Best Companion. / Containing, / Spelling, Reading, Writing, and Arithmetick, / in an easier Way than any yet published; and how to qua- / lify any Person for Business, without the Help of a Master. / Instructions to write Variety of Hands, with Copies / both in Prose and Verse. How to write Letters on Bu- / siness or Friendship. Forms of Indentures, Bonds, Bills of / Sale, Receipts, Wills, Leases, Releases, &c. / Also Merchants Accompts, and a short and easy Me- / thod of Shop and Book keeping; with a Description of the / several American Colonies. / Together with the Carpenter's Plain and exact Rule: Shew- / ing how to measure Carpenters, Joyners, Sawyers, Bricklayers, Plaisterers, / Plumbers, Masons Glasiers, and Painters Work. How to undertake each / Work, and at what Price; the Rates of each Commodity, and the com- / mon Wages of Journeymen; with Gunter's Line, and Coggeshal's De- / scription of the Sliding-Rule. / Likewise the Practical Gauger made Easy; the Art of / Dialling, and how to erect and fix any Dial; with Instructions for Dying, / Colouring, and making Colours. / To which is added, / The Poor Planter's Physician: / With Instructions for Marking on Linnen; how to Pickle and / Preserve; to make divers Sorts of Wine; and many excellent Plaisters, and / Medicines, necessary in all Families. / And also, / Prudent Advice to young Tradesmen and Dealers. / The whole better adapted to these American Colonies, than any / other Book of the like Kind. / By George Fisher, Accomptant. / The Tenth Edition, Revised and Corrected. / Philadelphia: Printed by B. Franklin and / D. Hall, at the New-Printing Office, in Market Street, 1753.

Title, 1 leaf; iii–v; 1–384; five plates. 17 cm.

The fifth plate and various other pages are torn, and partially wanting.

510

SAUNDERS, RICHARD. Poor Richard improved for 1754. B. Franklin and D. Hall [1753]

515

PENNSYLVANIA. Votes and Proceedings of the House of Representatives of Pennsylvania, beginning Oct. 14, 1707. Volume the Second. B. Franklin and D. Hall, 1753.

Two copies. See also nos. 499 and 530.

527

SAUNDERS, RICHARD. Poor Richard improved for 1755. B. Franklin and D. Hall [1754]

530

PENNSYLVANIA. Votes and Proceedings of the House of Representatives of Pennsylvania, beginning Oct. 14, 1726. Volume the Third. B. Franklin and D. Hall, 1754.

Two copies. See also nos. 499 and 515.

541

EVANS, LEWIS. Geographical, Historical, Political, Philosophical, and Mechanical Essays. Philadelphia: Printed by B. Franklin and D. Hall, 1755.

Lacks the folding map.

544

[IBID] Second edition. Philadelphia: Printed by B. Franklin and D. Hall, 1755. And Sold by R. & J. Dodsley, in Pall-Mall, London.

Lacks the folding map.

An early acquisition of the Library.

591

BARCLAY, ROBERT. The Anarchy of the Ranters, And other Libertines. B. Franklin and D. Hall, 1757.

Two copies, the second of which contains Pike's *Epistle* (602) bound at the end.

602

PIKE, JOSEPH. An Epistle to the National Meeting of Friends, in Dublin. B. Franklin and D. Hall, 1757.

Removed from a bound volume. For another copy, see Barclay's *Anarchy of the Ranters* (591).

607

SAUNDERS, RICHARD. Poor Richard improved, being an Almanack for 1758. B. Franklin and D. Hall [1757]

620

HALL, DAVID. A Mite into the Treasury. B. Franklin and D. Hall, 1758.

634

DELL, WILLIAM. The Doctrine of Baptisms, Reduced from its Ancient and Modern Corruptions. Fifth edition. B. Franklin and D. Hall, 1759.

640

RUTTY, JOHN. The Liberty of the Spirit and of the Flesh Distinguished. B. Franklin and D. Hall, 1759.

Two copies.

646

DELL, WILLIAM. The Trial of Spirits, Both in Teachers and Hearers. B. Franklin and D. Hall, 1760.

Top of three leaves cut off affecting "The" on title page and page numbers.

648

LAW, WILLIAM. An Extract from a Treatise Called, The Spirit of Prayer. B. Franklin and D. Hall, 1760.

Two copies, the second bound with two other pamphlets not Franklin imprints.

658

CONTINUATION of the Account of the Pennsylvania Hospital, from May 1, 1754, to May 5, 1761. B. Franklin and D. Hall, 1761.

666

PENNSYLVANIA. Anno Regni Georgii III. Secundo. General Assembly holden at Philadelphia, Oct. 14, 1761–Feb. 17, 1762. B. Franklin, 1762.

Title page is damaged affecting several words. With this is bound 667.

667

[IBID.] And from thence continued to Mar. 26, 1762. B. Franklin, 1762.

Pages 205–210 mutilated and p. 211 wanting. Bound with 666.

673

WOOLMAN, JOHN. Considerations on Keeping Negroes. Part Second. B. Franklin and D. Hall, 1762.

With this has been bound "An Extract from John Woolman's Journal in Manuscript, Concerning the Ministry." n.p., n.d. 7 pp.

680

DUCHÉ, JACOB. The Life and Death of the Righteous. / A / Sermon, / Preached at / Christ-Church, Philadelphia, / On Sunday, February the 13th, 1763, / At the Funeral of / Mr. Evan Morgan. / By Jacob Duché, M.A. / One of the Ministers of the united Churches of / Christ-Church, and St. Peter's, Philadelphia, / and Chaplain to the Right Honourable the Earl / of Stirling. / Philadelphia: / Printed by B. Franklin, and D. Hall. / MDCCLXIII.

Title, 1 leaf; dedication, [iii]–iv, 5–24. 19 cm.

693

LIBRARY COMPANY OF PHILADELPHIA. The Charter, Laws, and Catalogue of Books, of the Library Company of Philadelphia. B. Franklin and D. Hall, 1764.

701

SAUNDERS, RICHARD. Poor Richard improved, being an Almanack for 1765. B. Franklin and D. Hall [1764]

Incomplete, three leaves missing, i.e. pp. [5–6], [33–36]

723

SAUNDERS, RICHARD. Poor Richard improved, being an Almanack for 1766. B. Franklin and D. Hall [1765]

Two copies, the second lacking pages [33–36.]

728

LETCHWORTH, THOMAS. A Morning and Evening's Meditation, Or, A Descant On The Times. A Poem. B. Franklin and D. Hall, 1766.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Abstract of **Blood Group Substance from Intestinal Mucosa and its Precipitation from Borate.**—CHARLES A. ZITTLE (*Archives of Biochemistry*, **17**: 195, 1948). A polysaccharide fraction from phosphoesterase preparations from calf intestinal mucosa was obtained by precipitation with borate and the relation of this polysaccharide to the blood group substances was demonstrated. Since the relation of the Type 14 *pneumococcus* polysaccharide to blood group substances is well established, Type 14 serum (horse) was used to show this relation. Intestinal mucosa has been shown, therefore, to provide an additional source for the blood group substances. The borate procedure provides a convenient and apparently specific method for the separation of blood group substances from proteins.

CURRENT TOPICS.

Biological Effects of High Energy Radiations.—Study of the biological effects of high-energy radiations, such as 100-million-volt X-rays, will by undertaken by Union College in a research project financed by the Atomic Energy Commission for the Office of Naval Research, President Carter Davidson announced recently. The scientific work will be directed by Dr. Leonard B. Clark, Chairman of the Union College Department of Biology.

The General Electric Company will contribute by making available at its own expense certain resources of its Research Laboratory, including use of the 135-ton betatron for producing X-rays up to 100 million volts, which will be used to radiate the biological specimens.

In announcing the project, President Davidson pointed out that while X-rays of one million volts energy are used in a number of hospitals for cancer therapy, far more penetrating radiations are now available from new machines. Their biological effects, however, are still little known, and it is expected that the new program will shed light on these problems.

Biggest of the radiation generators in the Laboratory, said Dr. C. G. Suits, G-E Vice President in charge of research, is the 100-million-volt betatron. Still more powerful equipment is now under construction. For lower energies there are betatrons and synchrotrons of 80 million, 50 million, and 10 million volts, as well as X-ray generators of more conventional sort for voltages of two million, one million, 400 thousand and still lower figures, down to 1000.

Dr. Clark outlined some of the problems to be attacked. "With X-rays of voltage under one million," he explained, "the effect on living cells seems due to ionization of the atoms of the tissue itself or of the media surrounding the cells. Outer electrons are knocked off these atoms and chemical changes are produced.

"However, at about one million volts, there begins to be evident another effect—the so-called pair formation—in which units of radiation, called photons, are converted by the electric field surrounding the nucleus of the atom into pairs of particles, electrons and positrons. At 100 million volts, almost all the energy absorbed produces this effect, and it is desirable to see whether there is any difference in the biological response produced as compared with low-voltage rays. Also, at such high energies one element may be transmuted into another, and it is barely possible that such transmutations may be biologically important.

"Another thing we want to study," he continued, "is that of possible long-range effects, for they should be known before any application is made to human patients. This can be done by exposing experimental organisms to doses of the radiation too weak to be immediately harmful. Then we can check them perhaps a year later to see if any adverse symptoms have developed."

The contract with the Office of Naval Research calls for three main subjects of investigation.

First is the study of the relations between biological effects and the wavelengths of X-rays from one million to 100 million volts. It is known that the wavelengths of ultraviolet radiation vary in their biological effectiveness, as in preventing rickets. It may be that X-rays from one million to 100 million volts may show similar variations in biological effectiveness.

The second topic assigned is the study of the depth-dose effect of such X-rays in tissues or tissue-like masses of living organisms, as blocks of compressed yeast cells. It is known, for example, that the greater the energy of the X-rays, the smaller is the proportion of rays scattered at the surface and the larger the percentage that penetrate deeply into the body. This brings about a difference in the effects produced at different levels within the tissue, and the distribution of this effect should be known.

The third principal subject concerns a study of the correlation of different particular physiological processes in the same living cells exposed to X-radiation. It is planned to make parallel studies of the effect of X-radiation on a number of physiological processes in the same cell.

R. H. O.

Ceilometers for Army and Navy.—Ninety ceilometers, electronic air safety devices which automatically measure and record the altitudes and densities of clouds, are being produced for the Army and Navy, General Electric's Lighting and Rectifier Divisions have announced.

The ceilometer, already in use by weather bureaus, the Armed Forces, and several civilian air fields, operates equally well by night or day. A super-high-intensity quartz mercury lamp, smaller than a cigarette, mounted at the focal point of a searchlight mirror, sends skyward a beam of light.

Clouds overhead reflect the beam back to earth, and a detector unit, which includes a photo-electric tube pickup, and which is tuned to the same frequency as the light source, automatically analyzes the reflections, and transmits to a recording device a record of cloud heights.

The detector, placed about 1000 ft. from the projector, scans the searchlight beam by swinging slowly from a horizontal to vertical position and back again five times an hour.

Ceilometers make possible an accurate and rapid check of cloud ceilings at all times, expediting landing and takeoff instructions, according to the engineers. Data recorded by the instrument are especially vital to safe flying at airports handling heavy traffic.

One year ago, 125 of the devices were delivered to the Army and Navy for installation at the nation's military fields.

R. H. O.

From 19 Per cent to 90 Per cent.—Typical of the contribution rural electrification makes to the industrial economy of the Nation is that of the 28 rural electric cooperatives in Ohio, says the Rural Electrification Administration. The first Ohio co-op that was financed by REA began operating June 15, 1936, says the U. S. Department of Agriculture.

These 28 Ohio co-ops, all of them financed by loans from the Rural Electrification Administration, own and operate 24,287 miles of line. They serve 96,093 consumers, including 820 rural churches and 260 rural schools.

Collectively, they paid $251,825 in taxes during 1947. They provide employment for 541 persons. They own and operate 171 automobiles and motor trucks.

Since the first loan to an Ohio rural electric co-op was approved by REA in September, 1935, REA has allocated a total of $36,486,272 to its Ohio borrowers. They have repaid $7,901,507 in principal and interest, including $1,167,573 on principal in advance of the date due.

Shortly before the establishment of REA in May, 1935, only 48,048, or 18.8 per cent, of the farms in Ohio were receiving central station electric service. Today, 198,512 Ohio farms, or 90 per cent of the number in the State, receive electric service.

Protect Forests from Fire.—Efforts to prevent fire damage to forest lands probably can never be perfect but the record shows they do greatly reduce the number of fires and hold down the destruction.

Last year's record clearly points out this fact, says the U. S. Department of Agriculture. The figures were compiled by the Forest Service. These show that 543,329,000 acres are now under organized forest fire protection. Unprotected acreage is listed at 111,026,000 acres, about 1 acre out of every 6.

This one-sixth of the acreage had 60 per cent or 3 out of 5 of all the fires on forest land (120,429 out of 200,799). More than seven times as many unprotected acres were burned over, and the damage on the unprotected acres was much larger than it was on the five times as many acres that were under organized protection.

The report emphasizes that 90 per cent of our forest fires are man-caused and therefore preventable.

Radio Signals Sent from Rocket at 72-Mile Height (*Science News Letter*, Vol. 54, No. 6).—A war-developed device, for sending automatic radio signals giving scientific information from a roaring rocket high above the earth, operated successfully at the White Sands Proving Grounds, Las Cruces, New Mexico, in a test just revealed, from an altitude of nearly 7 miles while traveling at over 2800 miles an hour.

The device that provides scientific data from aloft to ground-based recording instruments is known as a telemeter. In this test what is known as the Aerobee telemetering system was used. It has had many successful trials, but this is the first time that this miniature telemetering system has reached an altitude of 71.78 miles above the earth and attained a maximum velocity of 2830 miles an hour.

The information collected and sent to the ground-based recording instruments includes flight characteristics, motor performance and missile aspect, data on cosmic ray intensity, the quality of sunlight above the atmospheric blanket, and changes in the earth's magnetic field. The system has been used to transmit 24 different kinds of information. The records sent are continuous.

The system used in this test was evolved during the war by scientists of Princeton University and of the Applied Physics Laboratory of Johns Hopkins University at Silver Spring, Md. The application to supersonic missiles, those that travel faster than sound, was made at the Applied Physics Laboratory.

18 JAN 1949

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 246 NOVEMBER, 1948 No. 5

A STUDY OF CHEMICALLY DEPOSITED SILVER MIRRORS.

BY

RAYMOND MORGAN[1] AND RALPH D. MYERS.[1]

INTRODUCTION.

The production of silver mirrors by the chemical deposition of the metal on glass has become an art rather well developed by industry. Thousands of square feet are produced daily; nevertheless a knowledge of the nature of the chemical and physical actions is rather limited.

The object of this research was to study the nature of mirrors produced by various methods in order to gain information, particularly on: (1) the structure of the silver coatings; (2) the presence of impurities in the coatings; and (3) the associated chemical compounds in the cases of spoilage.

At present, there are used commercially modifications of three processes; (1) the Rochelle Epsom Salts, hot table process; (2) the Weber, cold pouring process; and (3) the spray method using hydrazine compound as a reducer for silver ammonia nitrate solution.

The spray process from the standpoint of mass production is much to be desired. However, the quality of the mirrors produced by this method has been thought by some to be inferior.

In particular it has been contended:

1. A mirror produced by the spray method exhibits less adhesion to the glass.
2. Spoilage is more conspicuous in mirrors produced by this method.
3. There is a difference in the reflectivity of the silver coating.

The spray process is very rapid and there is a question as to whether either the size of the particle or the structure of the silver mirror film

---

[1] Department of Physics, University of Maryland, College Park, Md.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute

varies basically with the speed of precipitation. Also, do the mirrors differ chemically due to the inclusion of impurities?[2]

In the production of silver mirrors, the glass surface is usually sensitized by the use of stannous chloride before the deposition of the mirror. There has been no exhaustive study of what happens in the use of the sensitizer. It plays a most important part in the production of mirrors. As a matter of fact, it is difficult to produce good commercial mirrors without the use of the sensitizer.

Extensive work on silver mirrors was done by V. Kohlshutter and E. Fishchmann (**1**).[3] However, the technique of the methods that they employed was not suitable for determining the physical nature of the silver films. Kohlshutter concluded that the surface adjacent to the glass consisted of colloidal silver and was amorphous in nature. As to the nature of colloidal silver, P. v. Scherrer (**2**) describes work on colloidal systems of silver which were found to crystallize with the same atomic arrangement that prevails in the macrocrystalline metal. Wartenberg (**3**) studied the effect of the sensitizing agent and concluded that in the silver process, the oxidizing agents have no effect on the efficiency of the sensitizing solution.

PROCEDURE.

Use was made principally of X-ray and electron diffraction. The electron diffraction patterns were made of thin films by allowing the beam to pass through the film and of surfaces by allowing the beam to strike the surface at grazing incidence. For convenience we shall refer to the first as patterns by transmission and to the second as patterns by reflection. A number of other techniques, also, were employed to a limited extent such as: (1) adhesion of the silver to the glass; (2) optical constants of the glass-silver interface; (3) spectrum analysis; (4) influence of certain corroding agents; and (5) effect of aging the mirrors in a vacuum in the presence of an active drying agent.

The following formulae were used for silvering:

Hot Table Process.

Solution 1.

454	g.	Silver Nitrate, Peacock Lab., Inc., c.p.
355	ml.	Ammonium Hydroxide, 28.42 per cent, Bakers, c.p.
5.45	1.	Distilled Water

Dissolve the silver nitrate crystals in the water. Add ammonium hydroxide slowly with stirring, filter clear, and store in clean glass bottle.

---

[2] Extensive work has been done on the inclusion of extraneous materials in electroplated silver, particularly by G. A. Hulett and T. W. Richards with the silver voltameter. The extraneous materials found included in the case of silver mirrors chemically deposited on glass are enormously greater than found in the case of electroplated silver films.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

Solution 2.

1590	g.	Rochelle Salt, Merck's U.S.P.
114	g.	Magnesium Sulfate, Merck's U.S.P.
3.64	l.	Distilled Water

Dissolve the chemicals in water and filter the solution. Store in a clean glass bottle.

When ready to pour mirrors, mix:

256	ml.	Solution 1
256	ml.	Solution 2
3.64	l.	Distilled Water

and pour on glass which has been leveled on the heated silvering table.

Cold Pouring Process.

Solution 1.

114	ml.	95 per cent Pure Grain Alcohol
114	ml.	Distilled Water
14.2	ml.	Sulfuric Acid, sp. gr. 1.84, Bakers, c.p.

Solution 2.

14.2	ml.	Solution 1
1.14	l.	Distilled Water

Bring to boil in glass container and add:

114	g.	Rock Candy, Merck's

Boil 10 min.

Solution 3.

21.6	l.	Distilled Water
114	g.	Sodium Hydroxide, 95 per cent Merck's U.S.P.

Dissolve thoroughly.

Solution 4.

1.14	l.	Distilled Water
128	g.	Silver Nitrate, Peacock Lab., Inc., c.p.
398	ml.	Ammonium Hydroxide, 28.42 per cent, Bakers, c.p.

Dissolve the silver nitrate in the water, add the ammonium hydroxide while stirring, and filter clear; store in a clean bottle.

Solution 5.

Mix equal volumes of solutions 3 and 4, always pouring 3 into 4. (This solution should *not* be stored—explosion may result.) When ready to make mirrors: mix 3.55 ml. of solution 2 with 568 ml. of solution 5 and pour on glass prepared for silvering. In carrying out the entire process, great care should be taken to prevent explosion, especially on hot days.

Spray Process.

Solution 1.

114	g.	Silver Nitrate, Peacock Lab., Inc., c.p.
227	ml.	Ammonium Hydroxide, 28.42 per cent, Bakers, c.p.

Add distilled water to make 4.55 l. Filter and store in clean glass container.

Solution 2.

42.5	g.	Hydrazine Sulfate, Peacock Lab., Inc., Silver Spray Reducer
45.5	ml.	Ammonium Hydroxide, 28.42 per cent, Bakers, c.p.

Dissolve chemicals in distilled water and dilute to 4.55 l. Filter and store in clean glass container.

When ready to make mirrors:

Equal volumes of solutions 1 and 2 are applied by spray gun to the prepared glass on an almost vertical easel. An air pressure of about 40 lb. was used. The spray gun was supplied by the Peacock Laboratories, Inc., of Philadelphia, Pennsylvania.

Preparation of Specimens.

The glass regularly used in preparing test mirrors was single strength, Grade B American window glass. A limited number of specimens for special study were made with microscope slides and cover glasses. The window glass was cut to 6-in. squares for silvering.

The surfaces of the glass were cleaned with a paste of Spanish whiting and city water, applied by a cotton swab. The whiting was rinsed off with water and the entire surface swabbed with the sensitizing solution of stannous chloride. The stannous chloride sensitizer contains 0.5 g. stannous chloride (Peacock Lab., Inc., c.p.) per 1000 ml. of distilled water. An excess of the sensitizer is applied by use of a rubber sponge to the clean surface. It is then rubbed in evenly over the surface. Then, by means of a soft bristle brush, the sensitized surface is scrubbed with a good supply of distilled water. It is usually noticed that once the unwanted excess of stannous chloride is removed, the brush does not glide over the glass as easily as it did at the beginning. Finally, the surface is rinsed with the distilled water and the glass is then silvered without allowing the surface to dry even in spots, before application of the silvering solution. A few samples were made, omitting the use of the sensitizing solution.

For the silvering by evaporation, the glass surface was similarly cleaned, but after removing the whiting, the glass was rinsed with distilled water and acetone, and dried in a bell jar. Care was used to prevent contamination by dirt and grease.

Tests Performed.

Duplicate mirrors for different thicknesses were made by each process. Mirrors were classified as thin when they were fairly transparent and thick when about the weight of commercial mirrors or perhaps a little heavier. Electron diffraction studies were made on both surfaces of the silver coating; that is, the rear surface or the surface remote from the glass and the glass-silver interface, or the surface adjacent to the glass. For the diffraction studies, two or more specimens (8 × 4 mm.) were made from each square to afford multiple checks on observation.

Tests were made on the adhesion of the silver films to the glass by stripping with Scotch tape. A film completely removed from the glass by the tape was considered to have poor adhesion, one yielding spotty removal of the silver was classed as of fair adhesion, and mirrors were rated as having excellent adhesion if the silver film could not be affected by the Scotch tape.

The effect of aging was studied by placing the mirrors in a vacuum for four months in the presence of a dehydrating agent. Some specimens were heated in a vacuum to about 550° C. to note any changes that might be produced.

A number of mirrors were corroded with H_2S, NaOH, NCl, HNO_3, H_2SO_4, and NaKTa, and the surfaces and corrosion products were examined.

The influence of the sensitizing agent was studied in connection with the corroding agents and by the comparison of mirrors produced with and without the agent.

For the removal of the silver foil from the glass in order to study the glass-silver interface, several adhesives were tried. In general, no method gave satisfactory results since there was always a large number of failures. Three rather successful methods were used.

A drop of low melting point wax was allowed to fall on the back of the mirror and harden. A small piece of rectangular sheet metal (8 × 4 mm.) was then warmed sufficiently and pressed into the wax until there was left remaining a very thin layer of wax between the sheet of metal and the silver surface. The whole was allowed to cool and after trimming off the excess wax, the plate was lifted from the glass. In a number of cases the silver foil would adhere to the layer of wax and would expose the glass-silver interface. It often happened that the silver foil would become insulated by the layer of wax and would accordingly take on a charge in the diffraction camera and prevent taking a picture.

TABLE I.—*List of Interplanar Spacings d/n in Angstroms.*[a]

Rear Surface						Glass Silver Interface			Data from Literature					
Spray		Rochelle Epsom Salts		Weber		Spray	Weber	R. E. S.						
Thin	Thick	Thin	Thick	Thin	Thick	Scotch Tape	Cu-Plated	Cu-Plated	Ag	SnO_2	SnO	Ag_2O	AgCl	NH_4Cl
Plate No. 22 b	Plate No. 21 b	Plate No. 39 a	Plate No. 117 b	Plate No. 36 b	Plate No. 34 a	Plate No. 18 b	Plate No. 50 a	Plate No. 48 b						
d/n I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I
							3.79 w							3.85 m
										3.40 vs				
												3.35 vvw		
		3.19 m											3.20 m	
3.08 m				3.09 m	3.13 vw			2.91 m			2.97 s			
2.76 s		2.73 s										2.72 vs	2.77 s	2.72 s
2.68 m				2.69 s	2.67 vvw		2.67 vw	2.67 m		2.67 vs	2.66 m			
														2.22 w
2.36 s	2.34 s	2.42 w	2.37 m	2.39 w	2.35 m	2.41 w	2.38 vw	2.36 m	2.36 s	2.35 m	2.39 w	2.36 s		
2.04 s	2.01 s		2.03 m		2.03 w	2.13 w	2.01 vvw	2.06 w	2.04 s					
1.90 s		1.94 vs		1.92 vs									1.96 s	1.92 m
					1.82 vvw	1.84 vvw	1.87 w				1.89 w			
		1.78 vvw						1.77 m		1.77 vs	1.79 m			1.72 w
		1.65 w		1.65 m				1.64 vvw				1.67 m	1.67 m	
1.59 vvw		1.59 m		1.57 s			1.59 vvw				1.59 m		1.60 m	1.57 m
							1.53 w							
1.45 s	1.44 s	1.46 w	1.45 m	1.44 s	1.45 m	1.47 w	1.42 vvw	1.47 w	1.45 s	1.43 s	1.48 w	1.42 m		
		1.37 vw										1.36 w	1.38 w	1.37 w
		1.30 s								1.32 w	1.33 w		1.27 vw	1.29 vw

[a] Intensity of reflection indicated by: vs—very strong, s—strong, m—medium, w—weak, vw—very weak, vvw—very very weak

TABLE I.—*Continued.*

Rear Surface						Glass Silver Interface			Data from Literature					
Spray		Rochelle Epsom Salts		Weber		Spray	Weber	R. E. S.						
Thin Plate No. 22 b	Thick Plate No. 21 b	Thin Plate No. 39 a	Thick Plate No. 117 b	Thin Plate No. 36 b	Thick Plate No. 34 a	Scotch Tape Plate No. 18 b	Cu-Plated Plate No. 50 a	Cu-Plated Plate No. 48 b	Ag	SnO_2	SnO	Ag_2O	AgCl	NH_4Cl
d/n I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I	*d/n* I
1.24 vs	1.22 vs	1.25 m	1.24 m	1.23 vs	1.26 m	1.26 m	1.31 w	1.25 m	1.23 s	1.21 m	1.22 w		1.24 m	1.22 w
											1.19 w			
1.18 vw	1.16 w	1.17 vvw		1.14 s	1.18 vvw		1.14 vw	1.15 vvw	1.18 w	1.16 w	1.17 w	1.18 vvw	1.13 w	1.16 vw
		1.12 m								1.10 w	1.07 w	1.08 vw		1.11 vw
1.05 vvw	1.02 vvw	1.06 vvw		1.06 vw	1.09 vvw		0.98 vw	1.02 vvw	1.02 vw	1.05 vw	1.03 w	1.05 vw	1.06 vw	1.03 w
		0.98 vvw										0.96 vvw	0.98 vw	
0.93 m	0.93 s	0.93 m	0.93 w	0.94 s	0.93 w	0.93 vw		0.94 w	0.94 w				0.94 vw	
		0.92 m		0.92 s					0.92 w			0.91 vvw	0.92 w	0.92 vw
		0.87 vw		0.87 vvw									0.87 vw	0.87 vw
0.83 w	0.83 w	0.83 w	0.84 w	0.83 m	0.84 vw	0.83 vvw	0.84 vw	0.84 vvw	0.83 vw				0.84 vw	
0.78 w	0.78 w	0.79 vvw	0.78 vw	0.79 vvw	0.79 vw	0.79 vvw		0.80 vvw	0.79 w					
				0.77 vvw										
0.72 vvw	0.72 vvw	0.73 vvw		0.73 vw	0.73 vvw		0.73 vvw	0.73 vvw						
				0.72 vw										
0.69 m	0.69 m	0.69 vw	0.69 w	0.69 m	0.69 vvw	0.69 vw		0.69 vw	0.69 vw					
				0.66 vvw			0.64 vvw							
0.62 vvw	0.64 vvw	0.64 vvw		0.64 vvw	0.62 vvw									
		0.62 vvw	0.62 vvw	0.62 vvw										
		0.60 vvw		0.60 vvw										
0.56 vvw	0.56 w		0.57 vvw	0.56 w	0.56 vvw	0.57 vvw	0.56 vvw							
0.54 vvw	0.54 vw	0.54 vvw		0.54 w	0.53 vvw									
0.53 vvw	0.53 vw	0.53 vvw		0.53 w										
0.49 vvw	0.49 vvw	0.49 vvw		0.51 vvw		0.50 vvw								

Some silver coatings could very readily be removed with Scotch tape but the flexible specimens were not easily mounted in the camera.

The best results were obtained by electroplating the rear surfaces of the mirrors with copper of sufficient weight to permit the copper and silver to be stripped from the glass as a unit. These pieces were then trimmed to fit directly in the specimen holder of the electron camera and mounted for reflection. The copper plating baths did not appear to affect the silver films since the interface of several mirrors, stripped by means of Scotch tape, produced the same diffraction patterns as companion films stripped by means of the electroplated copper.

The following method for the electroplating was used:

Oxalto Plating Bath.

20	g.	Copper Sodium Oxalate Dihydrate
15	g.	Sodium Sulfate
20	g.	Boric Acid
1	l.	Distilled Water

Dissolve chemicals in the water, and use at 20 ma. per square inch. (This solution may be stored and used over again until it becomes exhausted.)

Acid Plating Bath.

200	g.	Copper Sulfate Pentahydrate
75	g.	Sulfuric Acid
1	l.	Distilled Water

Dissolve chemicals in the water, and use at 40 ma. per square inch. (This solution may be used repeatedly until it becomes exhausted.)

When ready to plate on silver films:

Plate mirrors for 10 min. using the Oxalto Bath, and follow with Acid Bath plating for 12 hr. or until copper deposit has desired thickness.

RESULTS AND DISCUSSION.

A total of 486 electron diffraction pictures were taken from various specimens by both reflection and transmission. In Table I are given *d/n* data from various representative pictures.

From the rear surfaces of thick mirrors very few patterns were obtained that were not at least fairly distinct.

The strength and sharpness of the rings were greatest for the Weber process, next for the spray process, and least for the Rochelle epsom salt.

Also, the number of rings present in addition to the silver pattern was greatest for the Weber process, next for the spray, and least or practically zero for the Rochelle epsom salt. There is accordingly a definite indication that the structures of the silver coatings differ and also that the amount of extraneous materials present is greatest for the Weber process and least for the Rochelle epsom salt process.

In the case of very thin mirrors prepared by all three processes, it was regularly observed that the patterns obtained by reflection from the back surfaces were strong and sharp as contrasted with the patterns from the back surfaces of thick mirrors. This result is somewhat surprising since it was observed that the thin mirrors appear to possess greater metallic luster than the thick ones. In his study, Sivertz (4) likewise observed that the back surfaces of thin mirrors have a higher reflectivity than the back surfaces of thick mirrors. The fact that a surface of higher metallic luster gives a more distinct electron diffraction pattern than a surface of lower metallic luster is contrary to the observation of Germer (5) who found that in general, surfaces inclined to be smooth and possessing metallic luster do not give good diffraction patterns.

For comparison, the d/n values taken from Hanawalt tables for Ag and the compounds AgCl, NH_4Cl, Ag_2O, SnO, SnO_2 are given in Table I. In no case was a perfect line-to-line agreement obtained for any of the compounds such as was obtained for silver. However, the amount of any compound present was relatively small so that fair agreement is all that could be expected. The specimens from mirrors made by any one of the processes varied greatly, indicating that the extraneous materials were randomly distributed over the mirrors. It was indeed difficult to get two specimens that would give the same patterns except that ordinarily there would always be a silver pattern predominating.

The various pictures obtained from the glass-silver interfaces also varied greatly even from the same mirrors. In a very large number of cases either no patterns at all or just one, two, or three broad bands were obtained. Both the absence of lines and the broad bands are characteristic of polished metal surfaces. In other cases some very fine diffraction patterns were obtained. It was not possible, however, to establish by electron diffraction any differences among the glass-silver interfaces for the three processes of mirror making.

In a few cases excellent silver patterns of the glass-silver interface were obtained that were most extraordinary. The patterns indicated either a change in the lattice of the silver or an error in the reading of the voltage. From the conditions under which the camera is operated, it is practically impossible to make an error of sufficient magnitude in reading the voltage to account for the change. On the other hand, it is hard to explain the change in the silver lattice from 4.06 to 3.80 Å. If there is a change in the lattice of the magnitude noted, it would mean that the density of the silver is about 15 per cent greater. It would be expected that the structure of the silver surface adjacent to the glass might be changed, but the normal silver structure with a changed lattice would ordinarily not be expected.

Specimens from each of the three processes stripped from glass by copper plating were mounted in a camera adjacent to heating grids, and diffraction patterns were obtained by reflection from the glass-silver interface both before and after the grids were kept at a red heat (550° C.) for an hour. No changes in the patterns for any of the specimens were produced by the heat treatment.

Several pictures were made by reflection from the back surface of the silver and from the glass-silver interface for specimens when the sensitizing agent had been and had not been used. The patterns obtained when no sensitizing agent was used indicated the presence of silver alone.

A number of experiments were made in which the back surface of the silver on a mirror was treated with stannous chloride and a picture taken. No pattern at all was obtained. The surface was then washed with distilled water and faint rings were obtained. The surface was next washed with sodium hydroxide and a very satisfactory pattern of silver was obtained. This would indicate that the stannous chloride solution formed some kind of an amorphous layer over the surface.

In order to get an approximate idea as to how much of the stannous chloride remains on the glass surface after treatment, thin microscope-slide coverings were silvered and quantitative spectrum analyses were made of: (1) the glass not silvered; (2) the glass silvered but not sensitized; and (3) the glass silvered and sensitized.

The analyses showed a slight increase in the presence of tin when the sensitizer was used.

The suspected presence of a film between the silver and glass due to the sensitizing agent suggests that the optical constants of the interface might well be affected by the film. The validity of Drude's optical method in investigating transparent films on metals has been pointed out by Tronstad (6). Accordingly, observations by Drude's polarimetric method were made on the glass-silver interface of both sensitized and unsensitized mirrors. No difference for either the index of refraction or the reflecting power was found for the two surfaces.

Duplicate samples of all weights and types of silver mirrors were aged for four months by placing them in an exhausted vacuum-chamber in the presence of phosphoric acid anhydride. At the end of the period, visual inspection as well as an electron diffraction study, revealed no differences from freshly made mirrors. There was, however, a notable change in the adhesion of the spray process and the cold pouring process mirrors. These films, particularly the thick ones, were easily stripped from the glass by Scotch tape prior to placing in the vacuum. After vacuum aging, none of the mirrors made by any process, whether thick or thin, was affected by Scotch tape.

The spoilage occurs most frequently by the development of tiny white spots on the silver surface adjacent to the glass. The spots grow in size and often the silver film will start to peel from the glass. Quite frequently the back surface of the silver foil is free from any sign of spoilage, even when the glass-silver interface presents a decided spoilage appearance. There is thus a preferential spoilage effect, that takes place on the silver surface adjacent to the glass. This rather suggests the occurrence of some material between the glass and the silver, that accelerates the spoilage.

An effort was made to determine the nature of the spoilage process by making electron diffraction patterns of the glass-silver interface from spoiled mirrors. Several old mirror specimens supplied by the Hobbs Glass Limited were used in part in this test. As the specimens were taken from mirrors that showed increased spoilage both the number and sharpness of the lines generally increased. In one case where the mirrors had turned completely white and the silver was peeling from the glass, nice diffraction patterns of silver were obtained without the presence of any extra lines due to other materials. X-ray patterns made from the whitish silver scraped from the glass showed nothing but silver.

The failure to detect any chemical compounds associated with the spoilage of mirrors suggested the use of certain corroding agents and a study of the resulting compounds. A few mirrors were exposed to hydrogen sulfide gas. Electron diffraction patterns by reflection did not indicate the presence of any silver sulfide until the mirrors had been rather extensively corroded. Even though the corrosion effect on a surface was readily visible, the diffraction pattern consisted only of that of silver. However, when the surfaces became quite blackened and heavily corroded, there was present, along with the silver pattern, lines of silver sulfide.

Treatment with hot sodium hydroxide solution lifted the silver films from the glass. Even the thinnest films were loosened intact. The electron diffraction patterns by transmission showed the presence of silver only. The same result was obtained when a solution of the basic salt, sodium potassium tartrate, was used as the corroding agent.

Mirrors were corroded by means of hydrochloric acid until there was left on the glass nothing but a white powder. The X-ray diffraction pictures of the residues showed a rather strong pattern of silver and peculiarly enough nothing for silver chloride but a very excellent agreement for ammonium chloride. This would rather indicate the presence of some ammonium compounds on the silver which upon treatment with hydrochloric acid gave ammonium chloride. X-ray and electron diffraction pictures made of mirrors corroded by nitric and sulfuric acid showed the presence of many lines in addition to those of the silver pattern. The lines could not be identified as belonging to any known compounds.

During the corrosion by acids there was noted a marked tendency for the corrosion to take place preferentially at the glass-silver interface. White spoilage occurred and the mirror as such was destroyed before there was any appreciable effect on the rear surface, where the acids were applied.

The preferential spoilage process that was found so frequently to take place on the glass-silver interface suggested that the phenomenon may be due to the hygroscospicity of the sensitizing layer between the silver and the glass. A number of spray mirrors were placed under a bell jar in the presence of water. At first, the usual white spots developed and subsequently almost the whole of the mirror surfaces turned white. Also, the silver foil became more easily removable from the glass. In some cases it even peeled from the glass of its own accord. The presence of water apparently has a decided influence on the adhesion. Evaporated mirrors placed under the bell jar with the other mirrors showed no spoilage. However, in the case of very thin chemically deposited mirrors there was little tendency to show spoilage but the silver foil was found to have less adherence to the glass. The whitish nature would not be expected to be so pronounced due to the transparency of the silver film.

A marked difference between chemical and evaporated mirrors is the presence of foreign materials in the chemical mirrors. It, therefore, seems logical that the foreign materials may act as spoiling agents. The foreign materials, however, seem to occur in greater quantities on the backs of thin mirrors than on thick mirrors, and spoilage and poor adhesion is more prevalent in thick mirrors than in thin ones. The explanation of this may be that spoilage is a mechanical rather than a chemical process in the cases involved. The impurities imbedded in a layer adjacent to glass absorb moisture or suffer some mechanical change that causes a frosted appearance of a very thin layer adjacent to the glass. In thick mirrors this frosted layer is thick and is plainly visible. In thin mirrors the layer is not so thick and the mutilation is not so pronounced.

SUMMARY AND CONCLUSIONS.

A. Back Surface of Mirrors.

1. The back surfaces of silver mirrors deposited by chemical reduction by all three of the processes described are predominately crystalline silver with the presence of considerable quantities of various chemical compounds. The compounds ranging in amount from zero to about 10 per cent of the surface layer, varied from mirror to mirror and from place to place on the same mirror.

2. AgCl and NH_4Cl in varying amounts were found both together and separately on the rear surfaces of thin mirrors made by all three processes.

3. On Rochelle epsom salts thick mirrors, silver alone was usually found.

4. On thick mirrors made by the spray method there were frequently

present extra materials, but never in sufficient amounts to permit identification.

5. On Weber thick mirrors, considerable foreign materials were present. SnO_2, AgCl, or NH_4Cl were sometimes among them.

B. Glass-silver Interface.

1. The silver surface adjacent to the glass presented a crystalline nature in some cases and in other cases the nature of an amorphous or polished surface. The surface varied from mirror to mirror and from spot to spot in the same mirror.

2. There were present extra materials in all three types of mirrors, but not in sufficient quantities to make identification possible. However, in the case of Weber mirrors there was good indication of the presence of SnO.

3. The amorphous behavior may be due to one of three types of surface:

(*a*) The silver surface may take a truly amorphous structure due to the deposition on the amorphous surface of glass.

(*b*) The surface may behave as an amorphous one due to the smoothness produced by deposition of the silver on the smooth glass surface.

(*c*) The sensitizing solution of stannous chloride may produce an amorphous layer of stannic oxide or some other compound.

C. Adhesion.

1. The adhesion of freshly made mirror films is classed as follows:

Poor—Heavy and medium spray process mirrors
Fair—Thin spray process mirrors
Heavy Weber cold pouring process mirrors
Excellent—Very thin spray process mirrors
Medium and thin Weber cold pouring process mirrors
All R.E.S. hot table process mirrors

2. The adhèsion was greatly improved in the case of all spray and thick Weber processed mirrors by aging for four months in a vacuum in the presence of a strong dehydrating agent. There were no other detectable changes produced in the mirrors. It is believed that the improvement was due to drying and that there were no accompanying changes in structure or chemical composition.

D. Spoilage.

1. No evidence was found that ordinary spoilage is a chemical reaction.

2. The deterioration of mirrors is greatly accelerated by moisture.

3. Spoilage in the presence of moisture is either a mechanical or chemical process resulting from impurities imbedded between the silver and glass.

ACKNOWLEDGMENT.

This work was done under contract for the Hobbs Glass Limited of London, Ontario, Canada. The authors wish to express their appreciation to the Company for its cooperation and financial support. They also want to thank K. P. Gladney, formerly of that company, for his advice and continued interest in the work.

REFERENCES.

(1) V. KOHLSHUTTER AND E. FISHCHMANN, "Über das Spiegelsilber," *Liebig's Annals*, Leipzig, Vol. 387, p. 88 (1912).

(2) P. v. SCHERRER, "The Determination of the Internal Structure and the Size of Colloidal Particles by Means of X-Rays," in R. ZSIGMONDY, "Kolloidchemie," Third edition, Leipzig, Otto Spamer, p. 387 (1920).

(3) VON H. VON WARTENBERG, "Über die Beschleunigung der Glasversilberung mit Zinnchlorür," *Z. anorg. Chem.*, Vol. 190, p. 185 (1930).

(4) C. SIVERTZ, private communication.

(5) L. H. GERMER, "Diffraction of Electrons by Metal Surfaces," *Phys. Rev.*, Vol. 43, p. 724 (1933).

(6) LEIF TRONSTAD, "The Validity of Drude's Optical Method of Investigating Transparent Films of Metals," *Transactions of the Faraday Society*, Vol. XXXI, September, 1935, pp. 1151–1158.

ELECTRON LENSES OF HYPERBOLIC FIELD STRUCTURE.*

BY

REINHOLD RÜDENBERG.[1]

Part II.

SUMMARY.

Increasing lens voltage, higher than the incident electron voltage, at first blocks the beam and then converts the electron lens into a curved mirror, the focal properties of which are analyzed.

The embodiment of hyperbolic lenses is discussed by means of the boundary conditions which may be rigorously or approximately satisfied in actual designs. The effects of lateral apertures and windows on the imaging qualities of the field are considered in detail.

Expressions are derived for velocity-, angular- and aperture-defects, the latter being merely a surface effect of the hyperbolic field. They all depend on a few parameters only of the lens field and are relatively small in magnitude. An experimental verification of the field structure with partial compensation of the defects shows favorable imaging qualities.

Two-dimensional fields, constituting electron cylinder lenses, are considered in a similar way, leading to analogous formulations for the cardinal properties.

7. CURVED MIRROR EFFECT.

For an electrostatic field of such power that it exceeds the blocking condition of Eq. 46 the transmission constant B is imaginary, and the term of Eq. 31 with the constant C becomes controlling. By effect of the double-valued cosh-function, this term represents a mirror action in the z-direction (**8**).[2] In the r-direction, the oscillation of Eq. 30 holds true also in this case.

By elimination of the time t from Eqs. 30 and 31 and use of Eq. 32, the trajectories are

$$\frac{z}{C} = \cosh\left[\sqrt{2}\left(\cos^{-1}\left(\frac{r}{A}\right) - \varphi\right)\right];$$

or

$$\frac{r}{A} = \cos\left[\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{z}{C}\right) + \varphi\right]. \qquad (83)$$

Figure 22 shows the development in space of these electron paths, each defined by a certain value of the phase angle φ. All the trajectories penetrate the field up to an abscissa $z/C = 1$ and then return on another branch of the cosh-function. Thus, the plane $z = C$ represents a mirror surface at which the individual rays are reflected, their incidence and emergence, however, being tangential to this surface. The actual reflecting action is produced in the volume of the field, quite differently from the abrupt reflection of light at a surface. The individual ray for $\varphi = 0$ returns exactly on the incident path. All the other rays are incident on one side and reflected on the other side of this $\varphi = 0$ path.

---

* Paper read at the Philadelphia meeting of the Electron Microscope Society of America on December 11, 1947; Part I appeared in the October, 1948, issue of this JOURNAL, p. 311.

[1] Gordon McKay Professor of Electrical Engineering, Graduate School of Engineering, Harvard University, Cambridge, Mass.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

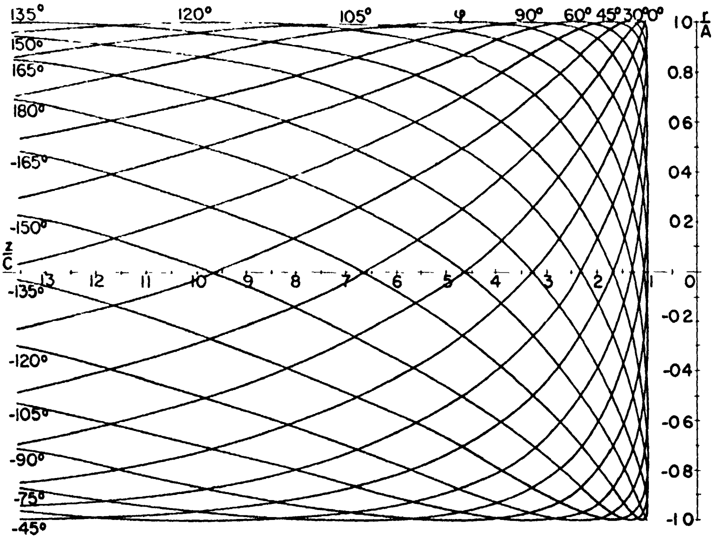


FIG. 22. Electron trajectories in hyperbolic mirror field.

The velocity of the electrons in the radial direction is given again by Eq. 38. In the axial direction, however, the velocity is

$$v_z = \frac{dz}{dt} = \mu C \sinh \mu t. \tag{84}$$

Eliminating t by Eq. 31 yields

$$\frac{v_z}{\mu C} = \sinh\left(\cosh^{-1}\left(\frac{z}{C}\right)\right) = \pm\sqrt{\left(\frac{z}{C}\right)^2 - 1}. \tag{85}$$

Figure 23 shows that the axial velocity again follows a hyperbola, only that compared with Fig. 9 the axes are turned by 90°. Hence, the axial velocity of any incident electron becomes zero and reverses its sign at the plane $z = C$, where merely radial velocity remains. Only for the trajectory $\varphi = 0$ do both axial and radial velocities vanish at the point of reflection, as seen from Fig. 22.

An electron entering the field at the plane $z = -Z$ with axial velocity V proceeds with velocity v_z according to Eq. 85, as shown by

Fig. 23. This gives the boundary condition

$$V = \pm \mu \sqrt{Z^2 - C^2}. \tag{86}$$

Using Eqs. 31 and 41 for μ and V gives

$$E_V = p(Z^2 - C^2). \tag{87}$$

Hence the reflection constant of the mirror is

$$C = \sqrt{Z^2 - \frac{E_V}{p}} = \sqrt{Z^2 - \left(\frac{V}{\mu}\right)^2}, \tag{88}$$

which is in contrast to Eq. 44 for the transmission constant of a lens.

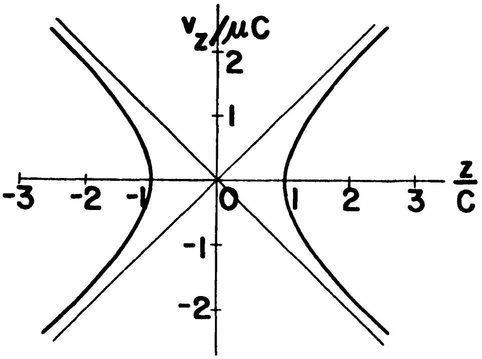


FIG. 23. Velocity distribution in mirror field.

Expressed by the parameters of the hyperbolic field, as shown in Fig. 5, the reflection constant becomes, with use of Eq. 26,

$$C = \sqrt{Z^2 - \frac{E_V}{E}\left(Z^2 + \frac{R^2}{2}\right)} = \sqrt{\left(1 - \frac{E_V}{E}\right)Z^2 - \frac{1}{2}\frac{E_V}{E}R^2}. \tag{89}$$

This again depends merely on three parameters, the half-thickness Z, the aperture-radius R and the voltage-ratio E/E_V which always must be larger than unity.

Since the electrons never can reach the center plane, it is not necessary to build up the complete field structure as is done in Fig. 5. In many cases it will be sufficient to limit the mirror on the rear side by an equipotential cone, as shown by Fig. 24(*a*). In this case the radius of the annular hyperboloid is $R = 0$, and the reflection constant thus will be

$$C = Z\sqrt{1 - \frac{E_V}{E}}. \tag{90}$$

It even might be sufficient to provide merely for one of the inner lateral hyperboloids, as shown by Fig. 24(*b*), with its vertex at a distance Z_1 from the center, which of course must be smaller than the distance C of the reflecting surface. In this case there is no annular hyperboloid, and therefore the difference of potential between two lateral hyperboloids determines the voltage E here applied. Thus, with use of Eq. 12 we have for $r = 0$

$$E = \Phi - \Phi_1 = p(Z^2 - Z_1^2) \tag{91}$$

and the power of the field is

$$p = \frac{E}{Z^2 - Z_1^2}. \tag{92}$$

The reflection constant of the unilateral mirror field is now by Eq. 88

$$C = \sqrt{Z^2 - \frac{E_V}{E}(Z^2 - Z_1^2)} = \sqrt{\left(1 - \frac{E_V}{E}\right) Z^2 + \frac{E_V}{E} Z_1^2}. \tag{93}$$

For $Z_1 = 0$ this value becomes identical with Eq. 90 when using the asymptotic cone.

We see from Fig. 22 that, similar to the lens trajectories of Fig. 6, at every point of the mirror field two incident trajectories are intersecting and that these same rays intersect again at other abscissas at the same distance from the axis. All these conjugate points are here on one side of the center plane. Any object point, therefore, appears imaged by the reflecting electron rays at consecutive image points, all having a magnification ±1. If the incident electron beam is uniform over the cross section, the image formation of the electrostatic mirror is free of aberration within the entire volume of the field.

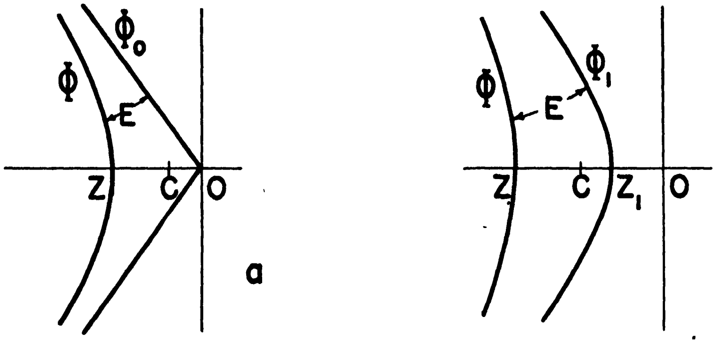


FIG. 24. Equipotential boundaries for electron mirror, (*a*) with rear cone, (*b*) with rear hyperboloid.

The cardinal points of a reflecting field which is limited by a plane radial boundary can be derived in a corresponding way as formerly used for lens fields. The condition of entrance of a parallel beam at $z = Z$, as in Fig. 25, is $r/A = 1$. Thus, from Eq. 83 we have for determination of the phase angle

$$\frac{Z}{C} = \cosh\left[\sqrt{2}(0 - \varphi)\right]; \qquad \varphi = \frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{Z}{C}\right). \tag{94}$$

These rays, therefore, follow trajectories within the mirror field, as given by

$$\frac{r}{A} = \cos\left[\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{z}{C}\right) + \frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{Z}{C}\right)\right]. \tag{95}$$

After reflection the electron paths are recessive and leave the lens field at the same boundary, $z = Z$, as indicated in Fig. 25. The exit radius, therefore, is given by

$$\frac{r}{A} = \cos\left[\sqrt{2}\cosh^{-1}\left(\frac{Z}{C}\right)\right]. \tag{96}$$

From there on the electron path is rectilinear, intersecting the axis at the focal point.

The focus will be at the surface of the field if in Eq. 96 $r/A = 0$, giving the condition

$$\cosh^{-1}\left(\frac{Z}{C}\right) = \frac{\pi}{2\sqrt{2}}; \qquad \frac{Z}{C} = 1.68; \qquad \varphi = \frac{\pi}{4}. \tag{97}$$

Herein the phase angle φ is obtained by the second Eq. 94. Hence, the focus lies outside the mirror field only if the width of the field is smaller than 45°. This can be graphically verified by means of Fig. 22.

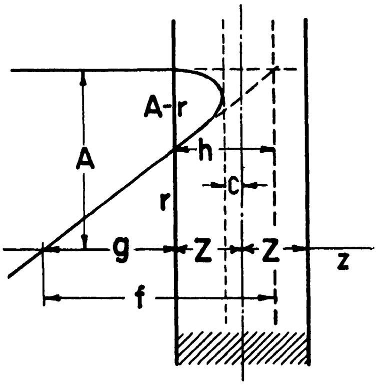


FIG. 25. Parallel incident ray and cardinal mirror points.

The slope of the rays is given by the derivative, the expression for which by Eq. 83 is generally

$$\frac{dr}{dz} = -\frac{A}{C}\,\frac{\sin\left[\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{z}{C}\right) + \varphi\right]}{\sqrt{2\left(\left(\frac{z}{C}\right)^2 - 1\right)}}. \tag{98}$$

This is different in both numerator and denominator from the corresponding slope in the lens field, see Eq. 55. At $z = C$ the denominator is zero determining the position of the reflecting plane.

For rays of parallel incidence, insertion of the phase angle φ from Eq. 94 gives the slope of the emerging ray at the abscissa Z

$$\frac{dr}{dz} = -\frac{A}{C}\,\frac{\sin\left[\sqrt{2}\cosh^{-1}\left(\frac{Z}{C}\right)\right]}{\sqrt{2\left(\left(\frac{Z}{C}\right)^2 - 1\right)}}. \tag{99}$$

The back-focus of the mirror field as seen by Fig. 25 is

$$g = \frac{r}{dr/dz} \tag{100}$$

applied to the exit point of the ray. With use of Eqs. 96 and 99 we obtain,

$$g = \frac{\sqrt{2}C\sqrt{\left(\frac{Z}{C}\right)^2 - 1}}{\tan\left[\sqrt{2}\cosh^{-1}\left(\frac{Z}{C}\right)\right]}. \tag{101}$$

This value is independent of the amplitude A, indicating freedom from any aperture defect. Figure 26 shows the change of the back-focus with half-width Z of the mirror, both distances plotted as ratio to the reflection constant C.

The main factors of the numerator in Eq. 101 are with use of Eq. 89

$$C\sqrt{\left(\frac{Z}{C}\right)^2 - 1} = \sqrt{Z^2 - C^2} = \sqrt{\frac{E_V}{E}\left(Z^2 + \frac{R^2}{2}\right)}. \tag{102}$$

Formally, this has the same value as Eq. 59, only that here the voltage ratio E_V/E is smaller than unity. If the mirror field is provided only on the side of incidence, as in Fig. 24(*a*) or (*b*), the main factors of Eq. 101 are with use of Eq. 93

$$C\sqrt{\left(\frac{Z}{C}\right)^2 - 1} = \sqrt{\frac{E_V}{E}(Z^2 - Z_1^2)}. \tag{103}$$

This gives for the same voltage across the electrodes a smaller back-focus g due to the higher power of the field.

The emerging rays reflected from a parallel incident beam, as in Fig. 25, seem to come from a depth h beyond the surface, defining the position of the principal reflecting plane of the mirror. This depth is determined by radius and derivative at the exit point as

$$h = \frac{A - r}{dr/dz} = \sqrt{2}C\sqrt{\left(\frac{Z}{C}\right)^2 - 1}\cdot\tan\left[\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{Z}{C}\right)\right]. \tag{104}$$

Herein Eqs. 96 and 99 are used, and the cosine and sine functions are combined into the tangent of half the argument. Figure 26 shows the change of this value with increasing width of the field. Where h is larger than Z, the principal plane is on the rear side of the field center.

The focal length of the electron mirror, according to Fig. 25, is

$$f = \frac{A}{dr/dz} = \frac{\sqrt{2}C\sqrt{\left(\frac{Z}{C}\right)^2 - 1}}{\sin\left[\sqrt{2}\cosh^{-1}\left(\frac{Z}{C}\right)\right]}. \tag{105}$$

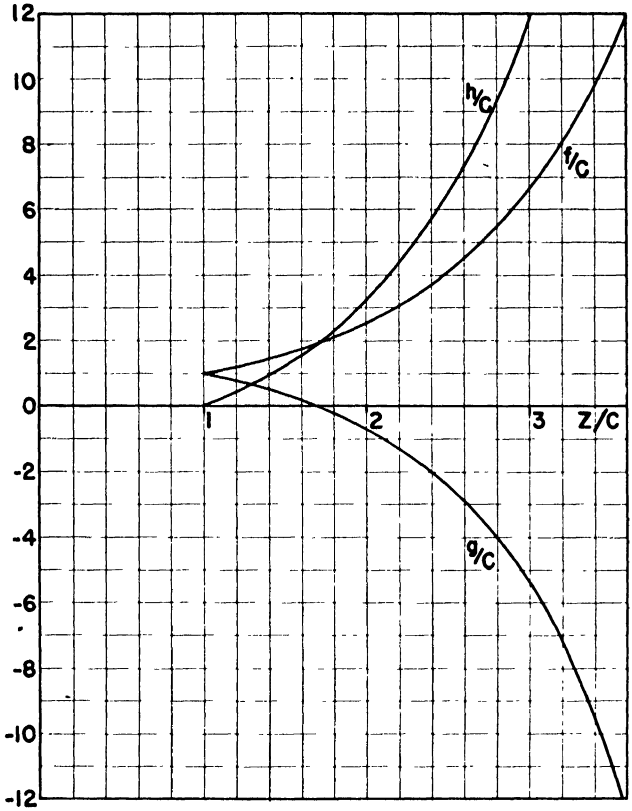


FIG. 26. Change of cardinal values with mirror thickness.

Its value as ratio to C is also plotted in Fig. 26.

When discussing these curves it should be noted that the abscissa, by Eq. 89, is

$$\frac{Z}{C} = \frac{1}{\sqrt{1 - \frac{E_V}{E}\left[1 + \frac{1}{2}\left(\frac{R}{Z}\right)^2\right]}}. \tag{106}$$

Thus, for values of E/E_V and Z/R which are near to the limiting condition (46), Z/C attains great magnitude. With higher-voltage and greater depth, however, Z/C decreases gradually towards unity. Electron mirrors of moderate value of Z/C, therefore, require considerable voltage E and small radius R with respect to Z.

In the range shown in Fig. 26, f is positive throughout and the focus lies before the principal plane. Thus, the field effect is converging, like that of a concave mirror in light-optics. For $Z/C = 4.66$, however, the sine in Eq. 105 changes sign. Hereby f becomes negative and a diverging electron mirror is formed, the properties of which are similar to those of a convex mirror in light optics. In the interval between $Z/C = 4.66$ and $Z/B = 4.55$, the focal length changes its sign many times and thus the performance here will be alternately that of a converging and diverging lens or mirror.

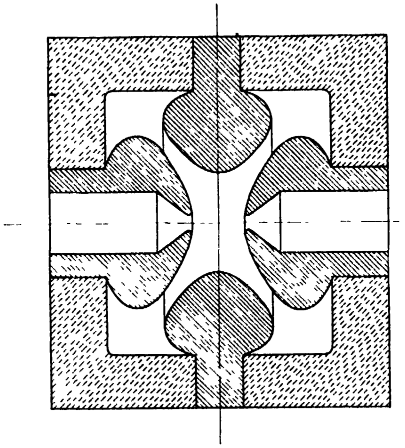


FIG. 27. Hyperbolic electrostatic lens, mushroom type.

If used for image formation of an object, the distances and the magnification for an electron mirror also follow Eq. 70 and 72. This can be derived by a method similar to that used for electron lenses and coincides with the ordinary relations for light-optical curved mirrors. In the electron case, however, the performance is that of a "thick" mirror.

If we consider a wide range of velocities of the incident electron beam, to cover both solutions of Eq. 31 which are separated by the blocking condition (46), we realize that a hyperbolic electron lens will act as a high-pass filter, and a hyperbolic electron mirror as a low-pass filter. They may be utilized singly or in combination for the purpose of selection of velocities.

8. BOUNDARIES OF THE FIELD.

There are several ways of arranging for the embodiment of an electron lens which insure the production of a hyperbolic field structure.

One method consists in building up some of the equipotential surfaces by conducting areas. Such hyperboloids may serve as electrodes or poles of the electron lens. If they are extended wide enough along the asymptotic cones in Fig. 2, they will secure the formation of the correct field structure in the interior space.

Figure 27 shows a mechanical setup of such a lens. The two lateral electrodes are at the same potential, and the intermediate annular electrode is charged to a different potential. The three electrodes may be kept in position by two insulating angular rings. The surfaces of the electrodes are shaped hyperboloidally according to any of the equipotential lines of Fig. 2, as expressed by Eq. 18. Axial apertures are provided in the lateral electrodes for the electron beam to enter and to leave the lens field. Far away from the beam the hyperboloidal boundaries of the active lens space are curved into softly rounded shapes of the electrodes in order that these will stand high voltage.

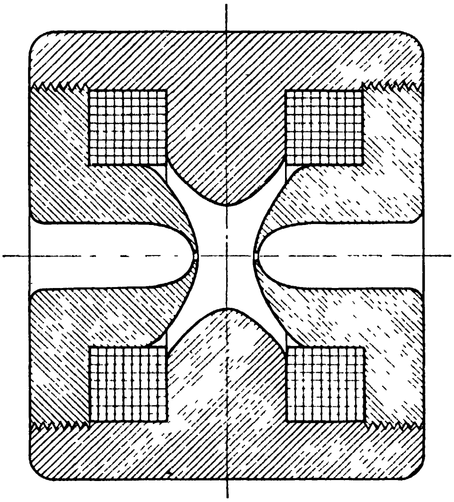


FIG. 28. Hyperbolic magnetic electron lens.

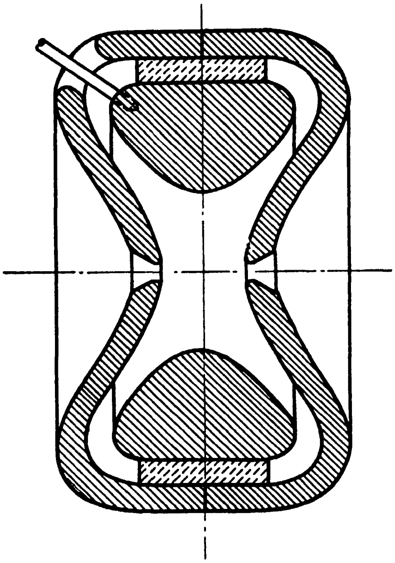


FIG. 29. Hyperbolic electrostatic lens, plate type.

A design of a magnetic hyperbolic lens is shown in Fig. 28. Two coaxial, current-carrying coils surround the lateral pole pieces exciting them in one polarity, which is opposite to the polarity of the intermediate annular pole. The inner surfaces of the magnetic circuit are shaped according to any of the hyperbolic lines shown in Fig. 2, except for an aperture in each of the lateral pole pieces. If the pole faces are made of highly permeable magnetic steel, the field in the interspace will have the structure of Eqs. 12 and 14. Instead of exciting such lens by electric current a permanent magnetization of the magnetic circuit may be used.

Another setup of an electrostatic hyperbolic lens is shown in Fig. 29. Here the lateral electrodes are formed as curved plates, joined together and fixed to the annular electrode by means of an insulating ring.

Another method of securing the distribution of a hyperbolic field is shown in Fig. 30. A number of concentric rings are arranged around the axis, each ring being charged to a potential given by Eq. 12 for the position z and r of the individual conducting ring. In Fig. 30 the necessary potentials of the various rings are indicated by the numbers, for a certain example.

Figure 31 shows a combination of equipotential surfaces and heteropotential rings, the latter arranged for correct termination of the field near the asymptotes. For each ring the potential, again, is determined by its radius r and its axial distance z, according to Eq. 12, and thus it is between the potentials of the main electrodes. By means of such intermediate rings the extension of the conducting surfaces along the asymptotes can be greatly reduced without detrimental reaction on the field structure near the beam. Therefore, in comparison to Fig. 27 there will remain a wider gap between the hyperboloidal electrodes, which is advantageous for the use of high lens voltage.

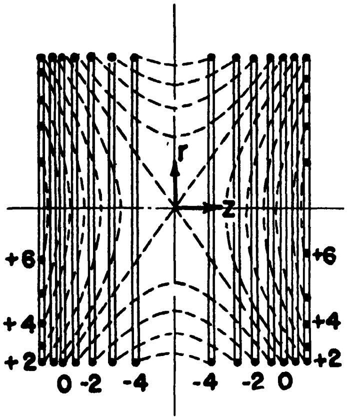


FIG. 30. Electrostatic ring lens of hyperbolic potential.

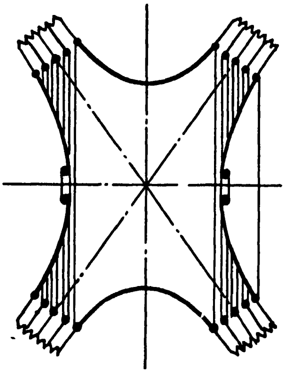


FIG. 31. Combined equipotential and ring hyperbolic lens.

A third method of producing a rigorous hyperbolic structure is shown in Fig. 32 by an example in which the field between the electrodes is terminated by the use of semi-conducting sheets. These are fixed here on the inner surface of two insulating rings, which may be used also for keeping the main electrodes at the proper distance. The voltage drop along the resistance coating can be adjusted so that the potential follows Eq. 12 along the sheet. Since the equipotential lines near the asymptotes of the field structure are nearly equidistant, a uniformly-conducting, high-resistance sheet may be used with good approximation. Any distorting influence on the field structure by the finite ends of the actual electrodes thus will be excluded. In consequence the gap between the electrodes can be kept wide, thereby increasing the high-voltage security of the lens.

Another example of a hyperbolic field structure, terminated by the means already described, is shown in Fig. 33. In this case the field section used lies entirely at one side of the origin and thus the two lateral equipotential hyperboloids are curved in the same direction. The field between these electrodes acts purely accelerating on the electron beam which may be produced by a photo-cathode on the left-hand side of this image converter. A cylindrical surface around the beam is formed by a co-axial insulating tube, coated with a semi-conducting layer. The resistance of this sheet varies in z direction so that the potential distribution along the surface will follow a quadratic law as prescribed by Eq. 12. For this purpose the unit resistance should be essentially proportional to the distance z. As indicated in Fig. 33, this can be secured by a decreasing thickness of the resistance coating with increasing z.

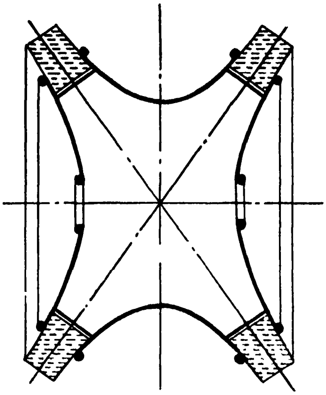


FIG. 32. Combined equipotential and semiconductor hyperbolic lens.

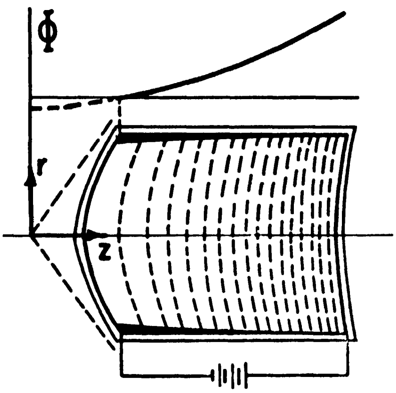


FIG. 33. Hyperbolic accelerating image converter.

In all those cases where an external electron beam enters or leaves the lens through the boundaries of the field, apertures must be provided in the centers of the lateral electrodes. An ideal way of arranging such an aperture for the electron beam, without causing any perturbation of the field structure, is the application of a window consisting of a thin conducting foil through which the electrons can pass. The foil may be coated for rigidity on a permeable material of low density. Arranged with very slight curvature as needed for the surfaces near the axis, such foils can be made of sufficient conductivity to act equipotentially and thus to secure the correct structure of the electrostatic field. Such a foil, to be sure, would show certain effects due to absorption and scattering of electrons in the beam. However, some metals exist which

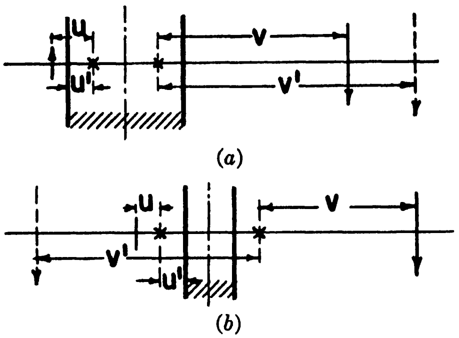


FIG. 34. Effect of lens windows on quality of image.

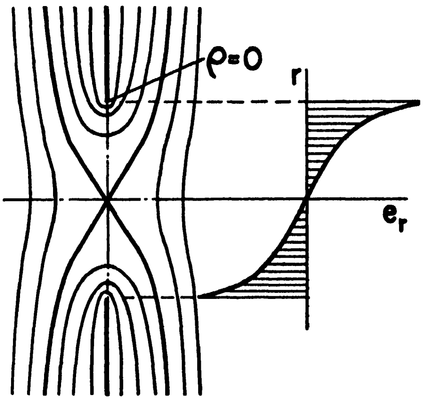


FIG. 35. Field structure in thin plate aperture.

do not show any marked pattern of structure in the electron beam, thus minimizing these detrimental effects.

Moreover, by proper placing of the windows, it is possible further to reduce these defects in the image formation. If the focus lies inside the lens field as in Fig. 34(*a*), due to a short focal length, a real image of the window would be produced not very distant from the image of the object, and the details of the latter might be greatly confused. However, if the focus is placed between object and window, as shown by Fig. 34(*b*), the image of the window will be virtual and will lie far away on the incident side of the lens. Thus, it will not substantially disturb the real image of the object on the exit side of the lens. Under such circumstances, even a granular window might give a diffuse background of the principal image, but no perturbation of its pattern.

If subsequent lenses are used, forming a compound instrument, the same viewpoints will hold for all of the front windows and for the inter-

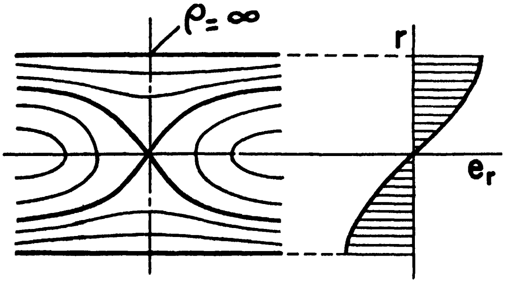


FIG. 36. Field structure in cylindrical aperture.

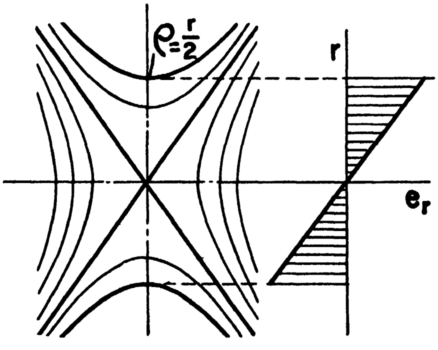


FIG. 37. Field structure in hyperboloidal aperture.

mediate images near the second or third electron lens. The rear windows will in all circumstances produce merely a diffuse background.

The conventional way of passing an external electron beam through the lens is the arrangement of openings in the centers of the lateral electrodes. It is well known, however, that such apertures produce additional lens fields of their own. Hence, they will give rise not only to supplementary focusing effects, but also to aberrations of the rays, due to the imperfect structure of their aperture field.

A proportional change of the radial field strength should be expected if the rim of the aperture were shaped between the two opposite extremes of Figs. 35 and 36 which have first been investigated by Ollendorff (3) and Gray (4). We know that a hyperbolic field, as shown in comparison by Fig. 37, will satisfy this condition. A rigorous application of the hyperbolic field structure to provide apertures in the lateral electrodes is not feasible. However, we realize by comparison of Figs. 35, 36, and 37 that the essential radial field strength will be widely linearized if we use the main geometric characteristic of a hyperbolic opening, namely its radius of curvature at the rim.

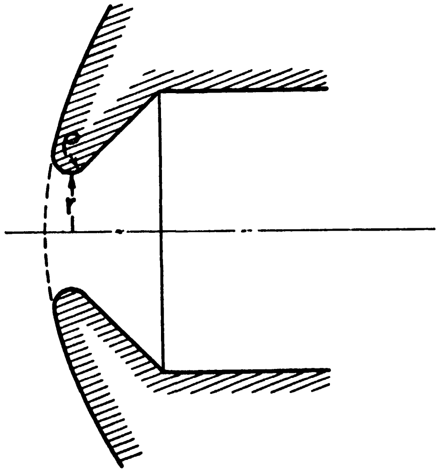


FIG. 38. Rounding off of lateral lens apertures.

Hence, if rounding off the lateral apertures by a radius in the meridian plane of half the value of the aperture radius in the radial plane, as indicated by Eq. 20, we will approximate a hyperbolic field structure within the aperture. Thereby we should avoid the formation of excessive field distortions and beam aberrations, as produced by the limiting cases of Figs. 35 and 36. Figure 38 shows the shape of a properly rounded off aperture in detail.

We have seen from the foregoing discussion that a uniform parallel electron beam incident on a hyperbolic field structure will not experience any aberration in connection with the transmission within the formative space of the field. However, at the entrance and exit of the field space it seems not to be possible to eliminate such perturbations entirely, but only to minimize them to a considerable degree. Hence, while volume aberrations of uniform beams are avoided in electron lenses as here described, surface aberrations remain which will be studied in greater detail. Further defects will be experienced if the beam entering the lens field is non-uniform in the velocity of its rays, either in magnitude or in the direction with respect to the lens axis.

9. VELOCITY ABERRATION.

If the incident beam is parallel but not homogeneous as supposed heretofore, containing electrons of a variety of speeds, there develops a velocity aberration of the rays throughout the entire structure of the hyperbolic field. Sometimes such a type of defect is designated as "chromatic aberration" in analogy to the light-optical phenomenon.

Our formulas for the cardinal distances hold true for any incident velocity. However, the transmission constant B, instead of being constant for the entire beam, now varies with the velocity voltage of the individual electron rays. We will derive the displacement of the focus with varying velocity and the corresponding change in focal length, as indicated in Fig. 39.

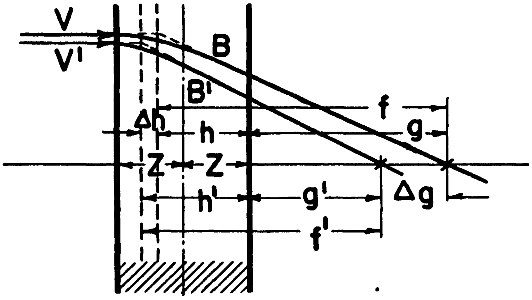


FIG. 39. Velocity aberration of electron rays.

For a lens of a given half-length Z we divide Eqs. 58 and 62 for the focal position and the focal length by Z and consider B/Z as the independent variable, which is directly determined by the incident velocity voltage E_V (see Eq. 49). In order to derive a simple interrelation, we substitute the right-hand side of Eq. 46 into Eq. 49 after denoting the blocking velocity voltage of Eq. 46 by E_{VO}. Thus we obtain the ratio of incident electron velocity voltage to incident blocking voltage

$$\frac{E_V}{E_{VO}} = 1 + \left(\frac{B}{Z}\right)^2. \tag{107}$$

From Figs. 15 and 17 there are plotted in Fig. 40 the ratios f/Z for the focal length and g/Z for the back-focus *versus* the ratio E_V/E_{VO} just mentioned. These curves show clearly the influence of the incident electron voltage E_V on the focal values while all other magnitudes are kept constant. In order to relate the incident electron voltage directly to the lens voltage E, rather than to the blocking velocity voltage, we derive from Eq. 46

$$\frac{E_V}{E} = \frac{E_V/E_{VO}}{1 + \frac{1}{2}\left(\frac{R}{Z}\right)^2}. \tag{108}$$

Thus, for different aperture ratios R/Z of the lens, the scales for E_V/E under the abscissa of Fig. 40 can be plotted. There is added a scale for Z/B corresponding to Eq. 107, all these abscissa scales being convenient for various purposes.

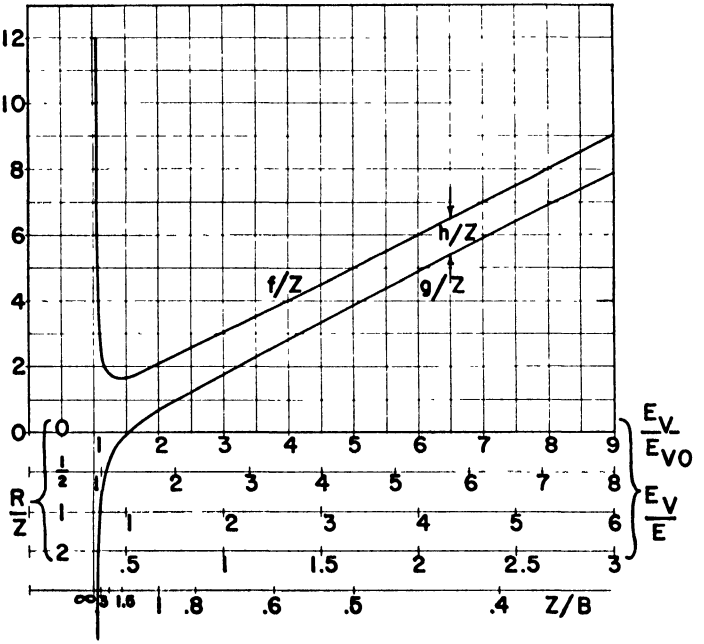


FIG. 40. Change of cardinal values with ray velocity.

For an incident electron voltage just equal to the blocking electron voltage, both focal length and back-focus are indeterminate. For rising electron voltage the back-focus g after initial oscillations increases steadily from negative values through zero to high positive values. The focal length f, however, decreases at first, reaches a minimum, and then increases, always remaining larger than the back-focus by the value h, the distance of the principal plane. The minimum of f/Z occurs at $E_V/E_{VO} = 1.44$, which corresponds to $Z/B = 1.5$ as seen from the scale at the bottom of Fig. 40. For this value, or for the corresponding values of E_V/E taken from the intermediate scales, the focal length is independent of small variations in the incident electron velocity.

However, the actual position of the focus, as given by the back-focus g, is for the most part of greater significance than f, as seen by discussion of Eqs. 70 and 72 for distance and magnification of the image. Hence, Fig. 40 shows that a truly akinetic or "achromatic" lens actually does not exist. However, in a certain range where f varies slightly and g varies not too much, the lens will have relatively favorable velocity qualities. On the other hand, for any given focal length or given magnification, the curves of Fig. 40 may be used to derive a lens of least total velocity aberration.

Whereas the graph of Fig. 40 shows the absolute values of the cardinal distances g and f for any spread of the incident electron velocity, another way of describing the velocity qualities of the lens is given by derivation of the differential aberrations.

For slight variation of the incident electron voltage we can express the displacement of the focus on the axis by

$$\Delta g = \frac{dg}{dB}\,\Delta B, \tag{109}$$

wherein the Δ's stand for a small change of the parameter. By Eq. 44 we correlate the change of B to the change of the incident voltage E_V

$$\Delta B = \frac{dB}{dE_V}\,\Delta E_V = \frac{\Delta E_V}{2Bp} = \frac{B^2 + Z^2}{2B}\,\frac{\Delta E_V}{E_V}, \tag{110}$$

wherein Eq. 43 also is used. On the other hand, differentiation of g from Eq. 58 yields

$$\frac{dg}{dB} = \frac{\sqrt{2}\cot\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sqrt{1+\left(\frac{Z}{B}\right)^2}} + \frac{2\frac{Z}{B}}{\sin^2\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{111}$$

If we substitute here for the sine and cotang their values from Eqs. 58 and 62, we obtain for the longitudinal velocity aberration of Eq. 109 the concise expression

$$\Delta g = \frac{1}{2}f\left(\frac{g}{f} + \frac{Z}{B}\frac{f}{B}\right)\frac{\Delta E_V}{E_V}. \tag{112}$$

Thus, the velocity displacement of the focus is proportional to the relative spread of the incident electron voltage and further directly depending on the values of back-focus and focal length ratios.

For example, with ratios $f/B \cong 2.1$ and $g/B = 0.6$, corresponding to $Z/B = 1.0$, the differential velocity displacement of the back-focus is

$$\frac{\Delta g}{f} = \frac{1}{2}\left(\frac{0.6}{2.1} + 1\cdot 2.1\right)\frac{\Delta E_V}{E_V} = 1.19\frac{\Delta E_V}{E_V}.$$

The change of the focal length with incident electron voltage is different from this value, since the principal plane is also depending on velocity, as indicated in Fig. 39. The differential aberration of f is

$$\Delta f = \frac{df}{dB}\Delta B. \tag{113}$$

Differentiation of Eq. 62 gives

$$\frac{df}{dB} = \frac{\sqrt{2}}{\sqrt{1+\left(\frac{Z}{B}\right)^2}\sin\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} + \frac{2\frac{Z}{B}\cos\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}{\sin^2\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{114}$$

Substituting again the trigonometric functions from Eqs. 58 and 62, and using Eq. 110 for ΔB, we obtain for the aberration in focal length

$$\Delta f = \frac{1}{2}f\left(1 + \frac{Z}{B}\frac{g}{B}\right)\frac{\Delta E_V}{E_V}. \tag{115}$$

The numerical values of the above example give

$$\frac{\Delta f}{f} = \frac{1}{2}(1 + 1\cdot 0.6)\frac{\Delta E_V}{E_V} = 0.80\frac{\Delta E_V}{E_V}.$$

This variation is smaller than the displacement of the focus, due to the shift of the principal plane.

We see from Eqs. 112 and 115 that both the velocity aberrations are depending on the focal length f as well as on the back-focus g. An approach to the blocking condition, $B = 0$, will make either aberration extremely large. On the other hand, the spread Δf of the focal length will be small for $g = 0$, the focus being in the lens surface, and even will vanish for $g = -0.61\ B$, the focus deep within the lens volume. The spread Δg of the back-focus, however, will not attain such small values.

10. HYPERBOLIC ABERRATION.

If the lens is built with equipotential boundaries in the region of the axis a hyperbolic aberration develops due to the hyperboloidal surfaces of the field. This type of defect is in some analogy to the "spherical aberration" of light-optical lenses.

In a parallel uniform electron beam of relatively large diameter entering the lens as in Fig. 41, the edge rays will traverse the field through a longer axial distance $\bar{Z}$ than the paraxial rays of length Z do. Therefore, they will experience a relatively greater deflection and a displacement of their focus. The half-width of the lens outside the axis is, by Eq. 18,

$$\bar{Z} = \sqrt{Z^2 + \frac{\bar{r}^2}{2}}, \tag{116}$$

wherein $\bar{r}$ denotes the radius of the incident ray. The symbol $Z = \sqrt{\Phi/p}$ shall now be restricted to the half-width of the lens on the axis.

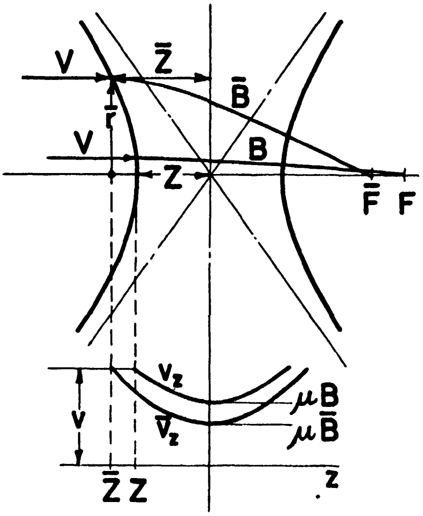


FIG. 41. Surface curvature of equipotential lens.

The transmission constant is depending on Z, as seen by Eq. 44. Thus, $\bar{B}$ will be smaller for extra-axial rays, namely

$$\bar{B} = \sqrt{\frac{E_V}{p} - \bar{Z}^2}. \tag{117}$$

Substituting $\bar{Z}$ from Eq. 116 and then B from Eq. 44 yields

$$\bar{B} = \sqrt{B^2 - \frac{\bar{r}^2}{2}}, \tag{118}$$

wherein the symbol B shall now be restricted to the transmission constant along the axis. Thus, for extra-axial rays $\bar{B}$ depends upon the radius of incidence.

A parallel incident edge ray, as in Fig. 41, will leave the lens at a smaller radius than it had at entrance. Thus, the exit abscissa will be smaller than the entrance abscissa, and this will lead to an unsymmetrical behavior of the lens, which is outside the scope of this paper. Therefore, we will consider here only the performance of symmetrical rays and central rays, as represented in Figs. 42 and 43. Both of these rays will pass through the same distance $\bar{Z}$ on the two sides of the lens.

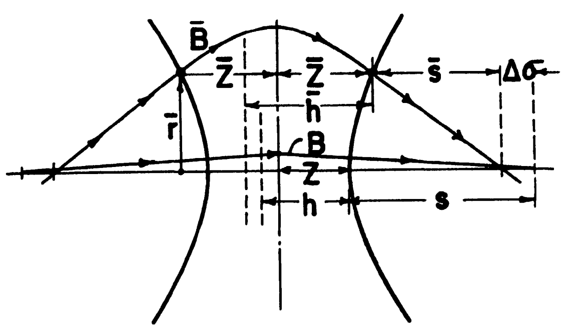


FIG. 42. Symmetrical ray in equipotential lens.

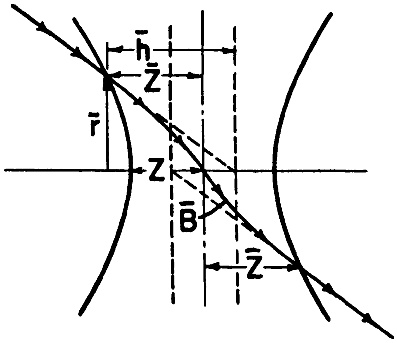


FIG. 43. Central ray in equipotential lens.

For increasing separation of the rays from the axis, the transmission constant $\bar{B}$ decreases, according to Eq. 118, and vanishes for a radius

$$r_0 = \sqrt{2}B. \tag{119}$$

For greater radii the electrons will be reflected, and thus the radius r_0 constitutes a natural limiting aperture. Substituting r_0 into Eq. 118 yields

$$\frac{\bar{B}}{B} = \sqrt{1 - \left(\frac{\bar{r}}{r_0}\right)^2}. \tag{120}$$

The transmission constant, therefore, follows a circle diagram over the radius as indicated in Fig. 44.

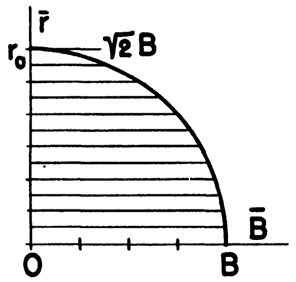


FIG. 44. Extra-axial transmission constant *versus* separation from axis.

Introducing the value of Eq. 119 into Eq. 116 yields

$$\frac{\bar{Z}}{B} = \sqrt{\left(\frac{Z}{B}\right)^2 + \left(\frac{\bar{r}}{r_0}\right)^2}. \tag{121}$$

For a radius $\bar{r}$ of 10 per cent of the limiting radius r_0 the transmission constant decreases by ½ per cent, for a radius of 20 per cent by 2 per cent and for a radius of 30 per cent by 4.5 per cent. The same percentage increase occurs in the axial path length of the ray through the lens, if the ratio Z/B is taken as unity.

We now can determine the cardinal distances for the edge rays of a wide beam by inserting in Eqs. 58, 61, 62 and 80 of sections 5 and 6 the extra-axial values $\bar{Z}$ and $\bar{B}$ instead of the axial values Z and B. All these formulas contain as a common factor the expression (59) which is for any ray, with use of Eqs. 116 and 118,

$$\sqrt{\bar{B}^2 + \bar{Z}^2} = \sqrt{B^2 + Z^2}. \qquad (122)$$

Thus, the finite beam radius has no effect on this factor, the value of which remains invariant with respect to $\bar{r}$. Hence, the width of the beam affects merely the argument $\bar{Z}/\bar{B}$ of the trigonometric-hyperbolic functions. With use of Eqs. 120 and 121, this is

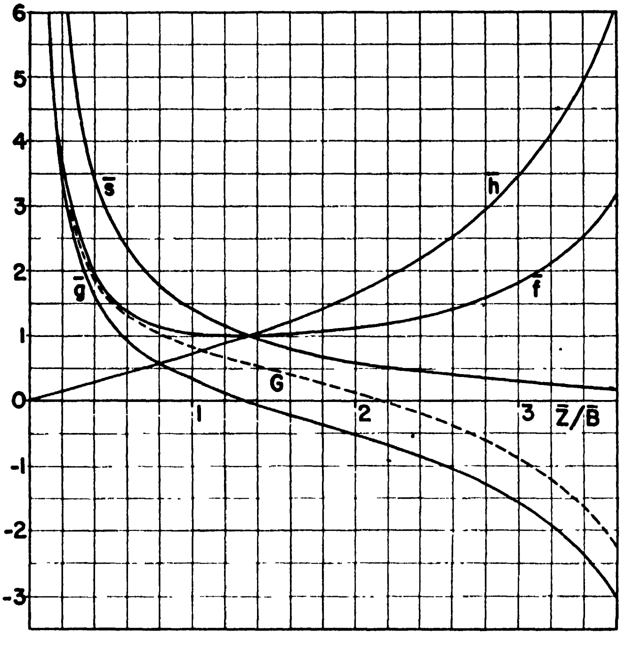


FIG. 45. Change of cardinal values with separation of ray from axis.

$$\frac{\bar{Z}}{\bar{B}} = \sqrt{\frac{\left(\frac{Z}{B}\right)^2 + \left(\frac{\bar{r}}{r_0}\right)^2}{1 - \left(\frac{\bar{r}}{r_0}\right)^2}}. \qquad (123)$$

For equal change of $\bar{r}/r_0$ this argument varies less with application of lenses thick in Z/B, than of thin lenses.

Figure 45 represents by the full-line curves the four characteristic functions according to which $\bar{f}$, $\bar{g}$, $\bar{h}$, and s change if the argument $\bar{Z}/\bar{B}$ varies over any range. They show directly the longitudinal aberrations of the cardinal distances depending on $\bar{Z}/\bar{B}$ if any axial value Z/B is chosen. Only the focal length $\bar{f}$, due to its sine function, experiences a minimum at the point $\bar{Z}/\bar{B} = 1.35$, indicating a lens free of focal length defect. This ratio, incidentally, would bring the focus into the boundary of the lens, $\bar{g}$ being zero here. However, $\bar{g}$ and $\bar{h}$ vary even at this point and thus the image formation, as determined in addition to f by either g (Eqs. 70 and 72) or h (Eqs. 71 and 73) yet will not be perfect for both paraxial and edge rays.

The image formed by the edge rays has position and magnification both different from those of the paraxial image, as indicated by Fig. 46.

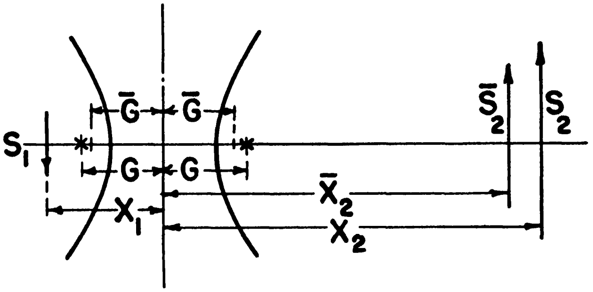


FIG. 46. Image formation determined by focal length and focal position.

In using Eqs. 70 and 72, we must note that the distances x and g are to be taken from the boundary of the lens, which is itself curved and thus is not suitable as a surface of reference. If we refer instead to the fixed center plane of the lens, as shown in Fig. 46, the new distances are X_1 and X_2 for object and image, and G for the back-focus, the focal length f remaining unchanged. The focal distance G to the center of the lens now is

$$G = Z + g = \sqrt{2}\,\sqrt{B^2 + Z^2}$$
$$\times \left\{ \frac{Z/B}{\sqrt{2\left(1 + \left(\frac{Z}{B}\right)^2\right)}} + \cot\left[\sqrt{2}\sinh^{-1}\left(\frac{Z}{B}\right)\right] \right\}. \qquad (124)$$

The factor in the brace is plotted as dashed curve in Fig. 45. Its rate of change is slightly smaller than that of g, since Z varies conversely to g.

Derived from Eqs. 70 and 72, the distance of the image from the center of the lens is

$$X_2 = G + \frac{f^2}{X_1 - G} \qquad (125)$$

and the magnitude of the image is

$$S_2 = S_1 \frac{f}{X_1 - G}. \tag{126}$$

For the paraxial rays the values of f and G are to be taken with argument Z/B and for the edge rays with argument $\bar{Z}/\bar{B}$, determined from $\bar{r}/r_0$ by Eq. 123.

Parallel rays of incident radius $\bar{r}$ have always a smaller exit radius than $\bar{r}$, as shown by Fig. 41, and therefore their aberration will be substantially smaller than that of symmetrical rays. For the most part, the image is mainly formed by rays traversing the lens somewhere between the symmetrical and central rays, as seen by Fig. 16. Then, the effective aberrations of hyperbolic lenses will be below the margins of the formulas, curves and figures as developed here.

If we restrict ourselves to symmetrical and central rays as in Figs. 42 and 43, the differential hyperbolic aberration also can be readily derived. This restriction has the further advantage that we cover the aberrations due to both f and g since $s = f + g$, as in Fig. 20. As is seen by Fig. 42, the axial spread $\Delta\sigma$ of the focal distance s is given by the difference of $(Z + s)$ taken on the axis and taken at the radius $\bar{r}$

$$\Delta\sigma = (Z + s) - (\bar{Z} + \bar{s}) = (s - \bar{s}) - (\bar{Z} - Z) = \Delta s - \Delta Z. \tag{127}$$

Herein, the change of Z counteracts favorably the change of s. From Eq. 116 we derive

$$\bar{Z}^2 - Z^2 = \frac{\bar{r}^2}{2} = (\bar{Z} + Z)(\bar{Z} - Z) \tag{128}$$

and, therefore, in sufficient approximation

$$\Delta Z = \frac{\bar{r}^2}{4Z} = \frac{1}{2}\left(\frac{\bar{r}}{r_0}\right)^2 \frac{B^2}{Z}, \tag{129}$$

where in the last expression the blocking radius r_0 of Eq. 119 is introduced.

For small aberrations, Δs may be expressed by the derivative of s with respect to the quotient Z/B, both these parameters contributing to the variation,

$$\Delta s = \frac{ds}{d(Z/B)} \cdot \Delta\left(\frac{Z}{B}\right). \tag{130}$$

The difference $\Delta(Z/B)$ may be derived from Eq. 123 analogous to the development of ΔZ above, yielding in good approximation

$$\Delta\left(\frac{Z}{B}\right) = \frac{\bar{Z}}{\bar{B}} - \frac{Z}{B} \cong \frac{1}{2}\left(\frac{\bar{r}}{r_0}\right)^2\left(\frac{B}{Z} + \frac{Z}{B}\right). \tag{131}$$

When differentiating Eq. 80 expressing s, we must note that according to Eq. 122 the value of the first factors remains invariant and thus only the cotang is variable with Z/B. This gives

$$\frac{ds}{d(Z/B)} = \frac{B}{\sin^2\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \tag{132}$$

Substituting now Eqs. 129, 131, and 132 into Eq. 127, we obtain as total hyperbolic aberration

$$\frac{\Delta\sigma}{B} = \frac{1}{2}\left(\frac{\bar{r}}{r_0}\right)^2 \cdot \frac{B}{Z}\left\{\frac{1+\left(\frac{Z}{B}\right)^2}{\sin^2\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} - 1\right\}. \tag{133}$$

The change of this longitudinal aberration with ratio Z/B is shown in Fig. 47. It has a flat minimum at $Z/B = 2.0$, giving a magnitude 2.89

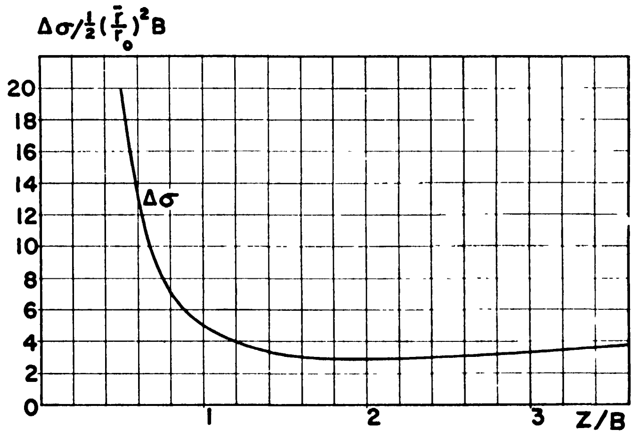


FIG. 47. Hyperbolic aberration depending on lens thickness.

for the factor which varies with Z/B. Hence the optimum lens thickness with respect to hyperbolic aberration is much larger than the optimum thickness with respect to focal length, see Eq. 63 and Fig. 17. This is due to the decreasing curvature of the lateral hyperboloids with thicker lenses. For values of Z/B beyond the range of Fig. 47 the value of $\Delta\sigma$ repeats to descend to further minima which, however, are higher than that shown in the figure.

The trigonometric sine can be transformed into a cotang and thus the contents of the brace in Eq. 133 develop to

$$\frac{1+\left(\frac{Z}{B}\right)^2}{\sin^2\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right]} - 1 = \left(\frac{Z}{B}\right)^2 + \left(1+\left(\frac{Z}{B}\right)^2\right)\cot^2\left[\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{Z}{B}\right)\right]. \tag{134}$$

Substituting herein the focal distance of the symmetrical ray by Eq. 80, we obtain as concise formulation for the hyperbolic aberration

$$\Delta\sigma = \Delta f + \Delta g = \frac{1}{2}\left(\frac{\bar{r}}{r_0}\right)^2\left(Z + \frac{1}{2}\frac{s^2}{Z}\right). \qquad (135)$$

This refers to the symmetrical ray and to the central ray of equal incident radius.

If we assume, as an example, $Z = 1.0B$, $f = 2.1B$ and $s = f + g = 2.7B$ and substitute for r_0 the value of Eq. 119, the longitudinal aberration is

$$\Delta\sigma = \frac{1}{2}\frac{\bar{r}^2}{2B^2}\left(1.0B + \frac{1}{2}\cdot\frac{2.7^2}{1.0}B\right) = 1.16\frac{\bar{r}^2}{B} = 2.44\frac{\bar{r}^2}{f}.$$

This is a favorably small numerical value, particularly if we note that it covers the defects both in f and in g. The possible minimum value is less than $\frac{1}{2}$ of this example, see Fig. 47. An equal spread $\Delta\sigma$ will appear on both sides of the lens. On the other hand, image formation by parallel rays will reduce substantially the effective hyperbolic aberration.

For $Z/B = 1.0$ the amplitude A of the symmetrical ray is determined, according to Eq. 33 and Fig. 6, by $\bar{r} = 0.81\,A$. Thus, the hyperbolic aberration of this example related to the amplitude radius of the ray will appear as

$$\Delta\sigma = 2.44\cdot 0.81^2\frac{A^2}{f} = 1.60\frac{A^2}{f}.$$

When discussing such numerical coefficients of aberration for different electron lenses, it is imperative to choose a comparable position of the apertures in question.

Since the aperture aberration of a hyperbolic lens is a pure surface effect due to the curvature of the lateral boundaries it should be possible, theoretically, to exclude this defect by using lateral electrodes of plane parallel shape so far as the transparent electronic apertures extend. These, however, either should be fed to secure a correct distribution of potential, as in Eq. 12, or they will cause a slight distortion of the hyperbolic field structure which may give rise to a certain aberration of its own.

If, on the other hand, the lens is built with geometrical openings in the lateral hyperboloids, their aberration must be added to the surface defect of the main lens field. This supplementary aberration is greatly dependent on the performance of the electric field within the opening and may be larger or smaller, and even of opposite sign, than the main field aberration.

11. ANGULAR ABERRATION.

So far we have supposed that the axial velocity of the electron rays incident to the lens is independently given, as it suggests itself by the solutions of the trajectories in Eqs. 30 and 31, different in character in the z and r direction. This condition is satisfied by homogeneous rays impinging in a parallel beam.

However, rays emitted from object points close to the lens as in Fig. 48 will have different axial velocity components, depending on the angle β of emission, even if the absolute velocity V over the entire central bundle of rays should be the same. This behavior leads to an imaging defect which in light-optics also would fall into the class of "spherical aberration."

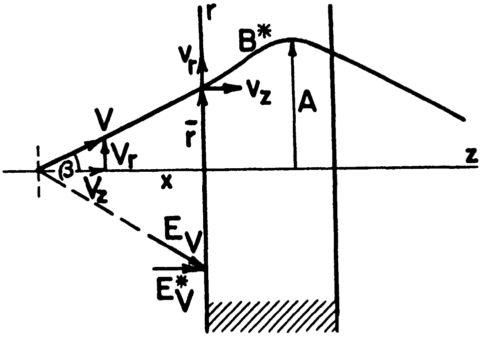


FIG. 48. Angular ray entering electron lens.

The radial and axial velocity components emitted are for each ray

$$V_r = V \sin \beta; \qquad V_z = V \cos \beta. \tag{136}$$

On the other hand, the velocity components within the lens, correlated by Eqs. 38 and 39 to the amplitude A and the transmission constant B, are

$$v_r = \nu\sqrt{A^2 - r^2}; \qquad v_z = \mu\sqrt{B^2 + Z^2}. \tag{137}$$

At the boundary of the lens the respective velocity components coincide and thus, for an object on the axis, there is

$$V \sin \beta = \nu\sqrt{A^2 - \bar{r}^2}; \qquad V \cos \beta = \mu\sqrt{B^{*2} + Z^2}. \tag{138}$$

Herein, $\bar{r}$ is the radius of incidence of the ray considered, and B^* designates the transmission constant of this ray of angular incidence, which is smaller than the paraxial constant B. The value of the extra-axial transmission constant results from the second Eq. 138 as

$$B^* = \sqrt{\left(\frac{V}{\mu}\right)^2 \cos^2 \beta - Z^2}. \tag{139}$$

For paraxial rays, we have with $\beta = 0$ and corresponding to Eqs. 42 and 43, the simple correlation

$$\left(\frac{V}{\mu}\right)^2 = \frac{E_V}{p} = B^2 + Z^2. \tag{140}$$

Therefore, the angular transmission constant B^* can be expressed by

$$B^* = \sqrt{(B^2 + Z^2)\cos^2\beta - Z^2} \tag{141}$$

or, with use of Eqs. 26 and 140, as

$$B^* = \sqrt{\frac{E_V}{E}\left(Z^2 + \frac{R^2}{2}\right)\cos^2\beta - Z^2}. \tag{142}$$

This shows directly the dependence on the paraxial ratio of velocity voltage, on the incident angle of the ray and on the geometrical dimensions of the lens.

For a certain angle β_o the angular transmission constant will vanish and for greater incident angle the rays will be reflected. The blocking value is given by

$$\cos\beta_o = \frac{Z}{\sqrt{B^2 + Z^2}} \quad \text{or} \quad \tan\beta_o = \frac{B}{Z}. \tag{143}$$

For example, a lens of $Z/B = 1$ will have a blocking angle of $\beta_o = 45°$. A very thin lens, towards $Z = 0$, will transmit all the diverging rays up to an angle near $\beta_o = 90°$. A very thick lens, towards $Z = \infty$, will reflect with $\beta_o = 0°$ the rays of any incident angle.

The tangent of the emission angle is geometrically, from Fig. 48, and analytically, from Eqs. 138 and 141,

$$\tan\beta = \frac{\bar{r}}{x} = \frac{v}{\mu}\sqrt{\frac{A^2 - \bar{r}^2}{(B^2 + Z^2)\cos^2\beta}}, \tag{144}$$

wherein x designates the distance of the object from the boundary of the lens. This formulation allows to determine the amplitude of the angular rays within the lens field. We obtain from the first and last expression (144), with use of Eq. 32,

$$\sin\beta = \frac{1}{\sqrt{2}}\sqrt{\frac{A^2 - \bar{r}^2}{(B^2 + Z^2)}}. \tag{145}$$

If then, $\sin\beta$ is expressed by the first and second term of Eq. 144, there results

$$A = \bar{r}\sqrt{1 + 2\frac{B^2 + Z^2}{x^2 + \bar{r}^2}}. \tag{146}$$

This amplitude is always larger than the incident radius $\bar{r}$. For example, an object before a lens at $x = Z = B$ yields, if $\bar{r}$ is small as compared to x,

$$A = \bar{r}\sqrt{1 + 2\frac{1 + 1^2}{1^2 + 0^2}} = \sqrt{5}\bar{r} = 2.24\bar{r}.$$

Thus, the amplitude of such a ray would be more than twice the radius $\bar{r}$.

Since the transmission constant decreases with increasing angle of incidence there will occur an angular aberration of the extra-axial rays. This defect is caused solely by the gradual decrease of the axial velocity component. Hence, the magnitude of the angular aberration in f, g, and h can be determined from our former equations for the velocity aberration by use of the angular transmission constant B^* of Eq. 141 or 142.

Or, we may directly use the axial component E_V^* of the extra-axial velocity voltage. According to the defining Eq. 41 the voltage is proportional to the square of the velocity and therefore is for angular rays, corresponding to the second Eq. 136,

$$E_V^* = E_V \cos^2 \beta. \tag{147}$$

Thus, for a bundle of rays emitted from a point on the axis, this velocity aberration is substantially depending on the angle β of incidence. Using Eq. 147 for the reduced axial voltage of the inclined rays we can apply the curves of Fig. 40 for obtaining the distribution of the cardinal values over the angle of incidence.

For direct use as the main abscissa of Fig. 40, we determine the ratio of any axial voltage component to the blocking velocity voltage by substituting Eq. 143 into Eq. 107, yielding

$$\frac{E_V^*}{E_{VO}} = \frac{E_V^*}{E_V} \cdot \frac{E_V}{E_{VO}} = \left(\frac{\cos \beta}{\cos \beta_o}\right)^2. \tag{148}$$

Thus, the angle β related to the blocking angle β_o is decisive.

The differential angular aberrations also can readily be derived. Equations 112 and 115 show that the velocity aberrations are determined by the relative axial voltage variation of the incident rays. This deviation is here, with use of Eq. 147,

$$\frac{\Delta E_V}{E_V} = \frac{E_V - E_V \cos^2 \beta}{E_V} = 1 - \cos^2 \beta = \sin^2 \beta. \tag{149}$$

Hence, the differential angular aberration will be:

for the back-focus

$$\Delta g = \frac{1}{2} f \left(\frac{g}{f} + \frac{Z}{B}\frac{f}{B}\right) \sin^2 \beta \tag{150}$$

and for the focal length

$$\Delta f = \frac{1}{2} f \left(1 + \frac{Z}{B}\frac{g}{B}\right) \sin^2 \beta. \tag{151}$$

Both these defects are depending on the sine-square of the angle of incidence.

Now, $\sin \beta$ is given by Eq. 145. Substituting therein $\bar{r}^2$ from Eq. 146

yields

$$\sin^2 \beta = \frac{A^2}{x^2 + \bar{r}^2 + 2(B^2 + Z^2)}. \qquad (152)$$

With the values of our former examples, $x = Z = B$ and $f = 2.1B$, and neglecting in the denominator $\bar{r}^2$ as small compared to x^2, there results

$$\sin^2 \beta = \frac{(A/B)^2}{1^2 + 2(1 + 1^2)} = \frac{1}{5}\left(\frac{A}{B}\right)^2 = 0.88\left(\frac{A}{f}\right)^2.$$

Thus, the angular aberration is with the same values as chosen after Eq. 112:

for the back-focus

$$\Delta g = 1.19 \cdot 0.88 \frac{A^2}{f} = 1.05 \frac{A^2}{f},$$

and for the focal length

$$\Delta f = 0.80 \cdot 0.88 \frac{A^2}{f} = 0.70 \frac{A^2}{f}.$$

Related to the incident radius $\bar{r}$ instead of the amplitude radius A of the rays, the aberrations would appear, with use of the value $A^2 = 5\bar{r}^2$ formerly obtained, as $5.3\bar{r}^2/f$ and $3.5\bar{r}^2/f$, respectively. Compared with the hyperbolic aberration as in the example after Eq. 135, but taken twice for the two sides of the lens, we see that the angular aberrations for $x = Z$ are of the same order of magnitude as the hyperbolic aberrations maximally occurring.

When comparing the numerical values of aberration as obtained here under the conditions stated, with values published in the literature (7), it should be noted that most of the latter data are limited to the spread of the focal length Δf and disregard the spread of the back-focus Δg. For thick lenses, however, Δg is different from Δf and is more important for the definition of the image. As realized from the minima of focal length in Figs. 40 and 45, it would be possible in principle for the lenses here considered to reduce to zero the aberrations Δf, both with respect to velocity voltage and radius of surface aperture. This would require a lens thickness near to $Z/B = 1.5$ for both of the defects. However, the spread in back-focus, Δg, would remain, thus retaining a defect in position and magnification of the image as seen by Eqs. 70 and 72.

The magnitude of our various aberrations is in the general order of those of the best electron lenses theoretically determined in the literature (7). The coefficient of r^2/f usually obtained for good lenses varies from 30 to 3 and very rarely down to 0.3, figures which are, however, always related to the spherical aberration Δf alone, and refer to various positions of r within or without the lens.

It seems to be possible to compensate to a substantial degree the surface and angular aberrations of the main hyperbolic lens field by a suitable structure of the electrostatic field in the lateral openings.

Such a hyperbolic electron lens was built and measured[3] in direct operation by a method derived from the lattice test as published by Spangenberg and Field (5). This yielded an aberration coefficient for Δf related to $\bar{r}^2/f$ of the order of 0.3 within the range of the possible errors of observation. This appears as a very small experimental value, indicating that the surface and angular aberrations of the main field were materially reduced.

12. TWO-DIMENSIONAL FIELD.

For a number of applications plane fields are employed acting on electron or ion rays like optical cylinder-lenses. If the field distribution is hyperbolic, as shown in Fig. 49, its lens performance can be readily treated analytically. The potential and stream functions of such a field are

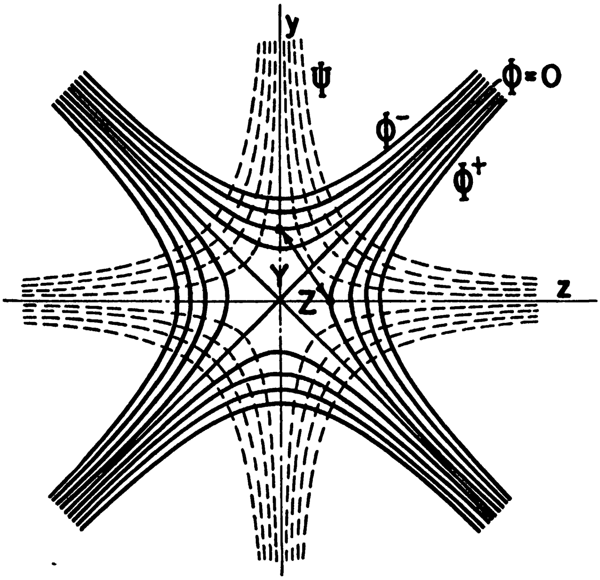


FIG. 49. Equipotential hyperbolas and field lines of cylinder lens.

$$\Phi = p(z^2 - y^2)\,; \qquad \Psi = 2pyz, \tag{153}$$

wherein y and z denote the coordinates and the constant p determines the power of the field. Equipotential lines and stream lines are ordinary hyperbolas, one system at an angle of 45° with respect to the other. The asymptotes of zero potential are under 45°, in contrast to the nearly 55° in the case of rotational symmetry.

---

[3] This was performed by Drs. G. Fleming and R. Rempfer at the Laboratories of the Farrand Optical Co., Inc.

The field strengths as derived by Eq. 1 from Eqs. 153 are

$$\mathcal{E}_y = 2py; \qquad \mathcal{E}_z = -2pz. \tag{154}$$

Thus, each field strength is proportional to its own coordinate and is independent of the other coordinate. Hence, the electron trajectories can be determined rigorously.

Derived from the first Eq. 153, the voltage between two equipotential hyperbolas of opposite potential is

$$E = \Phi^+ - \Phi^- = p(Z^2 + Y^2), \tag{155}$$

wherein Z and Y denote the minimum distances of the respective hyperbolas from the origin. Thus, Z is the half-length and Y is the half-width of the cylindrical hyperbolic electron lens. The power of the field structure, by Eq. 155, is

$$p = \frac{E}{Z^2 + Y^2}, \tag{156}$$

wherein the denominator can be derived from the hypotenuse over $\overline{ZY}$, as in Fig. 49.

The field strengths of Eq. 154 applied to Newton's law, as in Eq. 27, yield for the radial and axial positions of the electrons

$$\left.\begin{aligned} y &= A \cos(\nu t - \varphi) \\ z &= B \sinh \mu t + C \cosh \mu t \end{aligned}\right\} \qquad \nu = \mu = \sqrt{2 \frac{q}{m} p}. \tag{157}$$

These solutions resemble those for the circular lens in Eqs. 30 and 31, except that here ν and μ are equal.

By elimination of the time t, the trajectories of the electrons will be described by

$$\frac{y}{A} = \cos\left(\sinh^{-1}\left(\frac{z}{B}\right) - \varphi\right);$$

or

$$\frac{z}{B} = \sinh\left(\cos^{-1}\left(\frac{y}{A}\right) + \varphi\right), \tag{158}$$

if we limit ourselves to the transmitting case only, where C vanishes. Since the shape of these trajectories is independent of the amplitude A of the curve, all electron rays of parallel incidence upon one plane of the lens field will intersect at the same focal line of abscissa z. Hence, this two-dimensional hyperbolic lens field also is free from volume aberration for uniformly incident rays.

The radial and axial velocities of the electrons follow the same formulations as in the three-dimensional case, see Eqs. 38 and 39 and Fig. 9. Therefore, the same Eq. 44 develops for the transmission constant B in the two-dimensional case. Expressed by the power of the field, as in Eq. 156, the transmission constant here is

$$B = \sqrt{\frac{E_V}{E}(Z^2 + Y^2) - Z^2} = \sqrt{\left(\frac{E_V}{E} - 1\right) Z^2 + \frac{E_V}{E} Y^2}. \qquad (159)$$

For higher lens voltage E, this value decreases and is zero for

$$\left(\frac{E}{E_V}\right)_o = \frac{Z^2 + Y^2}{Z^2} = 1 + \frac{Y^2}{Z^2}. \qquad (160)$$

Beyond this blocking condition, the cosh of Eq. 157 forms the solution and the field will reflect electron rays.

For unipotential fields, where the lens voltage E equals the incident electron voltage E_V, the transmission constant is simply $B = Y$, being given by half the width of the slit between the negative electrodes.

The slope of the electron rays within the lens field, derived from Eq. 158, is

$$\frac{dy}{dz} = -\frac{A}{B} \frac{\sin\left[\sinh^{-1}\left(\frac{z}{B}\right) - \varphi\right]}{\sqrt{1 + \left(\frac{z}{B}\right)^2}}. \qquad (161)$$

For a symmetrical lens field of half-length Z the focal length is given by the subtangent of the trajectory, measured from the principal plane. This, similar to Eq. 62, is

$$f = \frac{A}{dy/dz} = \frac{B\sqrt{1 + \left(\frac{Z}{B}\right)^2}}{\sin\left[2 \sinh^{-1}\left(\frac{Z}{B}\right)\right]}. \qquad (162)$$

Because of the sine function in the denominator, the focal length has a minimum value depending on the ratio Z/B and follows a general trend like that of Figs. 17 and 18. All the other cardinal values, as back-focus and principal points, can be derived in the same way and are different from those with circular field merely by a factor $\sqrt{2}$ in the numerator and under the sine.

If the field is limited by lateral equipotential hyperbolic cylinders permeable to electrons, a surface aberration will occur which is little higher than in the three-dimensional case because the curvature of the lateral hyperbolas is slightly greater in the two-dimensional case.

ACKNOWLEDGMENT.

I wish to express my great appreciation to Professor Emeritus H. E. Clifford whose criticism and advice have constantly helped me in concisely formulating my work.

LETTER SYMBOLS.

a = distance from origin on asymptote
A = amplitude of trajectories
B = transmission constant of lens
C = reflection constant of mirror
d = differential
E = lens voltage between electrodes
E_V = velocity voltage of incident electrons
$\mathcal{E}$ = field strength
f = focal length
g = back-focus, from lens surface
G = back-focus, from lens center
h = distance of principal plane from surface
p = power of hyperbolic lens field
q/m = charge/mass of electrons
r = radial coordinate
R = aperture radius of annular hyperboloid
s = focal distance of symmetrical ray
S = size of object or image
t = time
v = velocity of electrons
V = incident electron velocity
x = distance of object or image from lens surface
X = distance of object or image from lens center
y = transverse coordinate
Y = half-width of cylinder lens
z = axial coordinate
Z = half-length of lens field
α = angle of asymptotes
β = angle of incidence
Δ = finite difference
ν, μ = frequencies of motion
ρ = radius of curvature
σ = longitudinal aberration
φ = phase angle of trajectory
Φ = potential-function
Ψ = stream-function

REFERENCES.

(1) G. G. STOKES, "On the Steady Motion of Incompressible Fluids," *Trans. Cambr. Phil. Soc.*, Vol. 7, p. 439 (1842); *Math. and Phys. Papers*, Vol. 1, p. 1 (1880).

(2) R. RÜDENBERG, "Apparatus for Producing Images of Objects," U. S. Patent 2,058,914, Priority of May 30, 1931; "Apparatus for Influencing the Character of Electron Rays," U. S. Patent 2,070,319, Priority of May 30, 1931.

——, "The Early History of the Electron Microscope." (Contains excerpt of above patents.) *Jour. Appl. Phys.*, Vol. 14, p. 434 (1943).

(3) F. OLLENDORFF, "Potentialfelder der Elektrotechnik," (a book), p. 295, Julius Springer, Berlin, 1932.

(4) F. GRAY, "Electrostatic Electron Optics," *Bell Syst. Tech. Jour.*, Vol. 18, p. 1 (1939).

(5) K. SPANGENBERG AND L. M. FIELD, "Simplified Methods of Determining the Optical Characteristics of Electron Lenses," *Proc. I.R.E.*, Vol. 30, p. 138 (1942).

(6) C. H. BACHMAN AND S. RAMO, "Electrostatic Electron Microscopy," *Jour. Appl. Phys.*, Vol. 14, pp. 8, 69, 155 (1943).

(7) C. MARTON AND S. SASS, "Bibliography of Electron Microscopy," *Jour. Appl. Phys.*, Vol. 14, p. 522 (1943); Vol. 15, p. 575 (1944); Vol. 16, p. 373 (1945); Vol. 17, p. 759 (1946).

(8) J. M. LAFFERTY, "Electron Reflectors with Quadratic Axial Potential Distribution," *Proc. I.R.E.*, Vol. 35, p. 778 (1947).

OPTIMUM RATIO BETWEEN CRANK ARM AND CONNECTING ROD FOR SMOOTH OPERATION OF RECIPROCATING ENGINES.

BY

W. S. KIMBALL, Ph.D.[1]

INTRODUCTION.

Maximum speed of the crosshead in reciprocating engines was discussed in a previous paper [2] as a function of the angular position of the crank arm, and of the ratio of crank arm, r to connecting rod length, c. An expression for the maximum velocity $V_m = \frac{\omega r(n+1)}{\sqrt{n^2+n-1}}$ was found in terms of $n = x/\rho = c^2/y^2$, where $y = r \sin \theta_c$ gives the critical angular position of the crank arm for maximum crosshead velocity, and ω is the angular velocity of the crank arm or shaft, taken to be constant here and in the previous paper. Graphs of these maximum velocities plotted against the critical angle θ_c, and against the crank arm connecting rod ratio, r/c, showed (Figs. 4 and 5, footnote 2) them to differ in magnitude and position from V_t the crosshead velocity at the cycle position where crank arm and connecting rod are at right angles. As to both position and magnitude this difference was definite but very small for usual engineering values of the ratio r/c.

The present article is a special study of the magnitude of the crosshead *acceleration* maxima, of their angular position in the engine cycle (Fig. 1) and of how these maxima vary for different values of the ratio of the crank arm (throw of the crankshaft) to the length of the connecting rod (Fig. 2). Two curious results emerge from this study. The first is that for all ratios of this r/c slightly larger than the usual 1/4 (from 1/3.79 and ranging up to unity), there are three maxima per cycle instead of the two at the ends of the stroke as with simple harmonic motions. And two of these three maxima are positive crosshead acceleration maxima, that is, in direction away from the crankshaft and towards the cylinder, and occurring where the crank arm is at two symmetric positions *before* and *past* the position away from crank opposite the cylinder at angular position $\theta = \pi$, the stroke end, where positive acceleration is maximum for simple harmonic motions.

The second curious result is that these two positive crosshead acceleration maxima have a smallest (equal) value, or minimum under variations in r/c when the ratio of crank arm to connecting rod is given by $r/c = 1/2.8 = 0.357$, these occurring at cycle angle $\pm 132°$,

---

[1] Lieutenant Commander, U. S. Naval Reserve; Department of Marine Engineering, U. S. Naval Academy, Annapolis, Md.

[2] W. S. Kimball, "Maximum Inertia Force and Maximum Speed of the Crosshead in Reciprocating Engines," JOURNAL FRANKLIN INST., Vol. 242, p. 309, October, 1946.

TABLE I.

$\rho = \frac{r^2}{c^2}$	$\sqrt{\rho} = \frac{r}{c}$	$u = \frac{y^2}{c^2} = \sin^2 \alpha$	$z = \frac{y^2}{r^2} = \sin^2 \theta$	$\sin \theta$	θ	Max. Positive Acceleration $\frac{V_m'}{\omega^2 r}$	Min. Positive Acc. at, $\theta = \pi$ $\frac{V'}{\omega^2 r} = 1 - \sqrt{\rho}$	Positive Acc. at $\theta = \pi/2$ $\frac{V'}{\omega^2 r} = \frac{1}{\sqrt{\frac{1}{\rho} - 1}}$
						Eq. (20)	Eq. (12)	Eq. (13)
$\frac{1}{14.365} = 0.0695 = \frac{5 - \sqrt{21}}{6}$	$\frac{1}{3.79} = 0.262$	0	0	0	$\pi - 0 = \pi$	0.7368	0.7368	0.274
$\frac{1}{12} = 0.08333$	$\frac{1}{2\sqrt{3}} = 0.2886$	0.01683	0.20195	0.4494	153°18′	0.7158	0.7114	0.3015
$\frac{1}{10} = 0.1000$	$\frac{1}{\sqrt{10}} = 0.3165$	0.0372	0.3721	0.6100	142°25′	0.7023	0.6835	$\frac{1}{3} = 0.3333$
$\frac{1}{9} = 0.1111$	$\frac{1}{3} = 0.3333$	0.0505	0.4545	0.6749	137°26.6′	0.6976	0.6666	$\frac{1}{\sqrt{8}} = 0.3535$
$\frac{1}{7.837} = 0.1276$	0.357	0.0703	0.551	0.7423	132°05′	0.6952	0.643	0.382
$\frac{2}{9} = 0.2222$	0.4714	0.1818	$\frac{9}{11} = 0.8181$	0.904	115°18′	0.737	0.5286	0.5347
$\frac{1}{4} = 0.2500$	$\frac{1}{2} = 0.5000$	0.2134	0.8536	0.9239	112°29.6′	0.7588	0.5000	$\frac{1}{\sqrt{3}} = 0.5773$
$\frac{4}{9} = 0.4444$	$\frac{2}{3} = 0.6666$	0.4242	0.9657	0.9827	100°40′	0.9844	$\frac{1}{3} = 0.3333$	0.8948
$\frac{2}{3} = 0.6666$	0.8165	0.6632	0.99476	0.99736	94°10′	1.4501	0.1835	$\sqrt{2} = 1.414$
$\frac{81}{100} = 0.8100$	$\frac{9}{10} = 0.9000$	0.80931	0.9991535	0.999565	91°42′	2.0819	$\frac{1}{10} = 0.1000$	$\frac{9}{\sqrt{19}} = 2.06433$
1.0000	1.0000	1.0000	1.0000	1.0000	90°	∞	0	∞

which is ± 48° away from the stroke end position $\theta = \pi$, where the positive maximum for simple harmonic motion occurs (see Fig. 3).

This ratio $r/c = 1/2.8$ is thus, in a certain sense, an optimum ratio, because minimum acceleration implies minimum jerk or inertia force for the crosshead and hence minimum engine wear and tear so far as these positive acceleration maxima are concerned. Also for this ratio, there is relatively small difference in the positive crosshead accelerations all the way around the outside half swing of the crank arm, thus making for smooth operation. Hence, $r/c = 1/2.8$ has been termed the optimum ratio for the smooth operation of reciprocating engines, although it refers only to positive crosshead accelerations, and negative acceleration maxima, Eq. 11, will be seen to have no minimum value, but increase linearly with r/c.

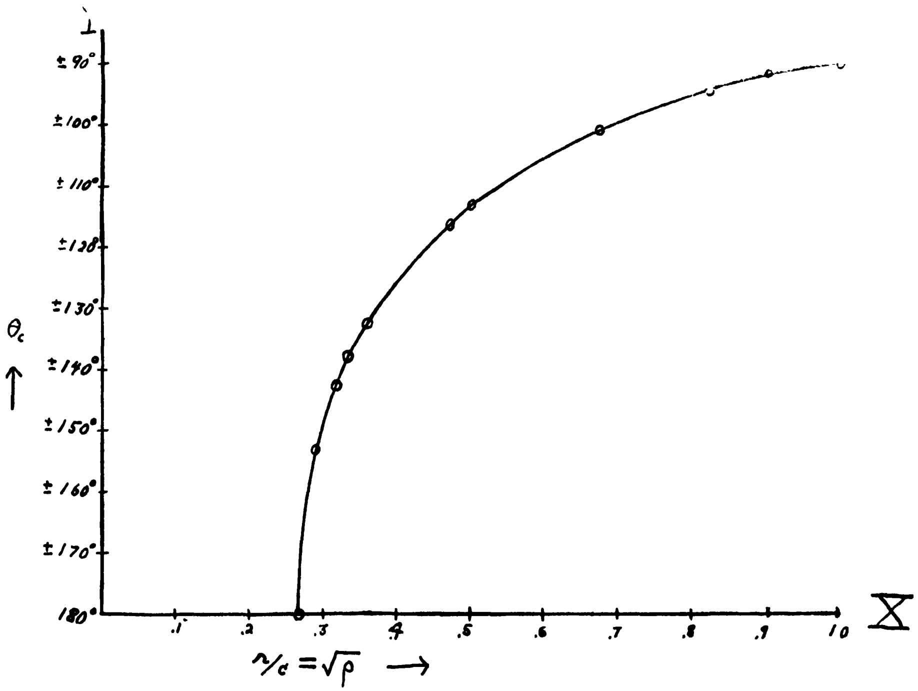


FIG. 1. Angular positions of the one or two positive acceleration maxima, showing where two occur at positions symmetric about $\theta = 180°$ for values of r/c greater than $1/3.79 = 0.2632$.

It does not appear that engines have been constructed with as large a ratio as this optimum $r/c = 1/2.8 = 0.357$, but its existence is here pointed out, and the position of its two smallest positive maxima at angular positions ± 48° away from the stroke end position $\theta = \pi$; and the fact that the magnitude of these maxima is only 69.52 per cent of the corresponding simple harmonic motion maximum, $\omega^2 r$ (see Table I) is here noted.

On the other hand, engines have been constructed where two positive acceleration maxima exist instead of a single one at $\theta = \pi$, as for simple harmonic motion. Thus the Liberty Motor, with a 7-in. stroke and a 12-in. connecting rod, has $r/c = 3.5/12 = 1/3.43 = 0.29$ and for this ratio Fig. 1 shows that positive acceleration maxima occur at ±153° or at angular positions ±27° away from $\theta = \pi$ at the end of the stroke. This engine thus has three crosshead acceleration maxima per cycle, including the negative one at $\theta = 0$ as well as the above two positive ones.

1. THE VELOCITY AND ACCELERATION OF THE CROSSHEAD.

Taking the center of the crankshaft as origin of coordinates, the position of the crosshead A, referring to Fig. 3 is given by

$$OA = x + w = \pm \sqrt{r^2 - y^2} + \sqrt{c^2 - y^2}, \tag{1}$$

where $x = OC = \pm \sqrt{r^2 - y^2}$ and $w = CA = \sqrt{c^2 - y^2}$ are the projections on the x axis of the crank arm $r = OB$ and the connecting rod $c = BA$. Note that w, the projection of c, is *always* positive and that the projection of r is positive and added to w in the first and fourth quadrants whereas it is negative, and subtracted from w in the second and third quadrants.

The velocity of the piston crosshead at A is given by the time derivative of (1):

$$V = -y\frac{dy}{dt}\left[\frac{\pm 1}{\sqrt{r^2 - y^2}} + \frac{1}{\sqrt{c^2 - y^2}}\right]. \tag{2}$$

Introducing $y = r \sin \theta$ and $d\theta/dt = \omega =$ constant for constant angular velocity of the shaft, we have:

$$\frac{dy}{dt} = r \cos \theta \frac{d\theta}{dt} = r\omega \cos \theta = \pm \omega\sqrt{r^2 - y^2}. \tag{3}$$

Then from (2) and (3)

$$V = -\omega y\left[1 \pm \sqrt{\frac{r^2 - y^2}{c^2 - y^2}}\right] = x' + w', \tag{4}$$

where we note that the signs of x' and w' check their tabulated values in Table I of the previous paper, the lower one of the paired signs always referring to the two quadrants at the left of the Y axis where x is minus and the upper of the paired signs referring to the two quadrants at the right of the Y axis where x is plus.

Likewise we may take the time derivative of (4) and obtain:

$$V' = -\omega\frac{dy}{dt}\left[1 \pm \frac{(y^4 - 2y^2c^2 + c^2r^2)}{\sqrt{(c^2 - y^2)^3(r^2 - y^2)}}\right], \tag{5}$$

or introducing (3):

$$V' = -\omega^2\left[\pm \sqrt{r^2 - y^2} + \frac{y^4 - 2c^2y^2 + c^2r^2}{(c^2 - y^2)^{3/2}}\right]. \tag{6}$$

Introduce

$$z = \sin^2 \theta = y^2/r^2 \qquad \text{and} \qquad \rho = r^2/c^2. \tag{7}$$

Then

$$V' = -\omega^2 r \left[\pm \sqrt{1 - z} + \frac{\rho^2 z^2 - 2\rho z + \rho}{\sqrt{\rho}(1 - \rho z)^{3/2}} \right] = x'' + w''. \tag{8}$$

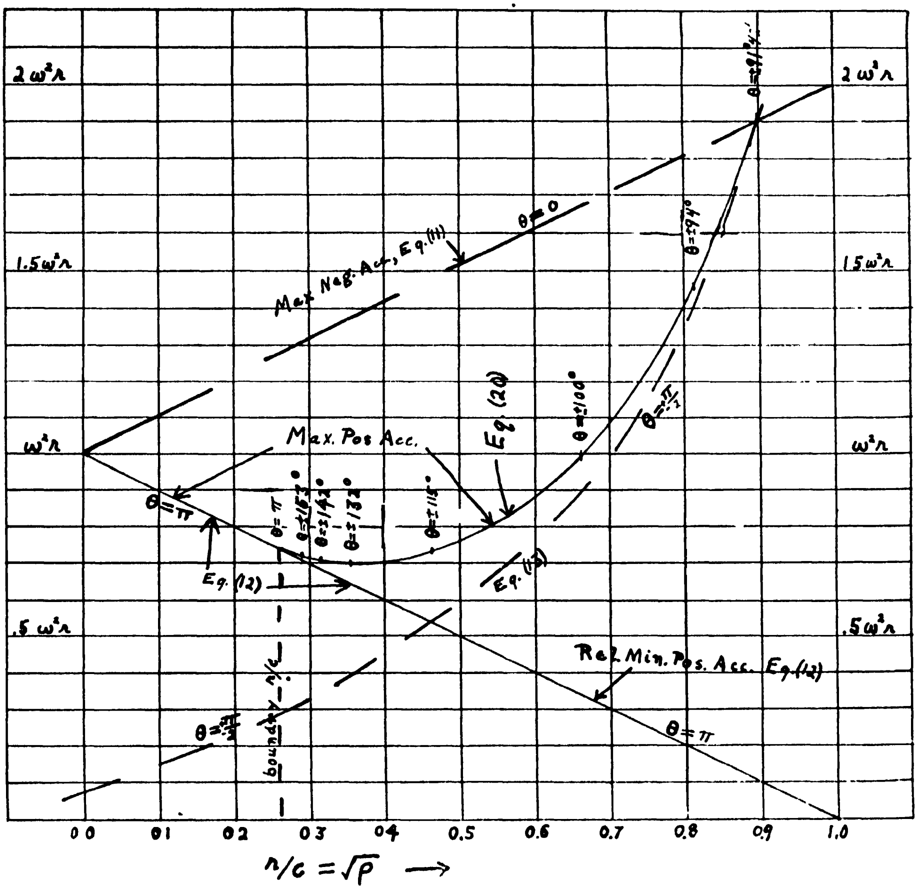


FIG. 2. Crosshead acceleration magnitudes as functions of r/c, the ratio of crank arm or throw of the crank, to connecting rod length, showing Eq. 20, Maximum Positive Acceleration, and Eq. 11, Maximum Negative Acceleration, and Eq. 12, Maximum, or Relative Minimum Positive Acceleration, and Eq. 13, for Positive Acceleration at $\theta = \pm \pi/2$, the limiting position of the maximum of Eq. 20 as the ratio r/c approaches unity.

Here the first term $x'' = -\omega^2 x$ is the simple harmonic motion formula for acceleration for the projection of r on OX and second one, w'' is the acceleration for the projection of the crank c, on OX and is seen to depend on ρ. The vanishing of (6) or (8) is the condition for maximum crosshead velocity treated in the previous[2] article. This vanishing occurs on the right of OY where x'' is minus and equal in magnitude to a positive w'' as given by (8) wherein hence $2\rho z = 2(y^2/c^2) = 2 \sin^2 \alpha$

is greater than $\rho + \rho^2 z^2$. A positive w'' always occurs for values of $\rho z = \sin^2 \alpha$ greater than the critical $\rho z = 1 - \sqrt{1 - \rho}$ which causes w'' to vanish.

2. CONDITION FOR MAXIMUM ACCELERATION OF THE CROSSHEAD.

The time derivative of (6) is

$$V'' = \omega^2 y \frac{dy}{dt}\left[\pm \frac{1}{\sqrt{r^2 - y^2}} + \frac{y^4 - 2c^2y^2 + 4c^4 - 3c^2r^2}{(c^2 - y^2)^{5/2}} \right], \tag{9}$$

which becomes when (3) is introduced

$$V'' = \omega^3 y \left[1 \pm \sqrt{\frac{r^2 - y^2}{(c^2 - y^2)^5}}\,(y^4 - 2c^2y^2 + 4c^4 - 3c^2r^2) \right]. \tag{10}$$

The vanishing of (10) is the condition for maximum acceleration and we note that this occurs where $y = 0$ as for simple harmonic motion, and the corresponding extremal values of (6) and (8) are, as previously[2] noted,

$$V_o' = -\omega^2 r \left(1 + \frac{r}{c} \right), \tag{11}$$

$$V_\pi' = \omega^2 r \left(1 - \frac{r}{c} \right), \tag{12}$$

and the subscripts refer to the angular position of the crank arm in the cycle. These are the maximum negative and positive values of V' throughout those engineering values of r/c, that are less than 1/3.79. Formula 11 always gives the maximum negative acceleration, but for values of r/c larger than 1/3.79 (see (16) below), the vanishing of the bracket in (10) accounts for the maximum positive acceleration, and when it does, then (12) gives a relative *minimum* V' and as r/c approaches unity this becomes zero and the acceleration maximum moves away from the position $\theta = \pi$ to the position $\theta = \pm \pi/2$ where it is, as previously shown, positive and infinitely large. For $\theta = \pm \pi/2$ the acceleration (6) or (8) is positive and given by

$$V'_{\pm\pi/2} = \frac{\omega^2 r^2}{\sqrt{c^2 - r^2}} = \omega^2 r \Big/ \sqrt{\frac{1}{\rho} - 1}, \tag{13}$$

which is seen to become infinite when $r = c$, and $\rho = 1$. The present investigation shows how this comes about from the vanishing of the bracket in (10), with accompanying shift of the angular position of V_m' from $\theta = \pi$ to $\theta = \pm \pi/2$ as given by (13).

The condition for maximum V' is also given by the vanishing of its partial derivative with respect to $z = \sin^2 \theta$, which gives its maximum under variations in the angle θ. Thus from (8) we have:

$$V_z' = \frac{\omega^2 r\sqrt{\rho}}{2}\left[\frac{\pm 1}{\sqrt{\rho - \rho z}} + \frac{\rho^2 z^2 - 2\rho z + 4 - 3\rho}{(1 - \rho z)^{5/2}}\right]. \tag{14}$$

When the crank arm is on the left of the Y axis, we use the lower sign in (14) and the bracket vanishes when

$$(1 - \rho z)^{5/2} = \sqrt{\rho - \rho z}(\rho^2 z^2 - 2\rho z + 4 - 3\rho).$$

Square this equality, cancel, transpose and factor out $1 - \rho$, and there remains:

$$\rho^4 z^4 + 2\rho^3 z^3 - 6\rho^2 z^2(1 + \rho) + \rho z(11 + 3\rho) + (1 - 15\rho + 9\rho^2) = 0. \tag{15}$$

This is the condition which determines the critical z (or angle θ) which maximizes the crosshead acceleration for a given ρ. Note that when the constant term $(1 - 15\rho + 9\rho^2)$ is zero, the equation is satisfied by $z = \sin^2\theta = 0$, so that the maximum occurs at $\theta = \pi$ and is given by (12). For this case

$$\rho = \frac{5 - \sqrt{21}}{6} = \frac{0.4174}{6} = 0.06956 = \frac{1}{14.365}, \tag{16}$$

$$\frac{r}{c} = \sqrt{\rho} = 0.26315 = \frac{1}{3.7914}.$$

Now introduce the variable

$$0 < u = \rho z = \frac{y^2}{c^2} = \sin^2\alpha < z < 1. \tag{17}$$

Then (15) becomes

$$u^4 + 2u^3 - 6u^2(1 + \rho) + u(11 + 3\rho) + (1 - 15\rho + 9\rho^2) = 0. \tag{18}$$

We may show that (15) and (18) have no real solutions when ρ is less than (16) because in this case the constant term $(1 - 15\rho + 9\rho^2)$ is positive. To see this, note that the positive term $11u$ is greater than all remaining negative terms put together, $-6u^2(1 + \rho)$:

$$11 > 6(1 + \rho) > 6u(1 + \rho). \tag{19}$$

This inequality obviously holds for $u < 1$ and ρ less than (16) and when (19) is multiplied by u, we have $11u$ greater than all negative terms combined, $-6u^2(1 + \rho)$; so that (15) never vanishes when $\rho = r^2/c^2$ is less than $(5 - \sqrt{21})/6$; that is, when $\sqrt{\rho} = r/c$ is less than 1/3.79 according to (16).

Thus the bracket in (10) or (14) never vanishes when the ratio of crank arm to connecting rod length is less than 1/3.79; and we note that this region includes the usual engineering practice where this ratio is about 1/4. Thus the old Navy triple expansion reciprocating steam engines had about 1 to 4 for the ratio of crank arm to connecting rod.

Also some of the latest radial engines like R3350, used in the B-29's, which has a 14-in. connecting rod with a stroke of 6.312 in., or $3.156/14 = 1/4.43$ for the ratio r/c in case of this comparatively long connecting rod. Maximum positive acceleration of the crosshead for the above cases is at $\theta = \pi$ and as for simple harmonic motion, there is also only this one angular position per cycle where maximum positive acceleration occurs.

The boundary ratio 1/3.79 is very close to 1/4 and many engines have been designed in recent years, having larger ratios than 1/3.79 between crank arm and connecting rod. Thus the Liberty motor has a 12-in. connecting rod and a 7-in. stroke, or 3.5-in. crank arm, or throw of the crank, with a ratio $3.5/12 = 1/3.43 = 0.29$, which is larger than the above boundary ratio 1/3.79 between crank arm and connecting rod. Thus for this engine there are two equal positive acceleration maxima (as well as one negative maximum) at symmetric angular positions in the cycle at about $\theta = \pm\ 153°$ as shown near the lowest circle of Fig. 1, and determined by the vanishing of the bracket in (10) or (14) instead of by (12).

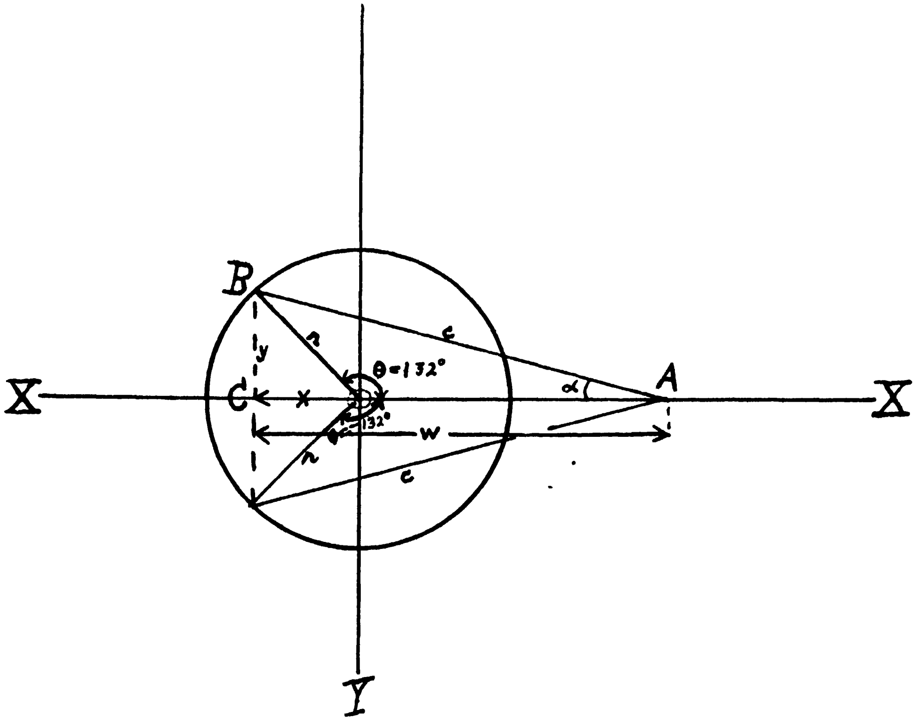


FIG. 3. The crank arm, r, and connecting rod, c, of ratio 1/2.8 shown at cycle angle ±132°, the positions of maximum positive crosshead acceleration per cycle, but the minimum value of this maximum under variations in r/c.

The Wright Whirlwind engine (Lawrence type) used by Lindbergh on his historic flight, had a 10.5-in. connecting rod and about 5.5-in. stroke or 2.75-in. crank arm or throw of the crank, giving a ratio $2.75/10.5 = 1/3.8$, which is almost at the boundary value $1/3.79$ separating the region where one positive acceleration exists at $\theta = \pi$ as in simple harmonic motion, from that in which two exist at symmetrical positions on either side of π as with the Liberty Motor.

3. MAXIMUM POSITIVE CROSSHEAD ACCELERATIONS.

The solutions of (15) or more easily (18) give the critical z (and hence angle θ) that maximizes the acceleration (6) or (8) for a given ρ, as this latter ranges from (16) to unity. These solutions may be obtained by Graeffe's method or by Horner's method. The maximum positive acceleration is then given by introducing these solutions into (6) or (8) using the minus (lower) sign in the first term since the crank arm is to the left of the Y axis where $\cos\theta$ is negative. Also introduce (17) for convenience in computations: Then

$$\begin{aligned} V_m' &= \omega^2 r\left[\sqrt{1-z} - \frac{u^2 - 2u + \rho}{\sqrt{\rho}(1-u)^{3/2}}\right] \\ &= \omega^2 r\left[\sqrt{1-z} + \frac{2u - u^2 - \rho}{\sqrt{\rho}(1-u)^{3/2}}\right]. \end{aligned} \tag{20}$$

For values of $r/c = \sqrt{\rho}$ ranging from 0.2632 to 1, Table I gives this maximum crosshead acceleration (20) and compares it with the smaller accelerations for each ρ at $\theta = \pm\pi/2$ and π. Note that for values of r/c greater than 0.8, as shown by the last three tabulated rows the maximum crosshead acceleration (20) approximates that for $\theta = \pm\pi/2$ its position when $\rho = 1$ as previously [2] shown.

4. MINIMUM VALUE OF THE MAXIMUM POSITIVE CROSSHEAD ACCELERATION.

Note from Table I that the smallest value of this maximum positive crosshead acceleration (20) is $V_m' = \omega^2 r\ (0.6952)$ and corresponds to $r/c = 0.357$ or $\rho = 1/7.837$. The condition for this minimum value of (20) under variations in ρ is given by setting the partial derivative of (8) with respect to ρ equal to zero (or the partial of (20) remembering that $u = \rho z$ by (17)). Thus:

$$\frac{\partial V'}{\partial \rho} = V_\rho' = \frac{\omega^2 r}{2\sqrt{\rho}}\frac{(\rho z^2 + 2z - 2\rho z - 1)}{(1-\rho z)^{5/2}} = 0 \tag{21}$$

or

$$\left.\begin{aligned} \rho^2 z^2 + 2\rho z &= (2\rho z + 1)\rho \\ u^2 + 2u &= (1 + 2u)\rho \\ \rho &= \frac{u^2 + 2u}{1 + 2u} \end{aligned}\right\}. \tag{22}$$

The condition for the minimum value under variations in ρ of the maximum crosshead acceleration given by (20) is the simultaneous vanishing of the partials of (8) with respect to z and with respect to ρ as given by (14) and (21); that is by solving (15), or more easily (18) simultaneously with (22).

When we introduce (22) into (18) we have a six degree equation in u, which has a double root of unity. Thus, when $(u - 1)^2$ is factored out, there remains:

$$4u^4 + 8u^3 - 18u^2 - 13u + 1 = 0, \tag{23}$$

for the condition on u (and hence ρ by (22)) which minimizes (20). The solution of this equation by Horner's method gives the critical $u_c = 0.0703$ and the other critical variables and minimum $V_m' = \omega^2 r$ (0.6952) given in Table I.

Figure 2 shows the graphs of the positive acceleration of the crosshead for various values of the ratios r/c of crank arm to connecting rod, at angular crank arm positions away from and opposite the cylinder, $\theta = \pi$, at right angles $\theta = \pm \pi/2$ and at such positions as where maximum acceleration occurs.

Note that for $r/c < 0.2632$ that is, throughout usual practical engineering ranges the maximum curve coincides with the curve (linear) for $\theta = \pi$, and so, maximum positive crosshead acceleration occurs away from and opposite the cylinder, and on the other hand for large values of r/c, exceeding 0.8, this maximum acceleration differs from that at $\theta = \pm \pi/2$ by less than 3 per cent, being scarcely distinguishable on the graph, and occurring within 5° of the right angle position for crank arm and cylinder axis where $\theta = \pm \pi/2$.

A further fact about the extremal positive crosshead accelerations to note is that when the maximum V_m' moves away from position $\theta = \pi$ with $r/c = 0.263$, we still have zero for its rate of change (10) at $y = 0$, where $\theta = \pi$, which hence then is a position of minimum (relative) acceleration. But it is a rather flat minimum and differs only slightly from the maximum for values of r/c anywhere near usual engineering practice. Thus for $r/c = 0.357$ which determines the minimum of the maximum $V_m' = \omega^2 r$ (0.6952) as given by (20), at $\theta = \pm 132°$, the relative minimum at $\theta = \pi$, is $V_\pi' = \omega^2 r$ (0.643). The corresponding crosshead inertia forces will thus differ by less than 10 per cent which would indicate a steady and smoothly operating cycle in this region of positive accelerations with crank arm rounding the half circuit away from the cylinder, and point to $r/c = 0.357$ with its minimum V_m' above as an optimum ratio for the reduction of crosshead wear and tear due to positive acceleration.

The opinions or assertions contained in this article are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department, or the Naval Service at large.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

MEASUREMENT OF RADIOACTIVE ISOTOPES.

The determination of the number of radioactive atoms in a sample of a radioactive isotope has become a matter of importance, because these isotopes not only have become an article of commerce but also are used in biological and medical experiments, and even in clinical treatment of diseases. Uniform quantitative results in all laboratories can be obtained only by using procedures that will yield absolute measurements or by the use of uniform standards of comparison that will give the same result whenever the determination is made.

The National Bureau of Standards has therefore issued standard samples of radioisotopes whose radioactivity has been accurately determined, and is planning to issue samples of additional radioisotopes as they become available. This follows logically the active role which the Bureau has long taken in the field of radioactive standards and measurements. It is furthermore in line with the Bureau's responsibility to furnish standards for precision measurement in all branches of the physical sciences.

Two factors are involved in the preparation and use of these samples. First, methods and standards that are correct in principle must be used. Second, a reasonable degree of accuracy must be attained in all steps of the process both of producing and of using the standards.

Essentially the measurement of a sample of radioisotope is the measurement of the number of radioactive atoms present. This, in turn, is dependent on the rate of disintegration, that is, the number of nuclear particles which are emitted in unit time from all atoms disintegrating within this interval of time. Counting the total number of disintegration particles, however, is not a simple procedure. The principal difficulty arises from the fact that these particles are emitted equally in all directions. It is therefore practically impossible to devise detecting equipment that will record all of them. Most of the difficulties in making quantitative determinations of radioisotopes, however, can be eliminated if standard sources of the isotope are available.

A standard source consists of a preparation of the isotope in a form convenient for use with the detector of radiation, and for which the disintegration rate is known from previous calibration. When the isotope to be measured is the same as that from which the standard is prepared, reliable results may be secured by observing only a few simple precautions. Other isotopes which have short half periods and a known disintegration scheme may also be measured with fair accuracy by means of beta-ray standards if it is known that a beta ray or a positron is emitted for every disintegration, and if the maximum energies of the beta-ray spectra of the standard and of the specimen are not too widely different.

---

* Communicated by the Director.

In the case of those isotopes for which a disintegration scheme is lacking, or for which the mode of disintegration does not permit measurement of disintegration rates, an alternative method of comparing the activities of sources is available if the isotope emits gamma rays. Although this method does not give disintegration rates, it can, when properly applied, yield reliable comparisons of sources in various laboratories. It is limited, however, to comparisons of the same isotope. To secure reliable results, a standard instrument and standard geometry are necessary. The roentgen is a convenient unit of gamma radiation, which is defined without reference to the energy of the gamma ray. An ionization instrument properly designed to measure roentgens will therefore satisfy the requirements for a standard instrument. To determine the strength of a radioactive source the roentgens per unit time must be measured at a standard distance. On the basis of these considerations, Bureau scientists have proposed that the unit of time be one hour and the distance one meter, giving as a unit for comparison of gamma-ray sources the *roentgen per hour at a meter* (abbreviated *rhm*).

At the same time the name *rutherford* (*rd*) has been proposed for that quantity of radioisotope which disintegrates at the rate of a million disintegrations per second. This unit, which is smaller than the classical *curie*, can be specified exactly and independently of natural constants such as the rate of decay of radium, which is the basis for the definition of the curie. The National Bureau of Standards has adopted an arbitrary value of 3.700×10^{10} disintegrations per second for the curie when applied to isotopes other than members of the radium family.

The use of the rutherford in data presupposes that a disintegration rate has been measured and that this rate is expressed in disintegrations per second. This procedure removes all confusion regarding units and renders data from different laboratories comparable on an absolute basis.

EXPANSIVE CHARACTERISTICS OF HYDRATED LIMES.

It is highly important that a mortar or plaster after setting in place shall not undergo any appreciable change in volume, because volume changes frequently result in damage or even complete failure of the finished job. The National Bureau of Standards has been interested for some time in a widespread type of plaster failure, characterized by the formation of large blisters or bulges in the finished coat and caused by an increase in volume of the hardened plaster. Laboratory studies led to the conclusion that the expansion is due to a slow chemical change taking place in one of the chemical compounds that is naturally present in many limes. As a result of these studies a limit for this constituent and others that might act in the same way has been included in recent specifications of the American Society for Testing Materials and the American Standards Association. The requirement is that the percentage of unhydrated oxides in hydrated limes shall not exceed 8 per cent by weight.

Recognizing that chemical composition alone does not always provide an indication of the soundness of hydrated lime, the Bureau continued the investigation to develop a suitable performance test. Because unsoundness does not usually show up for several years, an accelerated test has been proposed. The test involves preparation of a test bar by mixing the lime with portland cement, heating the bar (after it has set) in an autoclave, and measuring the expansion of the bar. An autoclave was specially modified to observe the behavior of test specimens during autoclaving.

Eighty commercial hydrated limes have been subjected to this treatment and to other tests for determining the effects of variables such as the amount of lime in the bars. In each case the relation between expansion and percentage of unhydrated oxides, the components that appear to cause this particular behavior, has also been determined. From these data, details of the test procedure have been worked out and a limit of expansion suggested.

HUMIDITY TEST APPARATUS.

A humidity test apparatus,[1] developed and built in the Mechanical Instruments Laboratory of the National Bureau of Standards, now makes available an instrument for research, calibration, and testing of hygrometers at temperatures below freezing. The use in recent years of radiosondes for the meteorological sounding of the atmosphere, an important acid in weather forecasting, has created a marked need for information on the performance of the humidity-sensing element in the radiosonde under conditions of flight. The humidity test apparatus was developed under the sponsorship of the Bureau of Ships, Department of the Navy, to provide an instrument for the investigation of radiosonde hygrometer elements at the temperatures and air velocities encountered during a flight.

The function of the humidity test apparatus is to produce air of known relative humidity at temperatures from 0° to −40° C. In the new divided flow, low temperature humidity test equipment developed by Arnold Wexler, a current of dry air is divided into two streams, one of which is maintained dry while the other is saturated with respect to ice; finally the two are recombined.

---

[1] For further technical details see, "Divided Flow, Low Temperature Humidity Test Apparatus," by Arnold Wexler, *J. Research NBS*, **40**, 479 (1948) RP1894.

A proportioning valve is used to divide the flow of moisture-free air in a known ratio. One part is passed through a saturator over a series of trays containing ice, until it is completely saturated. It is then mixed in a mixing chamber, with the other part that has been maintained dry, and allowed to exhaust through a test chamber into the atmosphere. The saturator, mixing chamber, and test chamber are kept immersed in a constant temperature bath. The hygrometer or other device, which is being subjected to air of known and constant humidity, is inserted into the test chamber.

The relative humidity in the test chamber is a function of (*a*) the fraction of air that passes through the saturator, (*b*) the total pressure in the saturator, (*c*) the total pressure in the test chamber, (*d*) the saturation pressure, and (*e*) the partial pressure of the water vapor in the test chamber. Under ideal conditions, which the apparatus closely approaches, all variables except one are eliminated, and the relative humidity becomes equal to the fraction of air that passes through the saturator.

The essential functional units of the apparatus are the drying system, the proportioning system, the humidifying system, the mixing chamber, the test chamber, the cooling system, and the thermoregulating system for temperature control.

The drying system serves to remove all water from the air entering the apparatus by freezing in a dry ice-varsol bath. The dry air then passes through the proportioning system, the major feature of which is the proportioning valve. This valve, which divides the air in a definite ratio, consists of six orifices of equal cross-sectional area so arranged that by a turn of the knob of the valve the incoming air can be divided to produce any of seven ratios, 0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1. The ratio is the fraction of air entering the valve that emerges through one exit channel.

The two air streams, upon leaving the proportioning system, flow through the humidifying system in parallel channels, thermally in contact with one another to allow heat interchange and temperature equilibrium between the two streams. Saturation of one stream is accomplished by allowing it to pass over a series of staggered ice-filled trays while the other stream passes over an identical path of dry shelves. The dry and saturated air are centrifugally mixed in a mixing chamber and discharged into the atmosphere through a $1\frac{1}{8}$ in. inside diameter tube that serves as a test chamber. The saturators, mixing chamber and test chamber are immersed in a bath whose temperature is closely regulated.

The over-all performance of the humidity test apparatus has been established by a series of gravimetric moisture determinations. For the temperature range of 0° to $-40°$ C. and for air velocities up to 1500 ft. per minute, this instrument produces air whose relative humidity is known within 3 per cent.

While particularly suited for the investigation and calibration of hygrometers, psychrometers, and development apparatus at temperatures below 0° C., the humidity test equipment may find further application in general research and testing where air of a known low dewpoint or moisture content is desired. With such apparatus, devices placed in the test chamber can be subjected to discrete and almost instantaneous changes in relative humidity. Lag studies may therefore be easily made. The velocity of the air passing through the test chamber can be adjusted to any value up to 1500 ft. per minute.

Although this instrument was designed primarily to fill a basic need in meteorology, the refrigeration and air-conditioning industries may also find it useful. In all these fields, humidity plays an important role. With the humidity test apparatus, it is now possible to calibrate readily and accurately the various humidity measuring instruments employed at low temperatures.

VARIABLE-RESISTANCE SPRING TRANSDUCER.

A highly sensitive mechano-electrical transducer, which transforms slight displacements into large changes of resistance, current, or voltage, is being developed by W. A. Wildhack and his associates of the National Bureau of Standards. The active element of the device is a helical or conical spring wound in such a way that the initial tension varies slightly along its length. Thus, when the ends of the spring are pulled apart, the turns separate one by one rather than simultaneously.

When the spring is entirely closed, it has an electrical resistance approximately that of a cylindrical tube. When it is completely open, its resistance is that of the total length of the coiled wire. Resistance can thus be varied over a wide range by stretching the spring. As the percentage change in resistance may be hundreds of times greater than the percentage change in length, displacements as small as 1/100,000 of an inch can be easily measured without the use of electrical amplifying devices. The spring transducer thus provides a sensitive means for conversion of any mechanical displacement to a change in an electrical quantity that can be precisely determined. When connected to another transducer which gives a mechanical displacement output (a bimetallic strip responding to temperature changes, for example), the combination gives an easily measurable electrical output. This type of use suggests numerous scientific and industrial applications, including strain gages, pressure elements, accelerometers, electric weighing devices, automatic temperature controls, dc-ac inverters, and voltage regulators.

The preferable construction for the transducer is a four-arm bridge of which each arm is a variable-resistance spring. An increase in applied tension elongates one pair of springs and shortens the other pair. The resistive unbalance of the bridge, as indicated by a galvanometer, thus gives a measure of the displacement that has occurred. With this arrangement, since the voltage can be nearly reversed through the bridge, the output voltage can be theoretically twice the input voltage.

The variation of the initial tension of the spring along its length may be accomplished in several ways: by conical winding, by varying the angle of feed of the wire on a uniform mandrel, or by varying its tension as it is wound. For greatest sensitivity the variation in initial tension is made quite small. To decrease contact resistance between successive turns of the closed spring, a high average initial tension is built into the spring, and the turns are coated with 0.0001 in. of gold. Thus far, nickel-alloy wire has been mainly used, because of its high resistivity and small change of mechanical properties with temperature.

The new transducer is undergoing further development at the National Bureau of Standards as part of a project on basic instrumentation for scientific research supported by the Office of Naval Research.

THE FRANKLIN INSTITUTE.

MEDAL DAY MEETING.

WEDNESDAY, OCTOBER 20, 1948.

The Annual Medal Day Exercises of The Franklin Institute were held in conjunction with the Monthly Stated Meeting on Wednesday, October 20th. This annual affair had been held in April in previous years, but upon recommendation of the Committee on Science and the Arts the Board of Managers voted to hold it on the third Wednesday in October.

There were approximately 375 members and friends of the Institute who attended the Dinner and the presentation of awards.

A reception in honor of the Medalists was held prior to the Dinner. The unveiling of the portrait of our past president, Dr. Charles S. Redding, took place at this reception. Dr. and Mrs. Redding were present together with the artist, Lieutenant Colonel J. J. Capolino. Mr. Nalle officiated at the ceremony of the unveiling.

Mr. Guy Marriner, Director of Music of The Franklin Institute, played the National Anthem, which opened the evening's exercises.

Mr. Richard T. Nalle, president of The Franklin Institute, greeted the assembly in opening the programme.

Since Medal Day was also a Stated Monthly Meeting of The Franklin Institute, the minutes of the previous meeting, which was held in May, had to be approved as printed. There was no dissent.

Mr. Nalle then reported that the evening's two Franklin Medalists had been nominated by the Board of Managers for Honorary Membership in The Franklin Institute. Mr. Nalle presented their names to the membership for voting. The motion was made, seconded, and carried.

The programme and list of Medalists for this year are listed below, and detailed information will be found in a subsequent issue of the JOURNAL.

PROGRAMME

Reception for Medalists	THE HOSTESS COMMITTEE
National Anthem	GUY MARRINER
Dinner	MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin	RICHARD T. NALLE President
Greetings	RICHARD T. NALLE President
Introduction of Former Medalists	THE PRESIDENT
The Institute in 1948	HENRY BUTLER ALLEN Secretary
Presentation of Awards	THE PRESIDENT
Advances in Virus Research	DR. WENDELL MEREDITH STANLEY
Progress in Aviation	DR. THEODORE VON KÁRMÁN

(Read by W. Laurence LePage, in the absence of Dr. von Kármán.)

MEDALISTS

Presentation of Cresson Medal (1848)
Edwin H. Colpitts, (*in absentia*)
Director
The Engineering Foundation

Presentation of Longstreth Medals (1890)
Raleigh J. Wise
Telefax Research Engineer
The Western Union Telegraph Company

Nicholas F. Arone
Switchgear Divisions Laboratory
General Electric Company, Phila.

Edwin H. Brink
Assistant Director of Research
Masonite Corporation

Presentation of Potts Medal (1906)
David Bigelow Parkinson
Research Engineer
Bell Telephone Laboratories

Clarence Anding Lovell
Switching Research Engineer
Bell Telephone Laboratories

Eugene J. Houdry
Founder
Houdry Process Corporation

Presentation of Levy Medals (1923)
Jan A. Rajchman
Research Engineer
Radio Corporation of America Laboratories

William H. Cherry
Physicist
Radio Corporation of America Laboratories

Presentation of Henderson Medal (1924)
Charles Duncanson Young
Vice President (Retired)
The Pennsylvania Railroad Company

Presentation of Wetherill Medal (1925)
Wendell Frederick Hess
Head, Department of Metallurgical Engineering
Rensselaer Polytechnic Institute

Presentation of Ballantine Medal (1946)
Ray Davis Kell
Director of Television Research
Radio Corporation of America Laboratories

Presentation of Franklin Medals (1914) and Certificate of Honorary Membership
Wendell Meredith Stanley
Professor and Chairman of the Departments of Biochemistry
Director of the Virus Laboratory
University of California, Berkeley, California

Theodore von Kármán, (*in absentia*)
Professor of Aeronautics and Director of Guggenheim Aeronautics Laboratories
California Institute of Technology

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, October 13, 1948.*)

Hall of the Committee,
Philadelphia, October 13, 1948.

Mr. Walter C. Wagner, *Acting Chairman*, in the Chair.

The following report was presented for final action:

No. 3192: Fowler Flap.

This report recommended the award of a John Price Wetherill Medal to Harlan D. Fowler, of Whittier, California, "In recognition of his many years of consistent endeavor directed toward a successful solution of the problem of variable lift airplane wings, culminating in the development of the Fowler Flap as an ingenious combination of variable wing area, variable wing camber and adjustable slot."

John Frazer,
Secretary to Committee.

MUSEUM NOTES

ACTIVITIES FOR SCHOOL YEAR 1948-49.

The work of the Educational Department is in full swing, booking reservations and conducting groups on visits to the Museum, Planetarium and Dairy Dell. In anticipation of the busy fall season, two separate mailings have been prepared and sent out to a mailing list which includes over 1500 schools and several hundred administrators. Student tickets and teachers' identification cards, in sufficient numbers to take care of the student enrollment and faculty of each school have been sent out, along with information concerning group visits.

The second mailing consisted of the schedule of special school demonstrations in the Fels Planetarium for the First Semester. Many reservations were made for these shows in the first week of school sessions, with one teacher asking, on the 14th of September, for a booking in the Christmas show.

Because of the extreme popularity of these planetarium shows, and the necessity in previous years of turning so many groups away, many additional lectures have been scheduled for this season. Demonstrations are now given on Tuesday, Wednesday, Thursday and Friday mornings and on Tuesday and Thursday afternoons.

The Department is continuing to sponsor, in cooperation with the Board of Education and Station WFIL, the school radio program "Science is Fun." In connection with this weekly program, a teachers' manual is prepared and mailed out to all schools on the mailing list. This manual is designed to help teachers in preparing their classes to listen more profitably to the broadcasts, and contains suggestions for experiments children can do at home or in the classroom, vocabulary, and material tying in study visits to The Franklin Institute with the subjects in the weekly radio programs. The 1948-49 manual was prepared and mailed out in the early part of the fall semester.

The Director of Education expects this year, as in the past, to have many conferences with teachers and department heads in connection with specialized teaching problems, and to continue to work with teachers colleges and schools of education.

Cooperation with the Philadelphia Science Council, the United Nations Youth Council and many other organizations will be continued this year.

Plans are being made with the cooperation of the *Philadelphia Inquirer* for a Science Fair in May, 1949. This will be held at The Franklin Institute and students in all schools in the Philadelphia area have been invited to plan exhibits. This will include primary grades through high school.

Another special program which has been planned for the Second Semester is a repetition of the Planetarium program in connection with Classics Week. In order to be able to accommodate more students this season, additional shows for Classics Week have been incorporated in the regular school planetarium schedule.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays, and Saturdays from 9 A.M. until 5 P.M., and Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

ARCHITECTURE AND BUILDING.

MURDOCK, L. J. Concrete Materials and Practice. 1948.
VOSS, WALTER C. Fireproof Construction. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BRANDENBERGER, E. Grundlagen der Werkstoffchemie. 1947.
DAVIDSON, MARTIN. The Mid-twentieth Century Atom. 1946.
FLAGG, JOHN F. Organic Reagents Used in Gravimetric and Volumetric Analysis. 1948.
FREAR, DONALD E. H. Chemistry of Insecticides, Fungicides and Herbicides. 1948.
KIRSCHBAUM, EMIL. Distillation and Rectification. 1948.
LEWIS, WINIFRED. Thin Films and Surfaces. No Date.
PIERCE, WILLIS CONWAY AND HAENISCH, EDWARD L. Quantitative Analysis. Third Edition. 1948.
A Symposium on the use of Isotopes in Biology and Medicine. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

SMULLIN, LOUIS D. AND MONTGOMERY, CAROL G. Microwave Duplexers. 1948.
BORDEN, PERRY A. AND THYNELL, GUSTAVE M. Principles and Methods of Telemetering. 1948.
COOK, ARTHUR L. AND CARR, CLIFFORD C. Elements of Electrical Engineering. Fifth Edition. 1947.
CROFT, TERRELL. American Electricians' Handbook. Sixth Edition. 1948.
DÜNNER, ERNST. Einfuhrung in die Elecktrotechnik. 1947.
SKILLING, HUGH H. Fundamentals of Electric Waves. Second Edition. 1948.

ENGINEERING.

ANDERSEN, PAUL. Substructure Analysis and Design. 1948.
MARIN, JOSEPH. Strength of Materials. 1948.
WHITE, ALFRED H. Engineering Materials. Second Edition. 1948.

FOOD.

ANSON, M. L. AND EDSALL, J. T. Advances in Protein Chemistry. Volume 4. 1948.
CRUESS, WILLIAM VERE. Commercial Fruit and Vegetable Products. Third Edition. 1948.

GEOLOGY.

FOSTER, EDGAR E. Rainfall and Runoff. 1948.
MCKINSTRY, HUGH EXTON. Mining Geology. 1948.

MANUFACTURE.

FASSETT, F. G., Editor. The Shipbuilding Business in the United States of America. Two Volumes. 1948.
FOUTS, E. L. AND FREEMAN, T. R. Dairy Manufacturing Processes. 1948.
MCCARTHY, JAMES REMINGTON. A Matter of Time. 1947.

MATHEMATICS.

MAC ROBERT, T. M. Spherical Harmonics. Second Edition. 1947.

MECHANICAL ENGINEERING.

HAM, CLARENCE WALTER AND CRANE, E. J. Mechanics of Machinery. Third Edition. 1948.
LORENZI, OTTO DE. Combustion Engineering. 1948.
RAUH, KURT. Praktische Getriebelehre. Two Volumes. 1931–1939.
SEVERENS, WILLIAM H. AND DEGLER, HOWARD E. Steam, Air and Gas Power. 1948.

METALLURGY.

SISCO, FRANK T. Modern Metallurgy for Engineers. Second Edition. 1948.

ORDNANCE.

BURCHARD, JOHN E. Rockets, Guns and Targets. 1948.

PATENTS.

PALMER, ARCHIE M. Survey of University Patent Policies. 1948.

PHYSICS.

BRAILSFORD, F. Magnetic Materials. 1948.
DAVIDSON, MARTIN, Editor. The Gyroscope and its Applications. Second Edition. 1948.
DEN HARTOG, JACOB PETER. Mechanics. 1948.
POYNTING, JOHN H. AND THOMSON, JOHN J. A University Text-book of Physics. Volume One. 1947.

PLASTICS.

NAUTH, RAYMOND. The Chemistry and Technology of Plastics. 1947.

PUBLICATIONS RECEIVED.

The Elements of Physics, by Alpheus W. Smith, 745 pages, 16 × 23 cm., drawings, illustrations, and plates. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

Thermodynamics, by Edward F. Obert. 571 pages, 15 × 24 cm., plates, drawings and tables. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.50.

Engineering Contracts and Specifications, by Robert W. Abbett. Second ed., 327 pages, 14 × 22 cm., tables. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.75.

Fundamentals of Electric Waves, by Hugh Hildreth Skilling. Second ed., 245 pages, 15 × 24 cm., drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $4.00.

Introduction to Applied Mathematics, by Francis D. Murnaghan. 389 pages, 16 × 23 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Atomic Energy, by Karl K. Darrow. 80 pages, 14 × 22 cm., tables and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $2.00.

Climatic Accidents in Landscape-Making, by C. A. Cotton. 354 pages, 14 × 22 cm., drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; no date. Price, $7.00.

Public Health Engineering, by Earle B. Phelps and Others. 655 pages, 14 × 22 cm., tables, drawings, and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $7.50.

The Mathematical Basis of the Arts, by Joseph Schillinger. 696 pages, 17 × 25 cm., charts and illustrations. New York, Philosophical Library., 1948. Price, $12.00.

Readings in the Physical Sciences, edited by Harlow Shapley and Others. 501 pages, 16 × 24 cm., drawings. New York, Appleton-Century-Crofts, Inc., 1948. Price, $3.00.

Symposium on Internal Stresses in Metals and Alloys, organized by The Institute of Metals. 485 pages, 14 × 22 cm., drawings, tables, and illustrations. London, The Institute of Metals., 1948. Price, 42s.

Handbooks of Textile Technology No. 3, The Identification of Textile Materials. 10 pages, 15 × 24 cm., illustrations and plates. Manchester, The Textile Institute, no date. Price, 5s (Paper).

BOOK REVIEWS.

INDUSTRIAL MANAGEMENT, by William R. Spriegel. Fourth edition, 656 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1947. Price, $5.00.

The objective behind any serious attempt to analyze the factors in industrial management is to study those administrative and managerial policies that have been most effective, to understand the physical phases of production, and to discover the proper balance among the managerial, physical and employee relationships. The fourth edition of this book, now at hand, uses this as a plan, and attempts to make its presentation in sympathy with the college student who desires to learn the latest practices and the theory behind them, and the executive who is interested in suggestions that may help him solve his current problems.

The work is laid out in a logical way, starting with organization structure involving types of organization, coordination, morale, product development and research, standardization, processes, and inspection. The plant and equipment are next taken up including the elimination of limiting or bottle-neck operations, production centers, layouts, internal transportation, the production line, materials handling, lighting, power, air conditioning, etc. Motion and time study is a subject treated as both an engineering and a personnel management function. The taking of time studies, establishing true values by time study and the utilization of time study data are given attention. An up-to-date and enlightening discussion is that on wage payment, particularly in the beginning devoted to the basis of industrial wages.

A similarly interesting section of the book is that which follows on buying, selling, and transportation. With regard to the sales department it is stated "The growth in the size and complexity of present day markets has forced management to give increasing attention to the organization of the sales force and to the formulation of policies to guide this organization." A considerable portion of the book is devoted to material production and control, and the final part of the book is on personnel administration and management. In the back of the book there are a bibliography and a comprehensive subject index.

The book covers a large field for the industrial manager as well as those in charge of the many departments under him. It is a thorough, clear, and modern presentation.

R. H. OPPERMANN.

THE MODERN METALLURGY OF ALLOYS, by R. H. Harrington. 209 pages, diagrams, tables, 14 × 21 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

Dr. Harrington, who is research metallurgist for the General Electric Company, has spent twenty-five years in the field of metallurgy. During this time he became aware that there seemed to be a lack of basic structure in the knowledge of the subject. There were ample data on the "whats" of metallurgy, but little on the "whys." His purpose in this book has been to supply some of these "whys," thus providing a foundation for the data that have been compiled.

The author's approach is both logical and fundamental, following what he considers the relative importance of the various topics. One of his major contributions has been the classification of terminology. Metallurgy, only about forty years old as a science compared with centuries of practice, has been beset by a multiplicity of theories and an overlapping in terms. Dr. Harrington has endeavored to express but one meaning with each term, choosing that which he felt was best represented by the word.

His first chapter constitutes a discussion and definition of heat-treatment terms grouped according to the three main alloy fields: "(A), allotropic modifications as in the iron-carbon system; (B) the cold-worked solid solutions, exemplified by cold-rolled brasses; and (C) precipitation-hardening alloys with the recently popularized copper-beryllium alloys as an example."

In the second chapter he presents a classification of ten types of equilibrium diagrams, accompanied by other diagrams showing the relationship of cooling curves and physical properties. He discusses likewise eight groups of reactions in the solid state, which are basic to heat treatments. The next chapter considers the role of strain in reactions of the solid state in alloys.

An interesting table which Dr. Harrington has prepared forms the subject of "A Metallurgist's Periodic Table." This includes data on crystal structure, lattice constants, closest atomic approach, melting point, resistivity, atomic radius, and energy levels for the different elements. The viewpoints of the physicist and of the chemist on metals are next presented.

A reverse viewpoint is offered under the title "The Metallurgist Looks at the Physics and Chemistry of Alloying." Here Dr. Harrington considers four factors in the formation of binary alloys, namely—atomic size, crystal-structure bonding, periodic grouping, and type of lattice. These four factors are then studied in connection with the alloys of the ten different binary alloy systems. A concluding chapter offers comments on porosity and particles.

In his correlation of practical data with modern theory, the author offers a presentation which should interest both the practical and theoretical metallurgist.

G. E. PETTENGILL.

Q. E. D. M. I. T. IN WORLD WAR II, by John Burchard. 354 pages, plates, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

An active participant himself, Mr. Burchard has written a comprehensive and readable account of the contributions of Massachusetts Institute of Technology toward winning World War II. The book is divided into several parts, each of which takes up a different phase of the activities.

After a brief introductory section, the author discusses M. I. T. contributions on a national level. Its men played a leading role in the various research—directing agencies as well as taking a part in the active research for the Army, Navy, and NDRC.

Then follows an account of the research carried on at M. I. T. itself. This was of various kinds, although the developments of the Radiation Laboratory have received perhaps the most attention in the public mind. Flash photos played an important role in the European invasion and Mr. Burchard discusses their wartime development in some detail as an example of some of the problems involved.

Another section points out that other M. I. T. men made their contribution by serving on the production front and on the psychological front. Not the least important activity was the continuance of the teaching function, as not only were there special courses for the Army and Navy, but there was the vital need of maintaining a continuing though diminished flow of trained engineers for the years ahead.

Mr. Burchard emphasizes that this is the record of only one institution and that it was paralleled by others. It is a fine record and one that adequately demonstrates the capacity of a private university to contribute to the nation's welfare as effectively in time of war as it does in time of peace.

G. E. PETTENGILL.

APPLIED THERMODYNAMICS, by Virgil Moring Faires. Revised edition, 480 pages, illustrations, 15 × 24 cm. New York, The Macmillan Co., 1947. Price, $4.75.

Textbooks are continually being revised, as experience in teaching dictates a more impressionable presentation, and in order to keep abreast of new knowledge. Each succeeding student, therefore, receives increasing benefits. The most successful textbooks are built on tried and improved theories of presentation. The book at hand is a revised edition and attempts to put forth the subject in a manner of conceptions which in many cases build up, one on the other, into a large integrated image. In doing this, liberties are taken to digress from the usual beaten path to enlist interest and give life and substance to an otherwise abstract theory.

There are twenty-three chapters in all. The first gives conceptions concerning the transformations of energy, the transfer of energy, the storage of energy, the first law of thermodynamics, and the consequent general energy equation. Immediately following this is a foreword to the student, placed here "because so few people read forewords." And it contains information that is workable and profitable regarding study habits. Then energy equations of a perfect gas and reversible non-flow processes of gases furnish a foundation for an understanding of thermodynamic cycles which follow. This deals with the Carnot and Ericsson cycles and important deductions regarding limitations of any power machine. At this point, the student is warned that it is necessary to master completely the analysis of cycles. The trend then leads into entropy and the second law of thermodynamics, internal combustion engines, compression and expansion of air, and the flow of gases which, by the way, is approached from the point of view of the energy equation only, excluding the general subject of fluid mechanics.

What may be called another section of the book starts with the subject of combustion and the products of combustion and proceeds through specific heats, liquids and vapors, processes of vapors, to steam power plant and auxiliary apparatus. This is followed by the study of thermodynamic cycles used as standards of comparison, the reciprocating steam engine, and the flow of steam in nozzles and turbines. The final chapters of the book are devoted to refrigeration, mixtures of vapors and gases, and the transfer of heat. In the back of the book there is a comprehensive subject index.

The book is a practical exposition of the principles of the subject, utilizing experience in teaching and modern methods of presentation.

R. H. OPPERMANN.

MAGNETIC CONTROL OF INDUSTRIAL MOTORS, by Gerhart W. Heumann. 589 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1947. Price, $7.50.

This is a subject which is rapidly attaining the specialized stage. Indeed, the application of motor controllers in the most efficient and proper manner often requires extensive knowledge and experience. Control equipment is recognized rightly as the director of motor performance and protector of the operator and the drive. Depending on the work to be done, controllers can be quite complicated and can become parts of integrated control systems involving a multiplicity of rotating and stationary power apparatus. This book is written from the viewpoint of the application engineer and user in treating on what control equipment can accomplish to the end that good judgment can be exercised in selecting the type of control to apply.

At the outset the principles of direct current and alternating current motor operation are given and upon this foundation there is built the discussion of contractors and relays. This leads into descriptions of circuit breakers, control transformers, voltage stabilizers, rectifiers and other equipment which are parts of control equipment. Separately then, there are taken up pilot devices, electronic devices, basic control circuits, amplidyne control circuits, solenoids, thrusters, brakes, intermittent duty controllers, and maintenance of control equipment. One of the most common and most interesting control set-ups in an industry plant is that of the synchronous motor. This is treated quite completely including field application by current-time relay, by slip cycle impedance relay, by slip frequency relay and by timing relay. Field removal is also given attention.

The book has a great many illustrations such as curves, wiring diagrams and photographs to which reference is made and which are very helpful. It is written in a language understandable by the average engineer and is quite complete in coverage. In the back is a subject index which adds considerably to the value of the book as a reference. The book will be useful in the plant engineer's office as a descriptive text.

R. H. OPPERMANN.

ELEMENTS OF ELECTRICAL ENGINEERING, by Arthur L. Cook. Fifth edition, 662 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1947. Price, $5.00.

This work is well known to a great many professors and former students as a treatise on fundamentals and their application in practice. There were four editions beginning in 1924 and ending in 1941. During these years considerable advancement was made both in the subject matter and in methods of presentation which necessitated the newer editions. The book at hand is the fifth edition, revised and rearranged.

The first part of 99 pages is devoted to electric and magnetic circuits, giving attention to the electric field, electric potential and current, the magnetic field, its density, hysterisis, induction, capacitance, and the various units involved. With this as a background, Part Two leads into direct current machinery, beginning with a discussion of parts and construction of dynamos, how they operate and what they do. This is an especially good approach, since both theory and practice are covered together. Then there are discussed losses and efficiencies, ratings and guarantees of machines. Control and protective devices are covered separately, followed by a treatment on batteries and systems of direct current transmission and distribution.

Part Three begins the subject of alternating currents, picking up the thread left by Part One. A genuine attempt is made to illustrate clearly the wave form as developed by a rotating machine, and then transferring this into a discussion on sinusoidal currents and voltages. This section includes the mathematical representation of vectors which is quite well illustrated for the solution of alternating current circuits. The properties of circuits involving resistance, inductance, and capacitance in parallel and in series and the solution of problems using the terms of admittance, conductance and susceptance are thoroughly covered. The final chapter in Part Three is devoted to polyphase circuits. The next part of the book on alternating current machinery starts with a description of the transformer, its physical make-up, vector representations and equivalent circuits. The same method is used with regard to polyphase induction motors, generators, synchronous motors, and synchronous convertors. Single phase motors are taken up separately in similar fashion and the basic types are treated. The rest of this section is devoted to transmission, distribution, switching, protective devices, and motor applications. The last or Part Five of the book, gives a concise but comprehensive treatment of fundamentals and characteristics of electron tubes, the functions and basic circuits of electron tubes and the principles of applications of electronic devices to industry.

This is a plain treatment with many exercises, which if studied diligently and the problems solved, will give a thorough knowledge of electric power machinery.

R. H. OPPERMANN.

THE CHEMISTRY AND TECHNOLOGY OF PLASTICS, by Raymond Nauth. 522 pages, drawings and illustrations, 16 × 23 cm. New York, Reinhold Publishing Corp., 1947. Price, $9.50.

The origin of the plastics industry can be traced back many centuries to the Egyptians and Chinese. As time went on, increasing investigation was made resulting in discoveries, many of which were not put into commercial use because the world was not ready for them. Some plastics like acrylics lay on the shelf for a century before being commercially practical. Today the search for plastics still goes on. As each new material is developed it may serve a particular purpose or have a specific merit, or a particular application. It is possible now to design a plastic material having properties to suit a specific purpose. With this rapid expansion it becomes increasingly difficult to visualize the field of plastics to come.

This book has five sections. The first, on thermosetting resins, has an introduction before discussing various resin compounds falling into this category. Later, attention is given individually to amino resins, thermosetting cast resins, amine-aldehyde resins, the viscous resins, and laminates. Summary data of a general nature appear at the beginning of the second section in the form of tables giving, for well-known types of plastics, adaptability, physical-chemical, and electrical properties, the effect of aging and exposure to light, the coefficient of thermal expansion. This is a fund of information, the location of which in this book can be questioned on the grounds of practical availability for quick reference by those not familiar with the detailed content and plan of the book. The second section on thermoplastic resins covers vinyl polymers and copolymers, and acrylic, polyamide, organic silicon and polystyrene resins. Special attention is given to the grouping of the cellulose plastics in section three, to which there is appended a chapter on regenerated cellulose. Synthetic rubber, natural resins, and plywood are discussed in section four. The last part of this includes veneer processes and molding. The final section of the book, devoted to mold design, presses and equipment, applies the chemical knowledge in the preceding sections to the processes and technology necessary to the finished product.

The book is well written, profusely illustrated, and gives a fund of information on the subject. Diagrams showing relationships and derivations are especially interesting. The appendices and subject index add to the value of the book.

R. H. OPPERMANN.

STEAM, AIR, AND GAS POWER, by William H. Severns and Howard E. Degler. Fourth edition, 509 pages, 15 × 23 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $4.75.

The fourth edition of this widely used textbook follows the well-established practices of the previous editions. It supplies the student of power engineering with the basic information which he will later use in application. The present, revised edition brings the material up-to-date with special attention paid to the gas turbine, the development of which has been unusually rapid within the past ten years.

No significant changes have been introduced in the chapters on steam and internal-combustion engines, although here the subject matter also has been rearranged and further clarified.

H. N. MICHAEL.

BOOK NOTES.

QUANTITATIVE ANALYSIS, by Willis Conway Pierce and Edward Lauth Haenisch. Third edition, 520 pages, 15 × 24 cm. New York, John Wiley and Sons, Inc. Price, $3.75.

Since the second edition of this text was written many changes have been made in the presentation of material to classes and this new third edition, almost entirely rewritten, incorporates these changes. Numerous laboratory exercises have been added and the questions and problems are mostly rewritten and almost doubled in number. The material is presented in four parts: Fundamentals, Volumetric Analysis, Gravimetric Analysis, Special Methods. There are several appendices, which contain useful tables and an index.

W. P.

VIBRATION AND SOUND, by Philip M. Morse. Second edition, 468 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.50.

In the revision of his work, the author, now director of Brookhaven National Laboratory, has brought up-to-date this important text on the theory of vibration and sound. Wartime developments have been included and the recent advances in the theory of room acoustics have necessitated considerable rewriting of Chapter VIII, Standing Waves of Sound.

In his discussion of the application of operational calculus and the Laplace transform methods to the study of acoustical transients, the author has made a major addition. Besides the new material in the text, the number of problems has also been considerably augmented.

G. E. P.

MECHANICS OF MACHINERY, by C. W. Ham and E. J. Crane. Third edition, 538 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.00.

This new edition has been revised to include new subjects and bring the material up-to-date. It contains more material than the previous editions, although the same general sequence that was featured in the earlier editions has been retained. The book is divided into two parts. Part One treats mechanism and Part Two, kinematics and dynamics of machinery. The book is arranged in seventeen chapters. Chapter 17 is on the gyroscope, which is new material for this edition. This is followed by an appendix, problems and an index.

W. P.

POPULAR GEMOLOGY, by Richard M. Pearl. 316 pages, illustrations, 14 × 21 cm. New York, John Wiley & Sons, Inc., 1948. Price, $4.00.

Mr. Pearl presents in his book a semitechnical survey of modern gemology. Its eight chapters are systematically arranged and the classifications are based, for the most part, on *Dana's System of Mineralogy.*

The bulk of the work is divided among the first three chapters which deal, respectively, with the recognition of gems, with faceted gems, and with cabochon gems. There are also chapters on silica gems, organic gems (pearls, coral, amber, jet), synthetic gems, and a highly interesting discussion on luminiscent gems.

Jewelers, mineral collectors, and the gem lover in general will find the material discussed in this book authoritative and up-to-date.

H. N. M.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Manometric Determination of Carboxypeptidase.—CHARLES A. ZITTLE. Carboxypeptidase has been estimated by formol titration with chloracetyltyrosine as the substrate (1). This enzyme retains much of its activity in the presence of formaldehyde and in the procedure mentioned formaldehyde is added at the beginning of the estimation. The amino group of the tyrosine released by hydrolysis is masked by the formaldehyde and the strong monochloracetic acid is readily measured. In the above procedure the acid is estimated by titration at the end of the experiment. This acid also could be conveniently measured manometrically with $NaHCO_3$ present, which would offer the advantage that the acid release could be followed throughout the experiment. The utilization of the manometric procedure is described herein.

EXPERIMENTAL.

Enzyme. The carboxypeptidase was prepared from frozen pancreas by the method described by Anson (1). Preparations of high activity were obtained but satisfactory crystallization did not occur. Recently additional literature on the crystallization of this enzyme has appeared (2,3,4). For the initial assays the precipitate which appears on dialysis was used: 1.0 cc. of a heavy suspension was dissolved with 1.0 cc. of 4.0 per cent NaCl and 1.0 cc. of 0.5 *M* $NaHCO_3$. The concentration was estimated by a nitrogen determination.

Substrate. 0.5 g. of chloracetyl-1-tyrosine was suspended in H_2O, brought into solution with *N* NaOH (about 2.0 cc.) and adjusted to a pH of 9 and a final volume of 25.0 cc. 20.0 cc. of this solution were mixed with 5.0 cc. of 35 per cent C.P. formaldehyde.

Procedure. The following solutions were placed in Warburg flasks: 0.2 cc. of 0.5 *M* $NaHCO_3$, 2.0 cc. of substrate, and water to make the final volume 3.5 cc. including the enzyme which was placed in the side arm. The measured pH was 7.5. The experiments were performed at 37° with an atmosphere of 5 per cent CO_2–95 per cent N_2. The final concentration of substrate was 0.036 *M;* a lesser concentration did not permit a maximum rate of hydrolysis. An experiment was performed in which alanine was introduced into a solution of formaldehyde and dilute $NaHCO_3$; the immediate release of acid, measured manometrically, indicated that the interaction of formaldehyde and amino group was very rapid. A typical experiment with carboxypeptidase is shown in Fig. 1. The rate of hydrolysis was proportional to the amount of enzyme used when readings were taken within the first ten minutes.

The subsequent decline in hydrolysis was probably due to the inhibitory effect of the monochloracetic acid (5) which was formed. The acid group released was sufficiently strong so that it was not completely neutralized by the amino group and it could be measured manometrically without the addition of formaldehyde. The measured acid release, however, was only about 40 per cent as great as when formaldehyde was added.

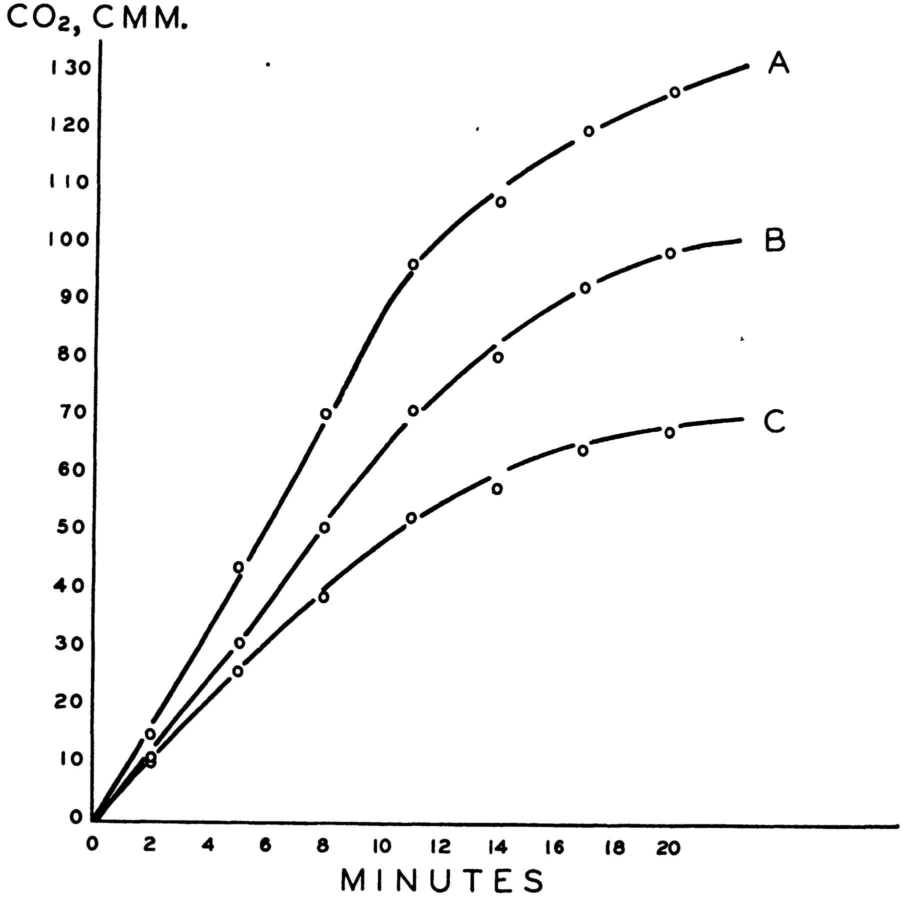


FIG. 1. Hydrolysis of chloracetyltyrosine by carboxypeptidase measured manometrically. The enzyme solution used contained 0.043 mg. N per cc.

Curve A: 0.8 cc. of enzyme solution
Curve B: 0.6 cc.
Curve C: 0.4 cc.

RESULTS.

In the experiment cited the enzyme which precipitated on the first dialysis of the pancreas extract was used. The activity was 2,600 cmm.

or 0.116 milliequivalent/1.0 mg. N/10 minutes. In the course of the subsequent purification the activity was raised to 0.179 milliequivalent/1.0 mg. N/10 minutes. Anson (1) defined a unit of carboxypeptidase activity as that amount of enzyme which produces 1 milliequivalent of acid in 1 minute. In terms of these units our enzyme contained 0.0179 units/1.0 mg. N. Anson's crystalline enzyme contained 0.081 units/1.0 mg. N. Whether the two methods of estimation give comparable units can only be decided when the crystalline enzyme is available.

DISCUSSION.

The decrease in activity of the carboxypeptidase with time probably is due to the inhibitory effect of the monochloracetic acid (5). Carbobenzoxyglycyl-1-phenylalanine does not have this drawback and it is more rapidly hydrolyzed (5,4). Formaldehyde exerts some inhibitory effect on carboxypeptidase (5) but procedures using it should give satisfactory comparative data.

SUMMARY.

A procedure for the manometric estimation of carboxypeptidase is described.

REFERENCES.

(1) M. L. ANSON, *J. Gen. Physiol.*, **20**: 663, 781 (1937).
(2) J. B. SUMNER and G. F. SOMERS, "Enzymes," 2nd Edition p. 186. New York, Academic Press (1947).
(3) F. W. PUTNAM and H. NEURATH, *J. Biol. Chem.*, **166**: 603 (1946).
(4) H. NEURATH, E. ELKINS, and S. KAUFMAN, *J. Biol. Chem.*, **170**: 221 (1947).
(5) K. HOFMANN and M. BERGMANN, *J. Biol. Chem.*, **134**: 225 (1940).

CURRENT TOPICS.

Natural Gas Stored in Underground Pipes. (*Compressed Air Magazine*, Vol. 53, No. 8).—Natural gas tapped from fields of the Texas Panhandle and piped nearly 1000 miles by the Public Service Company of Northern Illinois is being stored by the latter in a novel arrangement of buried pipe sections. This method of storage is the first of its kind in the country and originated when the company, faced with the necessity of enlarging existing facilities to meet increased consumer demand, decided to explore the possibility of storing gas underground instead of on the surface in conventional holders. Accordingly, in the fall of 1946, a pilot installation was made at Kankakee, Ill., with a capacity of 1,250,000 cu. ft. The success of this project led to the completion, recently, of a similar system on a 160-acre tract at Mount Prospect, Ill., where 40 million cubic feet of gas is stored underground.

Seamless pipe of high-carbon, molybdenum-alloy steel, similar to that used in the construction of the "Big Inch", was eventually selected. It has an outside diameter of 24 in. and a wall thickness of approximately ½ in. and was made by the National Tube Company of Pittsburgh, Pa. Random lengths were utilized, the average being about 40 ft. Because the high-carbon steel would have been difficult to weld in the field, both ends of each pipe section were forged at the fabricating plant into hemispherical shape and the solid hubs were drilled and tapped to receive 1½-in. threaded connections. After being stress-relieved, the unit was tested hydrostatically at 2800 psi. before being shipped. The volume of each container averages 109.5 cu. ft., the weight 5100 lb., and the capacity 25,000 cu. ft. of natural gas at 2240 psi. pressure.

Of particular interest in this type of storage is the effect of high pressure on the compressibility factor of natural gas. At the operating pressure of 2240 psi., natural gas deviates most from its expected performance, as calculated according to the laws for a perfect gas. Whereas computations indicate that 160 cu. ft. of perfect gas can, under a pressure of 2240 psi., be compressed into 1 cu. ft. of space, it is actually possible with the same pressure to force 230 cu. ft. of natural gas into the same space. This behavior of natural gas increases the capacity of the pipe sections by some 40 per cent, permitting the storage of much more gas than their actual volume would seem to indicate.

As the compressibility factor of natural gas becomes increasingly favorable with decreasing temperature, it is desirable to bury the pipe at a depth that will give reasonably low temperatures in winter and yet minimize seasonal changes. At 2240 psi. pressure, each 1°F. variation in temperature causes a pressure change of about 10 psi., one half of which is attributable to the change in the compressibility factor that accompanies the temperature change. Owing to seasonal temperature variations, some gas must be withdrawn from storage in spring and summer and a corresponding amount put back in fall and winter.

R. H. O.

The Transistor—A New Amplifier. (*Electrical Engineering*, Vol. 67, No. 8) —A new type of amplifying element known as a "transistor" which promises to have far reaching significance in electronics and electrical communication was demonstrated recently before members of the press by the Bell Telephone Laboratories, Inc., New York, N. Y. Like a vacuum tube, the transistor can serve as an amplifier or as an oscillator, yet it contains no vacuum, glass envelope, grid, plate, or cathode.

Two point contacts of the "cat's whisker" or detector type are made to a semiconductor (germanium was used in the samples demonstrated) 0.002 in. apart. The entire apparatus is housed in a small metal cylinder less than an inch long. The unit has been shown to produce amplification as high as 100 to 1 (20 decibels), and some test models have been operated as amplifiers at frequencies up to ten megacycles.

A number of interesting applications for the transistor were presented. In one demonstration it was used to amplify audio frequencies traveling between two telephones. · In another, a tubeless radio receiver using transistors as amplifiers, and a point contact rectifier for a detector, was operated satisfactorily. A transistor also was used to generate a standard frequency tone, thus demonstrating its capability as an oscillator.

The transistor is said to be little affected by ordinary changes in temperature (-50 to $+50$°C.), and such characteristics as life, ruggedness, and stability are expected to be much the same as those of point contact rectifiers though no data are available at the present time. Because the device is still in the experimental stage no data on cost are available either.

Electrically, the transistor differs somewhat from the ordinary vacuum tube. The input impedance is low (about 1,000 ohms) and the output impedance is high (about 10,000 ohms). The power handling capability of the units demonstrated was 50 milliwatts, and the power consumption is about 0.1 watt.

The amplification process in the transistor is caused by the fact that the input contact (called the "emitter") is surrounded by an "area of interaction." Within this area the electronic structure of the semiconductor is modified by the input current. If the output contact (called the "collector") is placed in this area, the output current can be controlled by the input current thus producing the basic mechanism of amplification.

R. H. O.

Silver Reclaiming Kit.—Gopher Laboratory, Minneapolis 1, Minn., now has available a Silver Reclaiming Kit designed especially for users of hypo in small quantities.

Active material is the same as that in the large Silver Reclaiming Cartridge Units, which have been in use by many large commercial photofinishers the past two years.

The Kit consists of 50 Silver Reclaiming Filaments, each of which will reclaim the silver from ½ gal. of film or X-ray hypo, or 1 gal. of print or photostat hypo.

The process is very simple, and merely consists in dropping one or more Filaments into the container of exhausted hypo, and allowing to stand overnight. The next day the hypo is poured off, and the silver-covered Filament lifted out. When the entire Kit has been used, the dried Filaments are sent to the Laboratory for refining. Upon receipt, the user is paid 100 per cent of the silver value, minus refining charges.

The Kit sells for $1.50 and will reclaim up to $15.00 worth of silver.

Steam Pipes for Hottest Turbine to Run Dull Red.—The 100,000-kw. turbine recently shipped by the Westinghouse Electric Corp. for installation at the Sewaren Generating Station of the Public Service Electric and Gas Co. of New Jersey will operate at such high temperatures and pressures that the steam pipes feeding it will glow a dull red. Steam at 1500 lb. pressure and 1050°F. will feed the new 3600-rpm. tandem compound double flow machine. This is the first steam power plant to be constructed for operation under such conditions.

While the highest temperature previously used in turbines is 1000°F., it is the last 50°F. that makes the job tougher. To withstand operating in the dull red region steam containers such as the throttle valves, steam chests and other high pressure parts of the turbine are constructed of 18–8 stainless steel. Fabrication of parts from this hard-to-work material has been a problem in itself.

The importance of the 50° rise in temperature is in the fuel savings. For each 50° increase, the quantity of coal required to produce one kilowatt hour of electrical energy is decreased by about 1.4 per cent. In 1915 the output of all our steam power plant turbines averaged only 525 Kwh. per ton of coal burned. With further developments towards increased plant efficiency, the output of all steam power plants by 1945 was 1550 Kwh. per ton of coal burned. These figures include both old and modern plant equipment in service during the year mentioned. This new turbine will produce about 2500 Kwh. of electrical energy from each ton of coal burned.

Gas Turbine for Electric Utility.—The first gas turbine for an electric utility in this country is under construction at the General Electric Company's Schenectady Works and will be shipped to the Southwest early in 1949.

A duplicate of the 4800-hp. locomotive gas turbine now undergoing tests, the 3500-kw. turbine generator set was purchased by the Oklahoma Gas and Electric Co. It will be installed at the Arthur S. Huey station, Oklahoma City.

The new gas-fired unit will be installed in an extension to the present station and operated on natural gas which is available in abundance in the area. The waste heat from the exhaust will be used with a separate heat exchanger to supplement the present boiler feed water heating system. This will result in additional kilowatt output from the station by releasing part of the heating load from the present installed equipment.

The "straight-through" or in-line arrangement of the compressor, combustion chambers, and turbine was selected for this application. It has been designed for long life and low cost fuels.

Because there are only two major moving parts, the turbine and compressor rotors, maintenance of the gas turbine is expected to be low.

The unit has been designed for maximum accessibility during maintenance and inspection periods, elements such as combustion chambers which might normally require maintenance being easy to replace.

The new gas turbine will be geared to a conventional 3600-rpm. totally enclosed generator with direct connected exciter. The only water required in the operation of the plant will be a small quantity for cooling the bearing lubricating oil and for cooling the a-c. generator.

R. H. O.

Operation Bootstrap.—The importance of "Operation Bootstrap," a means for producing a new atomic fuel while other atomic fuel is consumed making useful power, was emphasized by Bruce R. Prentice in a paper presented recently to the Pacific Convention of the A.I.E.E. Mr. Prentice spoke on "Design Criteria for Atomic Power Reactors."

"Perhaps the most important criterion of a major atomic power plant of the future will be its ability to 'breed' more new fuel than it burns," Mr. Prentice said. He also said that such an atomic plant, instead of just using up the world's supply of fuel as is done in ordinary fuel plants, would add to the world's supply at the same time it released useful energy.

"Of course, we never get something for nothing," Mr. Prentice pointed out, "and here the newly created fissionable material comes from the non-fissionable 'source material' uranium 238 or from thorium. However, neither of these is now fissionable or useful as fuel and they are vastly more abundant than the only natural atomic fuel, rare uranium 235. So in a very real sense, breeding will be a 'bootstrap' proposition of getting new fuel while burning other fuel making power."

Mr. Prentice hastened to emphasize caution in guessing when such a "breeder atomic power plant" might be a practical reality. He said that at present it looks theoretically possible and it may be demonstrated experimentally in two or three years. "Will atomic power be ours in 1965, 1975, or 2000?" he asked. He concluded that the answer will depend on future developments in technology, economics, politics, and international relations.

R. H. O.

70-Million Volt Synchrotron.—A 70-million-volt synchrotron, a type of atom smasher, is being built for Queens University, Kingston, Canada.

Under construction in General Electric's General Engineering and Consulting Laboratory, the machine will be used by the Canadian institution for nuclear research.

Newest among the "particle accelerators," as atom smashers are known to nuclear physicists, the synchrotron, like the betatron, accelerates electrons. There are significant advantages in this new type, particularly for the development of higher voltage machines.

Basic feature of the synchrotron is the principle of "phase stabilization" it employs. This means that accelerating electric impulses occur at exactly the right time to boost electrons as they circulate in the machine's vacuum doughnut, or acceleration chamber. The tendency for automatic synchronization gives rise to the name "synchrotron."

Although this electro-nuclear device utilizes an electro-magnet, the main acceleration is provided by an electrostatic field. When electrons have been accelerated by magnetic induction to an energy of 2,000,000 electron-volts, and are traveling at 97 per cent of the speed of light, a radio frequency oscillator turns on automatically. The oscillator is connected to a gap on the inside of the synchrotron doughnut, and circling electrons receive an increase in energy each time they go past.

At a predetermined time the path of the electrons is altered slightly so that they strike a small piece of metal inside the doughnut, producing an X-ray beam. These X-rays are many times more penetrating than those produced by conventional X-ray machines, or by radioactive substances such as radium.

Queens University's Prof. J. A. Gray and his collaborators, Dr. H. M. Cave and Dr. D. T. Roberts, together with other Canadian scientists, will employ the machine to conduct research with high-energy X-rays. The synchrotron will be installed in a special underground chamber to protect personnel from harmful radiations, and will be operated from a remote-control station in a nearby building.

R. H. O.

Special Committee on Technical Information Formed.—Formation of a Special Committee on Technical Information to promote effective exchange of research and development information among the departments of the National Military Establishment was announced recently by Dr. Vannevar Bush, chairman of the Research and Development Board, National Military Establishment.

The committee will study the problem of collecting, correlating, reproducing and disseminating technical information potentially useful in the research and development program of the National Military Establishment. Study and application of new methods and techniques to the problem of technical information organization, and promotion of active research in this effort, are expected to receive especial attention by the group.

At the first meeting of the committee, Dr. Detlev W. Bronk, chairman, pointed out the desirability of applying scientific methods to utilize more effectively the large body of information created by scientific activity.

"The handling of results of research in the matters of publication, dissemination and assimilation has not in the past shown an experimental or adventurous approach," Dr. Bronk stated.

Membership of the Special Committee on Technical Information includes: Professor John E. Burchard, Dean of Humanities, Massachusetts Institute of Technology; Herman Henkle, Director of the John Crerar Library; Lt. Col. F. L. Walker, Jr., Army; Captain W. H. Leahy, Navy; Colonel Bernard A. Schriever, Air Force. Norman T. Ball is executive director of the committee.

Symposium on Steroid Hormones.—A phase of science which had its roots at the University of Wisconsin returned to the university as a growing, important branch of life study at the University Centennial Symposium on Steroid Hormones, held in September.

Back in 1927, a Wisconsin scientist, Dr. Frederick L. Hisaw, began isolating chemical compounds he found in animals and people which he thought bore some relation to health.

This week, scientists from all over the world gathered at Wisconsin and confirmed his basic assumptions. They spent three days exchanging information on the findings they made—findings which indicate that these chemical compounds, complicated in structure, fatty in nature, and made for the most part in glandular organs, actually play a major part in the chemical processes of life, health, and disease.

They reported that they have isolated 56 different steroid compounds in human urine, but as one of the scientists pointed out, "we're not completely certain of the origin, the function, or the real importance of most of them."

Among the better known steroid hormones they listed vitamin D, morphine, bile acids, and cholesterol. It is cholesterol, the most abundant of all steroid compounds in animal tissue, which is the parent substance for all other steroids, they said.

Most of the work done thus far, the symposium of 600 experts in the field revealed, has been done on two of the chemical compounds, the male hormone and the female hormone.

The Symposium on Steroid Hormones was the third symposium in the University of Wisconsin's Centennial series. It was supported by funds through the university by the Wisconsin Alumni Research Foundation, and from the National Research Council Committee on Growth, acting for the American Cancer Society.

Dr. W. S. Gordon, chairman of the symposium arrangements committee, reported on the conclusion of the symposium: "It has crystalized ideas in this important field, and brought us all up-to-date on each other's work. Unfortunately, it vividly illustrated how much more work must be done in the field, too."

Exposure to Air Improves Some Wyoming-Type Bentonite.—New information about bentonite, a clay mineral used chiefly in the foundry and petroleum industries, has been uncovered according to an announcement made recently by Geological Survey Director W. E. Wrather.

Studies of Wyoming-type bentonite by Margaret D. Foster, Survey Geochemist, have revealed that the blue-gray subsurface material, now rejected in mining because of a deficiency in the necessary colloidal properties, becomes similar to high-grade material in important respects if permitted to oxidize. This blue-gray material is identical in chemical composition and iron content with the green near-surface bentonite now being mined, but it was found that the iron in the blue-gray material is predominantly in the unoxidized or ferrous state, whereas in the green material the iron has been oxidized to ferric iron, as previously suggested by M. M. Kenchtel of the Geological Survey.

The swelling test, one of the criteria by which the colloidal properties of bentonites are judged, indicated that the blue-gray material swelled or expanded in water only two-thirds as much as the green. It was found, however, that the iron in the blue-gray material oxidized relatively rapidly on exposure to air and that the swelling property also increased correspondingly. After ten months' exposure the iron in the blue-gray material had reached the same state of oxidation as that in the green material. At the same time the swelling property had increased also and had become equal to that of the green material.

This information should be of considerable value to those interested in mining the Wyoming-type bentonite because it suggests that by simple processing the blue-gray material now rejected might be converted to usable bentonite. Researches on the problem are continuing.

What's New in Equipment (*Factory Management and Maintenance*, Vol. 106, No. 9).—(1) Electrical wire with rubber insulation that is said to improve when soaked in water was developed for use underground and in locations where high moisture resistance is required. Wire is coated with natural rubber latex compound, called Laytex RUW, applied by the dip process. (2) Thermoplastic matting is suitable for use where acids or oils are present, or in locations where matting is given rigorous service. Manufacturer says it will lie flat and hug the floor. Manufactured in black and brown, it is 3/32 in. thick by 36 in. wide and comes in rolls 25 to 35 yards long. (3) Suitable for marking storage bays, aisles, stock locations, equipment clearance limits, and traffic lanes, marking paint is heavily based with fine silica, making it non-slippery. It comes in white and yellow, in 1- and 5-gal. cans. Manufacturer states that it resists strong cleaners, and retains its color and visibility after repeated scrubbings.

Czech Builds Road-Type Gas-Turbine Engine (*Product Engineering*, Vol. XIX, No. 9).—A Czech engineer, George Lakes, now working in the car repair shops at Hameln, in the British Zone of Germany, has become another entrant in the race to design a gas turbine for road vehicles. His engine is built from scrap; his only tool was a 6-in. lathe.

Apparently, Lakes' engine (about 150 hp.) combines, as the propulsive force directed into the turbine, a mixture of steam and burnt-fuel gas, at a maximum temperature of 985 F. The exhaust gas mixture is cooled in an air-cooled condenser, which should mean there is no heat-exchanger. Speed of rotation is 13,000 rpm. Calculated potential thermal efficiency is 20 to 30 per cent; thermal efficiency of the rough prototype is figured as only 5 per cent.

Line Shot Blast Room with Rubber. (*The Foundry*, Vol. 76, No. 9).—Installation of an abrasion resisting rubber compound on the walls and ceiling of its shot blast room solved a tough maintenance problem for a New England manufacturer of textile machinery. Two-inch wooden planking formerly lined the room, and under the punishment of shot from the hose against the rough castings, its ordinary life was about one month. It then had to be replaced, and the room shut down until the maintenance job was completed.

Since the ¼-in. fabric backed rubber compound manufactured by the B. F. Goodrich Co. was installed in May, 1947, there has been no maintenance problem and the installation shows no appreciable wear, according to the rubber company.

Quick Starting House Turbine-Generator Unit.—A 6000-kw. General Electric house turbine-generator unit designed for quick starting and capable of picking up full load instantly has been installed at the Pacific Gas and Electric Company's Kern Steam Plant near Bakersfield, Calif.

Under normal operation, the turbine floats on the line, carrying load in accordance with the steam demand of the deaerating heater of the main unit and exhausts through a 5-lb. extraction opening.

In case of a disturbance in the electrical system, the house turbine generator is automatically isolated from the main bus and picks up the entire station auxiliary load instantly.

The 6000-kw. turbine-generator is capable of going from a cold standstill to full speed and full load within 20 sec. with steam 1350 lb. gage and 950° F. total temperature. Extensive factory tests of the complete unit proved its capabilities for the unusual service for which it was designed.

Although designed for quick-starting, the unit is expected to be in operation practically all the time.

R. H. O.

28 FEB [illegible]

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 246 DECEMBER, 1948 No. 6

RECENT ADVANCES IN VIRUS RESEARCH.*

BY

WENDELL M. STANLEY, Ph.D.[1]

Viruses are small infectious agents that can cause disease in man, other animals, plants and bacteria. They range in size from about $10 m\mu$, a size slightly smaller than that of certain protein molecules, in an almost continuous spectrum of sizes up to about $300\ m\mu$, a size slightly larger than that of certain accepted living organisms. A given virus can multiply and cause disease only when within the cells of certain specific living organisms. No virus has been found to reproduce in the absence of living cells. During multiplication viruses occasionally change or mutate to form a new strain which in turn causes a new disease. Viruses were not discovered until 1892 when Iwanowski demonstrated that the causative agent of the mosaic disease of tobacco would pass through a filter that retained all known living organisms. Six years later Beijerinck proved that this agent was not an ordinary living organism and recognized it as a new type of infectious disease-producing agent—namely, a virus. The same year Loeffler and Frosch demonstrated that foot-and-mouth disease of cattle was caused by a virus. The discovery of the first virus disease of man, that of yellow fever, was made in 1901 by Reed and co-workers.

Since the original discovery of the infectious, disease-producing agent known as tobacco mosaic virus, well over three hundred different viruses, capable of causing disease in man, animals and plants have been discovered. Among the virus-induced diseases of man are smallpox, yellow fever, dengue fever, poliomyelitis, certain types of encephalitis, measles, mumps, influenza, virus pneumonia and the common cold. Virus diseases of animals include hog cholera, cattle plague, foot-

---

* Talk presented at the Medal Day Meeting at The Franklin Institute, October 20, 1948.

[1] Professor of Biochemistry and Director of the Virus Laboratory, University of California, Berkeley, Calif.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Linlithgow Library.
Imperial Agricultural Research Institute

and-mouth disease of cattle, swamp fever of horses, equine encephalitis, rabies, fowl pox, Newcastle disease of chickens, fowl paralysis, and certain benign as well as malignant tumors of rabbits and mice. Plant virus diseases include tobacco mosaic, peach yellows, aster yellows, potato yellow dwarf, alfalfa mosaic, curly top of sugar beets, tomato spotted wilt, tomato bushy stunt, corn mosaic, cucumber mosaic, and sugar cane yellow stripe. Bacteriophages, which are agents capable of causing the lysis of bacteria, are now regarded as viruses.

The viruses have been separated as a special group of infectious, disease-producing agents by means of several general properties, no one of which is, however, exclusively characteristic of viruses. Nevertheless, no great amount of difficulty has been encountered in the segregation of the virus group. Viruses are characterized by their small size, by their ability to reproduce or multiply when within the living cells of a given host, by their ability to change or mutate during multiplication and by their inability to reproduce or grow on artificial media or in the absence of specific living cells. The sole means of recognizing the existence of a virus is provided by the multiplication of the virus which is, of course, usually accompanied by manifestations of disease. Viruses spread from diseased to normal susceptible hosts by different methods. Some are transferred by direct contact, as when a diseased leaf is caused to rub against a healthy leaf by a gust of wind, or when a normal person or animal comes into direct contact with a diseased person or animal. Such viruses can usually be spread by indirect contact through the medium of non-specific animate or inanimate objects. Some viruses cannot be transferred by direct contact, but require an intermediate host such as a mosquito, louse, or leaf-hopper. In some cases a highly specific intermediate host is necessary, and a more or less definite period of incubation within this host may be required before the virus can be transmitted.

Because properties such as reproduction and mutation have long been considered characteristic of living entities, viruses were, for many years, regarded as living organisms somewhat smaller than ordinary bacteria. However, the isolation in 1935 of tobacco mosaic virus in the form of a crystalline nucleoprotein of unusually high molecular weight and the subsequent isolation of still other viruses in the form of high molecular weight nucleoproteins, some of which were also crystallizable, cast doubt upon the validity of classifying all viruses as organisms. With the exception of virus activity, the properties of some of the smaller viruses are quite similar to the properties of ordinary protein molecules, whereas at the other extreme with respect to size, the properties of the viruses are more nearly like those of accepted living organisms. The viruses, therefore, serve as a bridge between the molecules of the chemist and the organisms of the bacteriologist, and provide us with new reasons for considering that life, as we know it, owes its existence to structure, to a specific state of matter and that the vital phenomenon does not occur spontaneously, but is possessed in varying degrees by all matter. It is obvious that a sharp line dividing living from non-living things cannot be drawn and this fact serves to add fuel for discussion of the age-old question of "What is life?".

Attempts to learn something about the nature of viruses through studies on their general properties began with Beijerinck's work in 1898 and were continued in different laboratories for over thirty years without too much success. Although Beijerinck and Allard made important contributions, perhaps the most significant work was that of Vinson and Petre during the years from 1927 to 1931 when they showed that tobacco mosaic virus could be subjected to several kinds of chemical manipulations without loss of virus activity. Nevertheless, in 1932 the true nature of viruses was a complete mystery. It was not known whether they were inorganic, carbohydrate, hydrocarbon, lipid, protein or organismal in nature. It became necessary, therefore, to conduct experiments which would yield information of a definite nature. Tobacco mosaic virus was selected for these initial experiments because it appeared to provide several unusual advantages. Large amounts of highly infectious starting material were readily available and the virus was known to be unusually stable. Furthermore, it was possible to titrate or measure the amount of this virus in a preparation with ease and rapidity and with great accuracy. During the course of a wide variety of early exploratory experiments, it was found that the enzyme pepsin inactivated tobacco mosaic virus only under conditions under which pepsin is active as a proteolytic agent. It was concluded that tobacco mosaic virus was a protein or very closely associated with a protein which could be hydrolyzed by pepsin. With this as a lead, efforts were made to concentrate and purify tobacco mosaic virus by means of the methods previously employed in work with proteins. Soon, by means of a combination of procedures involving salting-out, isoelectric precipitation and adsorption on and elution from an inert material, a crystalline material was obtained which possessed the properties of tobacco mosaic virus. This crystalline material was found to be a nucleoprotein with rod-shaped molecules or particles about 280 by 15 $m\mu$ in size and with a molecular weight of about 40,000,000. Early skepticism that a virus could exist in the form of a crystallizable nucleoprotein has largely disappeared, chiefly because the results of a vast amount of experimental work have indicated that the virus activity is a specific property of the rod-shaped nucleoprotein.

Tobacco mosaic virus exists in the form of many strains which appear to have arisen by a process similar to that of mutation in higher organisms. Several of these strains have been obtained in purified form by means of differential centrifugation. Purified preparations obtained from plants diseased with different strains of tobacco mosaic virus were found to possess properties quite similar to, yet in every case distinctive from, those of purified preparations of the ordinary strain. Spectacular progress has been made in the establishment of the nature of the chemical changes which accompany the mutation of tobacco mosaic virus. The amino acid composition of purified preparations of eight strains of tobacco mosaic virus and of two types of influenza virus has been determined. The results obtained with the strains of tobacco mosaic virus indicate that the mutation of a virus can be accompanied by the elimination of one or more amino acids from the virus structure, by the introduction of one or more new amino acids into the virus structure or by a change in the concentration of one or more amino acids present in the virus structure. This work has great significance for it has provided the first information regarding the nature of the structural changes which accompany mutation. Extension of this work may reveal the exact nature of the chemical differences between virulent and avirulent virus strains, and provide important information regarding the mutation process in higher organisms.

Attempts have been made to change the structure of tobacco mosaic virus by means of known chemical reactions *in vitro* in an effort to secure chemically modified active virus. Although several types of chemical derivatives of this virus were produced and were found to possess full virus activity, the inoculation of such virus derivatives to normal Turkish tobacco plants always resulted in the production of ordinary tobacco mosaic virus. The results indicated that the chemical derivatives were converted to ordinary virus following their introduction into the cells of the plant, or, more probably, that the infecting molecules may not necessarily function as exact patterns for reproduction. Despite these results it still appears that it may be possible to make changes *in vitro* similar to those which occur in nature, and thus secure a heritable chemical modification. Obviously this is a field in which important new results can be anticipated.

Following the isolation of tobacco mosaic virus in the form of a crystalline nucleoprotein having individual molecules or particles about 15 by 280 mμ in size, studies were undertaken in several laboratories to determine if other viruses could be obtained in purified form, mainly by techniques involving high speed centrifugation. Some of these purified viruses are crystallizable nucleoproteins having either rod-like or spherical particles. Some are nucleoproteins which have, as yet, not been crystallized. Others are large particles consisting of nucleoprotein, lipid and carbohydrate and possessing, in some cases, a degree of morphological differentiation which resembles that of organisms. Still other viruses have, as yet, defied isolation and purification, possibly, in some cases, because of extreme instability. The viruses which have been purified possess varied shapes and form an almost continuous spectrum of sizes. The smaller rod or spherically shaped viruses appear to be simple nucleoproteins, some of which can be obtained in crystalline form. These appear to have chemical and physical properties which, neglecting virus activity, would tend to place them in the molecular world. The larger viruses have a composition and properties which are characteristic, not of molecules, but of organisms. The viruses have certainly provided a link between the molecules of the chemist and the organisms of the biologist. Yet there is no place at which a line can be drawn dividing the molecules from the organisms.

The viruses appear to form a continuous series with respect to structure, ranging from the smaller viruses, which are simple nucleoproteins with many properties similar to those of ordinary molecules, on through viruses with a gradually increasing complexity of structure, to the larger viruses, which, with respect to structure and properties, are similar in many respects to organisms. However, it must be remembered that the properties of only a relatively few purified viruses have been determined. In view of the possibility that these represent the more stable and more easily purified viruses, one cannot be certain that a true picture of the chemical and physical properties of viruses as a whole has been obtained as yet. Information regarding the mode of reproduction of viruses is needed most urgently. At present it is not known whether viruses reproduce by fission or by means of some new process. The solution of this puzzle would certainly represent a most important and significant advance, for the basic reactions characteristic of virus reproduction may well represent the fundamental process which characterizes all living things.

Westinghouse Announces Formation of Atomic Power Division.—Gwilym A. Price, President of the Westinghouse Electric Corporation, has announced the formation of an Atomic Power Division which will concentrate solely on the harnessing of nuclear energy for the production of useful power.

According to Mr. Price, the new Division "will be available to undertake atomic energy projects for the United States Government as well as to carry on independent studies. It will conduct research, development, engineering, and any necessary associated construction." It will work closely with the Westinghouse Research Laboratories and with other divisions of the company so as to benefit from their knowledge, experience, and facilities. Location and other details of organization will be announced later.

Hafnium Oxide.—The DeRewal International Rare Metals Company, it is believed, is the only firm in the United States that produces hafnium metal powder commercially. This firm has for the past three years been supplying hafnium oxide to various scientific institutions in this country, for use in research programs. The firm is located in Philadelphia.

Hafnium metal powder is prepared by subjecting a molecularly proportioned mixture of hafnium oxide and metallic calcium to a high temperature, in a closed system. The cooled reaction mixture is treated chemically to remove calcium compounds, after which the metal powder is separated by filtration. Thus obtained it is a heavy black powder, quite flammable when dry; when moist it is safe to handle.

Fast Break-in Procedure for Dodge Cars.—New Dodge cars do not require customary slow and tedious breaking-in because of the proven effectiveness of a protective oil-absorbing chemical coating applied to the cylinder walls of Dodge engines during the past year.

This outstanding advancement in engine manufacturing inaugurated by Dodge has virtually eliminated the possibility of scuffing or scoring piston rings, pistons, or cylinder walls and consequent damage to the engine during either the critical break-in period or after many thousands of miles of engine usage. As a result, new Dodge cars may be driven at a speed of 40 miles per hour for the initial 250 miles without engine damage. Car speed may then be increased at the rate of five miles per hour for each additional 25 miles traveled until the desired maximum speed is attained.

Scuffing or scoring occurs when extreme local heat or pressure resulting from abusive overspeeding or overloading of an engine causes piston rings, pistons, or cylinder walls to expand and pin-point areas of their metal surfaces to flow or drag. The combination of better lubrication and chemical properties of the Dodge protective coating prevents such a flow or drag of these metals and the serious damage which would result from their welding or bonding together.

In applying the protective coating, a chemical process creates on the cylinder walls thousands of minute pockets or pores that trap and retain oil, thereby providing better lubrication. Simultaneously, the process applies to the cylinder walls a protective oil-absorbing coating of manganese iron phosphate crystals that become chemically combined with the cylinder walls and which possess properties that prevent the flow of metal under conditions of extreme heat or pressure.

PROGRESS IN AVIATION.[*]

BY

THEODORE VON KÁRMÁN.[1]

The year 1948 will be recorded in the history of aviation as a remarkable year. Technical progress is, of course, not a matter of one year; it is the result of efforts and achievements over an extended period of time. There are, however, single facts which have symbolic significance. Such a single fact was the first break through the so-called "sonic barrier"; that is, the first flight of a manned airplane with a speed faster than sound, an event which was announced this year.

The speed of sound is the speed of propagation of pressure effects in the air and it appears somewhat paradoxical to move a vehicle faster than the effect which is produced by its motion. I am afraid, however, that the layman connects with this fact more of a kind of romanticism than is really justified. Since long ago, projectiles move with speeds which are several times greater than the velocity of sound. The artillery man has no difficulty in breaking through the sonic barrier, because projectiles pass through the so-called *transonic* speed range in such a short time that the difficulties which an airplane encounters do not appear. One cannot say that the laws of aerodynamics change if we leave the subsonic speed range and enter the supersonic range. The fundamental laws of classical mechanics apply equally to both ranges. It only happens that both from the mathematical and experimental points of view, the *application* of these laws is relatively easy if the flow of air is entirely subsonic or entirely supersonic. But both the mathematician and the experimental researcher encounter great difficulties if they have to establish practical rules which are valid in the transition range.

The process of transition between the subsonic and supersonic speed range is somewhat analogous to the transition from the so-called displacement ship floating in and on the water surface, to the speed boat riding on the step. In the first case, especially at low speeds, friction represents the main part of the resistance, while in the second case the boat struggles against the bow wave which its motion produces. However, the hydrodynamics of this transition process can be investigated without great intrinsic difficulties. From a practical point of view, it is significant that the man steering the boat *sees* what happens in the water, whereas the pilot of the transonic airplane does *not* see the changes in the air flow, shock waves and similar phenomena. The fact is that, in addition to increased drag, which can be overwhelmed by sheer power, significant and sudden changes occur in the stability and control characteristics of the plane, unforeseen by our previous aerodynamic knowledge. This circumstance caused the sacrifice of a number of gallant and experienced pilots both in this country and abroad.

---

[*] Talk presented at the Medal Day Meeting at the Franklin Institute, October 20, 1948. (Read by Mr. W. Laurence LePage, in the absence of the author.)

[1] Professor of Aeronautics and Director of Guggenheim Aeronautics Laboratory, California Institute of Technology, Pasadena, Calif.

We cannot say that we already have the right airplane for this extended speed range which was opened to aviation in the past year. Nor can we say that we already have acquired exact knowledge of the phenomena of *transonic* aerodynamics. As a matter of fact, the experimental airplanes in the hands of intelligent and courageous test pilots contributed more to such knowledge than experimental research in the laboratory. Maybe our learned boards and committees were somewhat slow in the creation of adequate research facilities.

Another significant fact which will remain in the memory of aviators, aeronautical engineers and economists is the air-bridge leading to the German capital of Berlin. Of course, it is a paradoxical fact to both engineers and economists that we spend calories in the form of gasoline in order to deliver by air calories in the form of coal. This, however, is only one more proof for the fact that the *homo politicus* did not reach yet the stage of the *homo sapiens* who was placed on this earth to find out the laws, and rule over the forces, of nature. On the other hand, this unique experiment of the air-lift to Berlin is more than a testimony to the capacities of the Air Forces, the achievements of the pilots and the extraordinary talent of organization of General LeMay and his collaborators. The possibility of continuous flow of goods by air, as it is done on the surface every day by the railroad and the trucks which fill our highways, is being demonstrated. The relative merit of airplane sizes, shortcomings in design from the cargo point of view, useful progresses, as for example the routine use of radar landing devices, have been revealed. It is up to the engineers and economists to draw the conclusions and determine the reasonable limitations of air transport for the years to come.

As you see, neither of the two facts I have mentioned, the break through the sonic barrier and the grand experiment of the air-lift, represents immediate solution of current problems of aviation. However, I believe they give significant glimpses into the future. Maybe the coming decades will witness the development of airplanes with enormous speeds, thus making contact between continents a matter of a few hours, for the increase of human understanding, and will witness the continuous flow of goods by air in an economic way, for the increase of human comfort.

A MODIFIED METHOD OF ITERATION OF THE PICARD TYPE IN THE SOLUTION OF DIFFERENTIAL EQUATIONS.

BY

GWOH-FAN DJANG, Ph.D.[1]

INTRODUCTION.

Differential equations in engineering and physics of nonlinear and difficult types which cannot be solved by ordinary methods may be solved by a method of iteration developed by Picard. The method is generally given in most books that have a section on numerical integration (1).[2] Starting from a given function fixed by the differential equation and the boundary conditions at the initial point, the dependent variable of the differential equation is obtained through a number of integrations by means of successive substitution. Differential equations of higher orders are solved as a number of simultaneous equations each of the first order. There are two main difficulties encountered when this method is put into practice: (*a*) The convergence of the successive approximations to the true solution is often slow when the interval used for integration is large. If the interval chosen is small, the number of steps required to cover a given range will be increased. The labor of computation in either case is often tremendous. (*b*) The method cannot be used in its original form if the boundary conditions of the problem in question are spread to a number of points. For the method to be applicable, auxiliary methods have to be devised so as to make all the values of the dependent variable and its derivatives definitely specified at the initial point (2).

The present paper suggests a modified form of iteration which converges somewhat faster than the Picard method and may be used directly whether the boundary conditions are spread or concentrated at a single point. It also solves differential equations of higher orders in a more straightforward manner than the method of simultaneous equations employed by Picard.

OUTLINE OF THE METHOD.

Let

$$F\left(\frac{d^ny}{dx^n}, \frac{d^{n-1}y}{dx_a{}^{n-1}}, \cdots, \frac{dy}{dx}, y, x\right) = 0 \qquad . \quad (1)$$

be a general differential equation of order n. The problem is to find a function

$$y = \Psi(x), \qquad (2)$$

---

[1] Professor of Physics and Fluid Mechanics, National Pei-Yang University, Tientsin, China.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

which satisfies Eq. 1 together with given boundary conditions. Now write Eq. 1 in the form

$$\frac{d^ny}{dx^n} = f\left(\frac{d^{n-1}y}{dx^{n-1}}, \frac{d^{n-2}y}{dx^{n-2}}, \cdots, \frac{dy}{dx}, y, x\right). \tag{3}$$

Assume

$$y_1 = \Psi_1(x) \tag{4}$$

as any convenient function that satisfies the boundary conditions. Putting y_1 into the right side of Eq. 3, one may write

$$\frac{d^ny_2}{dx^n} = f\left(\frac{d^{n-1}y_1}{dx^{n-1}}, \frac{d^{n-2}y_1}{dx^{n-2}}, \cdots, \frac{dy_1}{dx}, y_1, x\right), \tag{5}$$

the integration of which will give y_2 together with n arbitrary constants that can be determined from the boundary conditions. Repeating the process, one can obtain y_n from y_{n-1}. The process is completed when $y_n \doteq y_{n-1}$ to the degree of accuracy set beforehand. y_n is then the required solution.

The fact that we start with an arbitrary function, instead of a given function fixed by the differential equation as in the Picard method, has the advantage that the arbitrary function may be chosen as a solution of the differential equation with some of its unmanageable terms first dropped out. This usually can save us quite an amount of labor in the steps of iteration. A following example will illustrate this point.

COMPARISON OF THE TWO METHODS.

The merits of the two methods may best be compared by applying them to a few examples.

Example 1.

Let

$$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0, \quad \text{with} \quad x = 0, \quad y = 1, \quad dy/dx = 1,$$

be the differential equation to be solved. It is an example used by Levy and Baggot (3) for illustrating Picard's method. Solving the equation as two simultaneous equations

$$\frac{dy}{dx} = z, \qquad \frac{dz}{dx} = 6y - 3xz,$$

the successive approximations are:

$$\begin{aligned} y_1 &= 1 + x, \qquad \text{(using } z = 1\text{)},\\ z_1 &= 1 + 6x;\\ y_2 &= 1 + x + 3x^2,\\ z_2 &= 1 + 6x + \tfrac{3}{2}x^2 - 6x^3; \end{aligned}$$

$$
\begin{aligned}
y_3 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3 - \tfrac{3}{2}x^4, \\
z_3 &= 1 + 6x + \tfrac{3}{2}x^2 - \tfrac{9}{8}x^4 + \tfrac{18}{5}x^5; \\
y_4 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3 - \tfrac{9}{40}x^5 - \tfrac{3}{8}x^6, \\
z_4 &= 1 + 6x + \tfrac{3}{2}x^2 - \tfrac{3}{8}x^4 - \tfrac{9}{8}x^5. \\
y_5 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3 - \tfrac{3}{40}x^5 - \tfrac{3}{16}x^6.
\end{aligned}
$$

Using the present method, however, starting with

$$y_1 = 1 + x,$$

the successive approximations are:

$$
\begin{aligned}
\frac{dy_2}{dx} &= 1 + 6x + \tfrac{3}{2}x^2, \\
y_2 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3; \\
\frac{dy_3}{dx} &= 1 + 6x + \tfrac{3}{2}x^2 - \tfrac{3}{8}x^4, \\
y_3 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3 - \tfrac{3}{40}x^5; \\
\frac{dy_4}{dx} &= 1 + 6x + \tfrac{3}{2}x^2 - \tfrac{3}{8}x^4 - \tfrac{9}{80}x^6, \\
y_4 &= 1 + x + 3x^2 + \tfrac{1}{2}x^3 - \tfrac{3}{40}x^5 - \tfrac{9}{560}x^7.
\end{aligned}
$$

A casual glance will show that the present direct method gives results which converge faster than those of the Picard method. For equations of still higher orders, the difference would be even more pronounced.

Example 2.

Let us take

$$d^2x/dt^2 + p^2x = x^3, \text{ with } p^2 = \tfrac{3}{2}, \text{ and } t = 0,\ x = 0,\ dx/dt = 1,$$

as a second example. It is a differential equation for the oscillation of a simple pendulum. The equation has been used by Moulton (4) as a numerical example in the application of the Picard method. The work of computation in solving the two simultaneous equations is found to be quite heavy.

Using the present method, one may employ either $x_1 = t + t^2$, or $x_1' = (1/p) \sin pt$ as a starting function. Adopting $x_1 = t + t^2$ (note that in the Picard method one would use $x_1 = t$), we get the successive approximations:

$$
\begin{aligned}
x_2 &= t - 0.25t^3 - 0.125t^4 + 0.05t^5 + 0.1t^6 + 0.071t^7 + 0.017t^8; \\
x_3 &= t - 0.25t^3 + 6.874 \times 10^{-2}t^5 + 6.25 \times 10^{-3}t^6 - 1.964 \times 10^{-2}t^7 \\
&\quad - 9.375 \times 10^{-3}t^8 + 3.2 \times 10^{-3}t^9 + 5.119 \times 10^{-3}t^{10} + 1.551 \\
&\quad \times 10^{-3}t^{11} + (15 \text{ more terms}).
\end{aligned}
$$

Mechanical Hands.—Mechanical "hands," which in dangerous radioactive areas can perform delicate chemical experiments, operate machine tools, and do countless other tasks requiring great dexterity, were demonstrated recently by scientists of the General Electric Company.

Principal part of a device known as a remote-controlled manipulator, the "hands" also proved they could perform such common jobs as slicing an orange, twirling spaghetti on a spoon, pouring liquids from one receptacle to another, lighting a cigarette, boring a hole, and writing a name.

In actual use the "hands" would extend over a protective eight-foot-high wall into a radioactive area but would be operated by remote control from a room outside the area, thus insuring against danger to humans from radiation, the scientists said.

The actual "hands" are similar to the double hooks used by amputees. They are at the lower ends of two arms which descend vertically from two horizontal shafts eight feet high across the top of the wall. Vertical arms come down from these at the other ends, and carry at the bottoms the handles with which the operator controls the device. He is seated and views his mechanical hands, eight feet away, with the aid of binoculars and a four-mirror periscope which looks over the wall. His feet are on pedals which control the grasping action of the "hands."

The general motion of the hooks corresponds to that given the handles, as they are moved up or down, forwards or backwards, or from side to side. In addition the handles can be turned around three axes, by wrist movement, and this causes the hooks to do likewise. While most of the connections between handles and hooks are mechanical, the twisting of the wrists is accomplished electrically. This permits the artificial hands to do something real ones cannot. They can be twisted around completely, any number of times, which is particularly useful when unscrewing a nut, for example.

It is also possible, from behind the wall, to change "hands." A large one can be used for heavy jobs. If the task requires a more delicate touch, the hand can be lowered into a holder and removed. Then the arm is moved to pick up and attach a smaller hand. Another is provided, instead of hooks, with a pair of snips for cutting sheet metal. Even without practice, it is not difficult to do a simple task, such as removing the stopper from a bottle and pouring its contents into another bottle. With practice much more complicated things can be done, such as writing one's name and operating a drill press. In the latter case, the "hands" pick the proper-sized drill from the box, insert it in the chuck, place the work in position, turn on the switch and feed the drill.

Pedals are used to close the "hands" for grasping, because of the fact that most people are accustomed to the feel of a pedal in operating an automobile brake. The pedals themselves are of regular automobile type, and standard auto brake fluid and cylinders are used to transmit their force. It is possible to squeeze an object between the hooks much harder than the ordinary person can with his fingers. On the other hand, by a more gentle push on the pedal, a very light touch can be exerted, so as to break the shell of a hard-boiled egg without damaging the contents.

R. H. O.

ANALYSIS OF THE PARTICLE-SIZE DISTRIBUTION OF THE GRAINS PRODUCING A PHOTOGRAPHIC IMAGE.*

BY

R. P. LOVELAND,[1] H. M. MENIHAN,[1] AND A. P. H. TRIVELLI.[1]

INTRODUCTION.

In a previous paper Loveland and Trivelli (1)[2] reported an investigation of the formation of the grain-size distribution of the silver halide grains of photographic emulsions. Such a particle-size distribution is anticipatory to the events which lead to the formation of the photographic image. In the present paper the authors will deal with the grains of the photographic emulsion layer of approximate commercial thickness which, upon development, become the grains of the image.

About 25 years ago investigations of the sensitivity distribution among the grains of photographic emulsions were made utilizing photographic plates coated with a single layer of grains. This technique was simpler than the use of the relatively thickly coated commercial emulsion, since all the grains in the areas examined were uniformly exposed and uniformly developed. The gradual absorption of scattered light through the depth of multigrain-layer emulsions alters the distribution of the latent image among the different grain sizes. The single-grain-layer coatings also eliminate the complications of development with penetration of a thick coating, that is, exhaustion of developer and increase in the products of development. Undoubtedly, therefore, exposure and development of commercial multigrain-layer emulsions will produce a different size distribution of developed grains. The magnitude of this effect varies with the type of emulsion and the developer.

GENERAL METHOD.

Exposure to light and development are the agents that select those grains of a photographic emulsion that will develop and become the photographic image. The effective sizes of the grains are those of the undeveloped grains, although the ones selected can only be identified after development. Therefore, the size distribution of the silver halide grains which later develop was obtained statistically by using the normally coated film and the usual sensitometric exposure and development. For this, the size distribution of the undeveloped (residual) grains of the developed but unfixed film was subtracted from that of the original

---

* Communication No. 1175 from the Kodak Research Laboratories.

[1] Eastman Kodak Company, Rochester, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

undeveloped film for each exposure-development step studied. The size-frequency distribution of the halide grains *per unit area of emulsion layer* was necessary and was obtained by adjusting the *relative* size frequencies (per 1000) with the grain concentration of each film area in the sensitometric strip obtained by counts with dark-field illumination.

EMULSION.

A silver bromoiodide emulsion containing 2.56 moles per cent of silver iodide was used. The particle-size constants are given in the section, "Structure of the Photographic Emulsion." A photomicrograph of the grains of this emulsion at 2500 diameters is shown in Fig. 1.

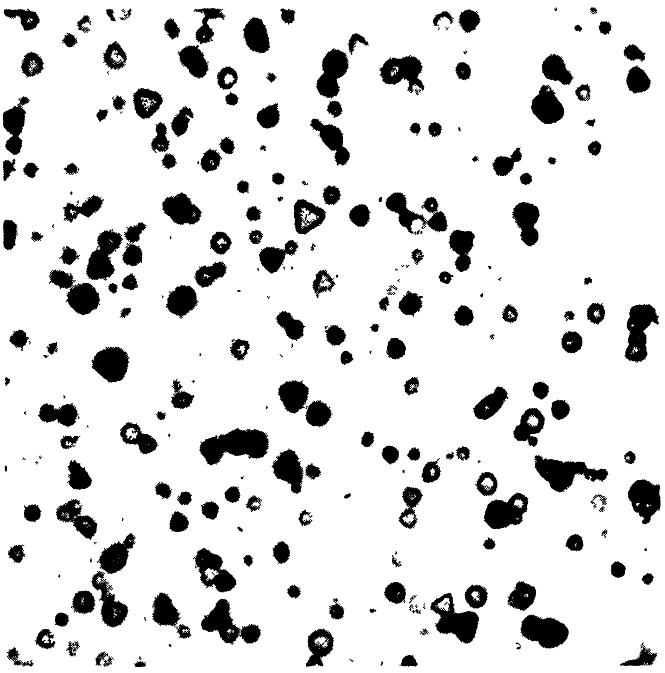


FIG. 1. Photomicrograph of grains of undeveloped silver bromoiodide emulsion containing 2.56 moles per cent of silver iodide (×2500).

Sensitizing was accomplished with a gelatin containing a sulfur sensitizer which gave a relatively low fog. The hydrogen-ion concentration of the emulsion was pH = 7.0. Upon development of the film in DK-50 at 18° C. for 3, 5, and 12 min., the fog densities were 0.03, 0.04, and 0.08, respectively.

Since partial development of individual grains might lead to quantitative errors, which would be particularly undesirable for this type of investigation, this factor was reduced to a minimum by suitable choice of emulsion and developer.

PROCESSING.

The exposure and processing of the film were carried out in the Physics Department of these Laboratories. The 35-mm. film strips were exposed in an Eastman Type IB Sensitometer with a variable-intensity-step scale of exposure through a filter of very narrow wave-band transmission and having an effective wavelength of 426 mμ. The exposure time was 5 min. The film was developed in a machine in DK-50 at 18° C. for 2, 3, 4, 5, 8, and 12 min. An acetic acid stop-bath was used.

The 3-, 5-, and 12-min. strips were selected for special study; the characteristic curves representing them are shown in Fig. 2, the exposures along the logarithmic axis being expressed in ergs.

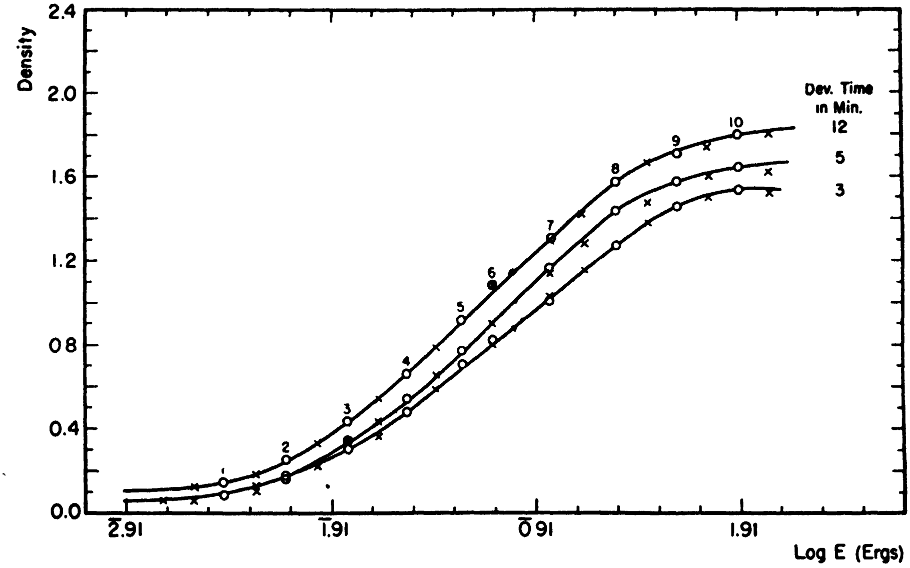


FIG. 2.

X Counts
O Size measurements

EXPERIMENTAL PROCEDURE.

All the film strips were exposed identically. Groups of these then received identical development to give the series of successive development times. Each of these groups having the same development was then further subdivided into two groups. The first subgroup of strips was then fixed in F-5 fixing bath and washed and dried as usual. Their densities were then measured and plotted to give the curves in Fig. 2.

The remaining strips of successive developing times were dried after the acid stop-bath without fixing and without being exposed to strong light. Areas of carefully determined dimensions were cut from the uniformly exposed steps on the strips to give a cross series varying in both exposure and development. The emulsion from these film areas was then melted and recoated into a single layer of grains. The technique used varied according to whether the purpose was (*a*) to determine the concentration of grains by counting the number per unit area, or (*b*) to determine the particle-size distribution, because the requirements for the coated plates differ, and better quality is obtained by doing them separately. Obviously the sampling techniques of quantitative analysis were applied for (*a*); the chief criteria for the second purpose are that the measured grains be representative and accurately in a plane, since sharpness of focus affects the apparent size. The developed silver grains were then removed from these plates with chromic acid. The unexposed area of the film was used to make the plates for the "original distribution" used as the base for subtracting the distribution of the light-affected grains of exposed areas from that of the original grains.

Except in a few cases where both the concentration and the size distribution could be determined from the same sample, they were determined on alternate steps of the exposed and developed strips treated with chromic acid. The densities which were represented by the removed silver grains could be determined from the $D/\log E$ curves at corresponding exposures. Those densities are indicated in Fig. 2 by crosses that correspond to equivalent areas from which samples for grain counts were taken. Correspondingly, those densities indicating the size-frequency determinations are represented by circles. In this way values of density, concentration, and size distribution could be obtained for each exposure step.

The complete size-frequency distributions were determined for ten exposure steps of each characteristic curve ranging from those producing almost fog densities to those producing nearly the maximum densities. Thus, there was a separate series for each development time. Approximately the following numbers of grains were measured to determine the size-frequency curve of the steps for each $D/\log E$ curve:

Step No.:	1	2	3	4	5	6	7	8	9	10
No. of Grains:	4000	3500	3000	2500	2300	2000	1800	1600	1200	800

Trouble was encountered from hardening of the gelatin during remelting of the emulsion, prior to recoating, although no hardener had been used in the emulsion for the first coating. This was partly due to hardening of the gelatin by the quinone, which was formed during development by oxidation of the hydroquinone developer, and partly due to irreversible drying of the gelatin, which was accentuated when the film was cut into small pieces.

After the cause of the hardening was discovered, the film was kept in sealed containers, and small pieces were cut just before use; however, this difficulty with the previously mentioned samples affected the quality of the recoating, particularly the dispersion into individual grains, and hence must have affected the accuracy of the analytical results.

THE RELATION BETWEEN DENSITY AND NUMBER OF DEVELOPED GRAINS.

Sheppard and Mees (2) state that in a photographic emulsion layer the number of developed grains increases with the exposure, being approximately proportional to the density. Nutting (3), and Silberstein and Trivelli (4) assumed these to be simply proportional to the number of developed grains. We have confirmed this assumption within the limits of experimental error.

Microscopic investigations of cross sections of multigrain-layer coatings have shown that in the majority of cases the exposed and developed grains lie principally near the upper layer, the limit of which is largely determined by penetration effects, whereas the undeveloped grains lie largely in a lower layer with a thickness determined by the coating thickness of the film; therefore, the accuracy of the statistical results of the determination of the number and distribution of the developed grains is dependent to a considerable extent upon a uniform thickness of film coating. Moreover, it follows that with an unequal coating thickness, the errors will be greater for the lower than for the higher densities.

The films used were coated on an experimental scale, a procedure which does not always give a thickness of coating as uniform as the commercial films. About 3000 or 4000 residual grains were counted in the dark field for each exposure step to correlate with the corresponding density:

$$N_{\mathrm{d}} = N_{\mathrm{orig}} - N_{\mathrm{res}},$$

where

N_{d} = number (concn.) of the developed grains per unit area;

N_{orig} = number (concn.) of the original grains before development per unit area; and

N_{res} = number (concn.) of residual grains left after development and the silver bleach per unit area.

The last two values were obtained by direct counting. The concentration of the original grains on the untreated emulsion layer, N_{orig}, is a constant,

$$N_{\mathrm{orig}} = (0.930 \pm 0.010) \times 10^9 \text{ grains per square centimeter of film.}$$

The probable error, 0.010×10^9, is a measure of the consistency of the counts.

The N_{d} values, corresponding to the lowest densities, were the most erratic, as was to be expected. Some negative values were even observed among the lower densities; however, when the values of N_{d} were plotted against the corresponding densities for each development time, it was found that all were apparently straight lines converging to a point at $N_d = -0.15 \times 10^9$, instead of the origin-locus. This seems to be due to an inherent experimental error, probably a thickness variation of the areas of the emulsion layer from which the samples for the counts of the original concentration were taken. In any case, a logical and simple procedure was adopted, that is, the constant, 0.15×10^9, was added to that of N_{orig}. The corrected value is

$$N_{\text{orig}} = 1.080 \times 10^9.$$

The D/N-relations for the different development times are shown in Table I.

The relation between density and number of developed grains per unit area is linear and passes through the origin. A linear formula was applied to the data in Fig. 3 by the method of least squares.

TABLE I.

Development Time, min.	D	N_d(obs.) $\times 10^{-6}$	N_d(calc.) $\times 10^{-6}$	$\Delta N_d \times 10^{-6}$
3	0.36	179	145	−34
	0.58	254	234	−20
	0.80	323	323	0
	1.03	428	416	−12
	1.38	518	558	+40
	1.50	607	606	−1
	1.52	623	614	−9
4	0.07	38	29	−9
	0.12	74	49	−25
	0.24	94	99	+5
	0.40	174	164	−10
	0.62	251	255	+4
	1.09	442	448	+6
	1.23	510	505	−5
	1.44	585	592	+7
	1.56	620	641	+21
	1.58	672	649	−23
5	0.34	170	134	−36
	0.65	253	257	+4
	0.90	371	356	−15
	1.14	402	450	+48
	1.28	472	506	+34
	1.48	601	585	−16
	1.60	604	632	+28
	1.62	696	640	−56
12	0.78	318	304	−14
	1.09	413	425	+12
	1.30	463	507	+44
	1.42	536	554	+18
	1.66	613	647	+34
	1.74	704	679	−25
	1.80	750	702	−48

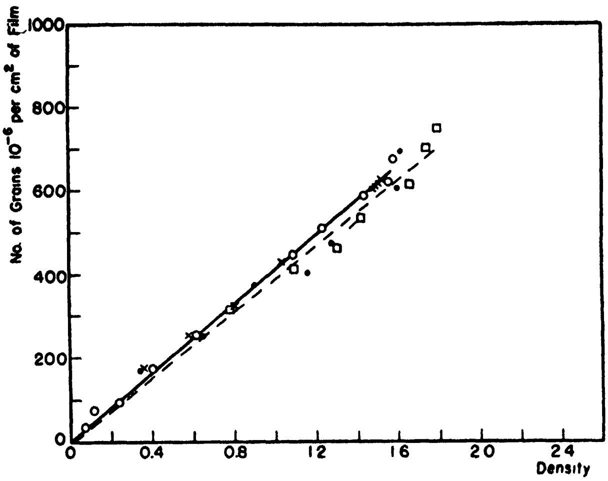


FIG. 3.

○○○ 4-min. obs.
——— 4-min. calcd.
××× 3-min. obs.
● ● ● 5-min. obs.
□□□ 12-min. obs.
- - - - 12-min. calcd.

The formulas obtained are given in Table II.

Of these, the formula for the 4-min. development time is supported most completely by the data.

For this emulsion and within the limits of accuracy of the experiments, the density-number of grains ratio, D/N, is a constant for different development times.

TABLE II.

Development Time, min.	Formula	
3	$N_d = 4.1 \times 10^8 D$	(1a)
4	$N_d = 4.0 \times 10^8 D$	(1b)
5	$N_d = 4.0 \times 10^8 D$	(1c)
12	$N_d = 3.9 \times 10^8 D$	(1d)

STRUCTURE OF THE PHOTOGRAPHIC EMULSION.

The two factors of the structure of the photographic emulsion can be considered to be (*a*) the grain concentration (total number of silver halide grains per unit area), and (*b*) the grain-size distribution. The grain concentration is

$$N = (1.080 \pm 0.010) \times 10^9,$$

where N equals the number of silver halide grains per square centimeter of film.

For the grain-size distribution, 4500 grains were measured; the size-frequency distribution is presented in Fig. 4. The most important particle-size constants, measured and expressed in the projective areas of the grains, are as follows:

Average particle size, $\bar{a} = 0.50\mu^2$
Medium particle size, $\bar{a}_{\mathrm{me}} = 0.33\mu^2$
Dispersion of sizes around the average, $\sigma = 0.43\mu^2$
Dispersion constant, $\bar{a}/\bar{a}_{\mathrm{me}} = 1.51.$

The dispersion, σ, is measured by the standard deviation. The dispersion constant, which is the ratio of the arithmetic average to the median, will be discussed later. The largest grain observed was $a = 4.85\mu^2$.

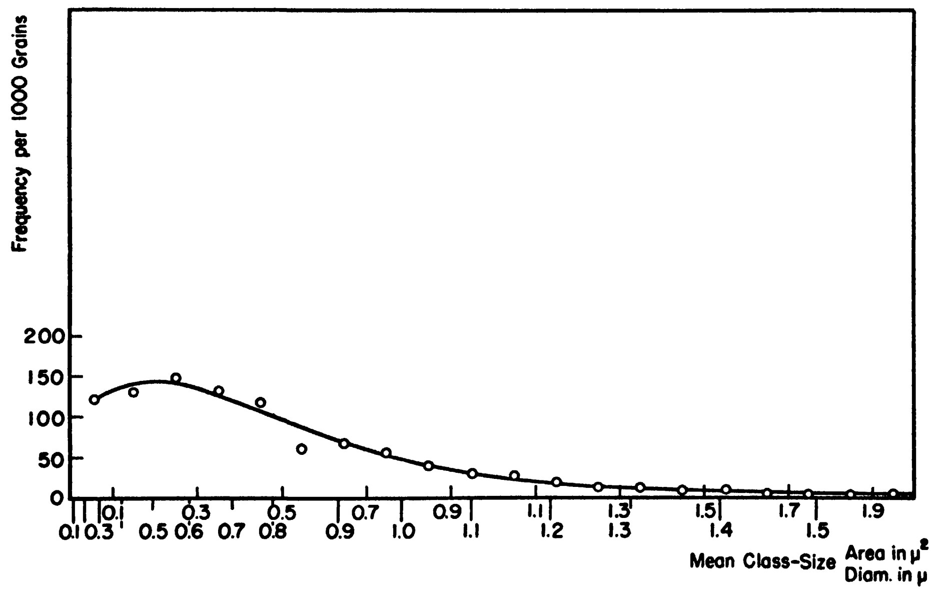


FIG. 4.

The projective area is a very advantageous attribute for the measurement and expression of particle size. First, it is the most important size attribute for this purpose and represents the light-collecting power of the grains. For multilayer emulsions, the total grain area, s, may assume primary importance, but with tabular grains it is about twice the former. Second, the particle size can be expressed and measured, irrespective of the shape of the grain, that is, the "shape factor" does not enter as a limiting assumption to the accuracy.

The normal function plotted on semilogarithmic paper in Fig. 5 shows the distribution to contain a "break" at about $0.9\mu^2$, corresponding to a frequency per 1000 of about 850. Therefore, the emulsion grain must have been formed principally on two types of nuclei (1).

Moreover, Fig. 5 shows that the observed frequency point of $0.55\mu^2$ in the arithmetic size-frequency curve must be an error or a local wave of rhythmic precipitation rather than the point of departure for a break. Experiments by the authors have shown that local fluctuations or waves in the size-frequency curves are quite reproducible within the same emulsion from one batch to another. A study of the various size-frequency curves of the various steps suggests, in fact, that the point of $0.45\mu^2$ should be the height of such a local wave, and the point at $0.55\mu^2$ is too low.

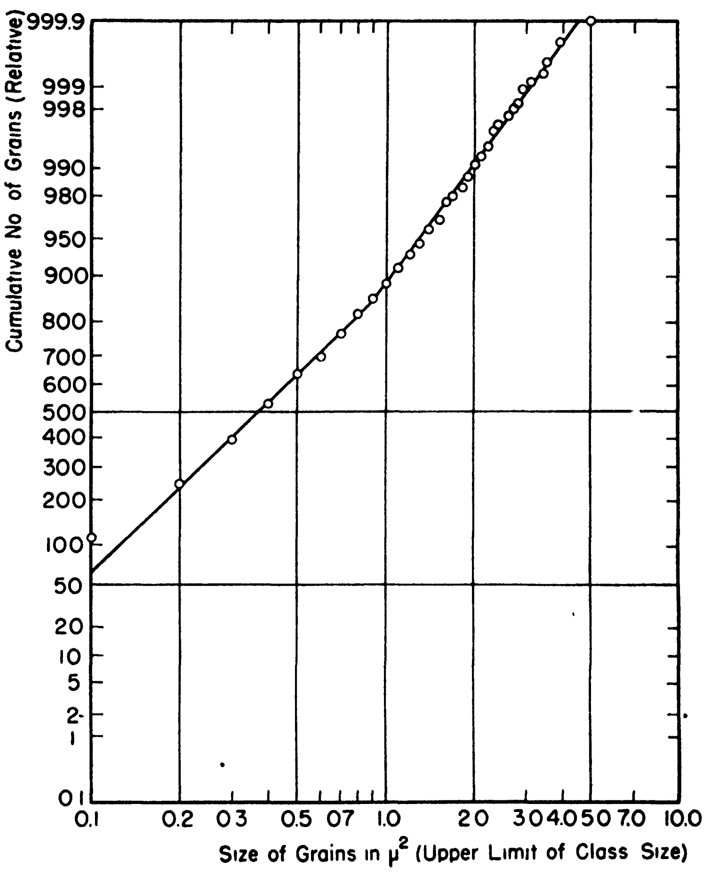


FIG. 5. Log normal function.

STRUCTURE OF THE DENSITIES.

The size-frequency distribution curves of the silver halide grains that will form the image by development are given in Figs. 6 and 7 for the exposure steps receiving 3- and 12-min. development, respectively. The curves are numbered 1 through 10 to correspond to the exposure steps represented by eyelets in Fig. 2. Each curve was obtained by sub-

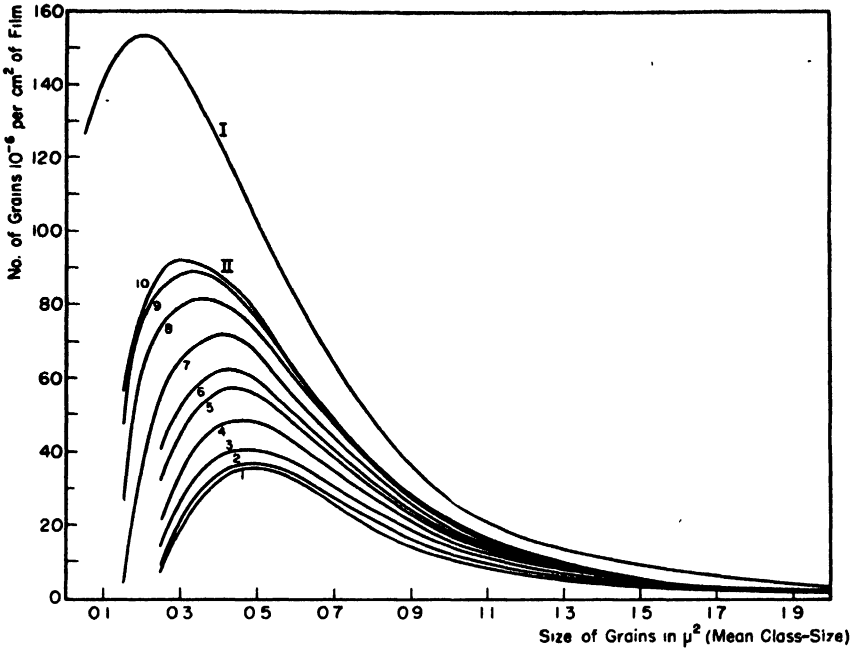


FIG. 6. Three-minute development.

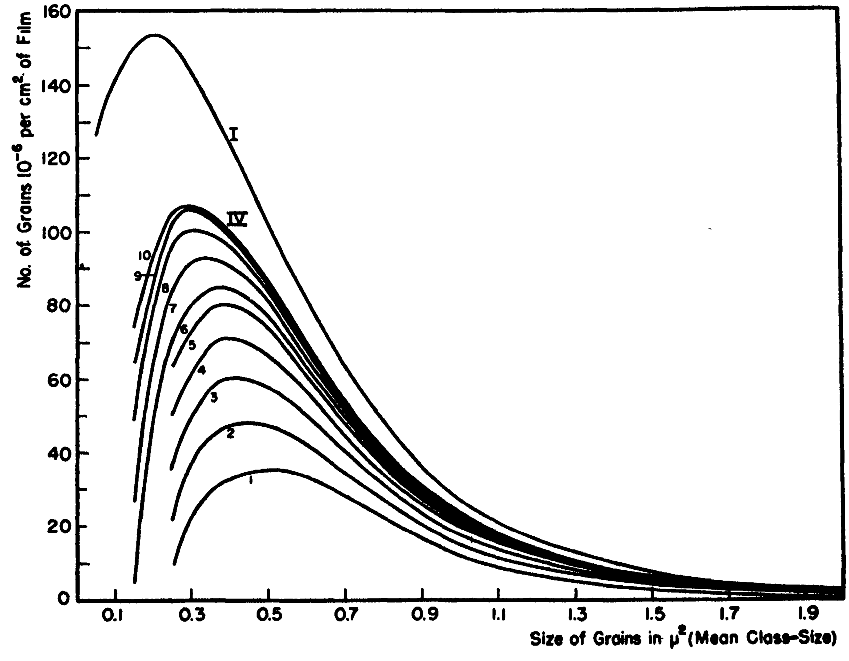


FIG. 7. Twelve-minute development.

traction of the size-frequency distribution data of the residual undeveloped grains from those of the original undeveloped emulsion, proceeding with each class-size progressively. It was first necessary to adjust the size-frequency data, putting the relative size-frequency curves (per 1000 grains) on an absolute basis by multiplying the successive frequency values of each distribution curve by $N_d/1000$, where $N_d = \int ydx$ is the total number of grains per unit area, as determined by the grain counts.

The frequency values used were obtained from the smoothed size-frequency curves, and the total numbers, N_d, were determined by Eq. 1b. In Figs. 6 and 7, Curve I is the size-frequency curve of the original emulsion, multiplied by 1.080×10^6, and also corresponds to the curve of developable grains if every grain in the emulsion developed.

To obtain sensitometric data after development, a photographic emulsion layer is exposed in the usual way. Then, after every exposure there will be a number of developable and undevelopable grains if, for simplicity, the effect of fog grains is disregarded. The density produced by development of all developable grains can be measured sensitometrically from the curve of maximum gamma. This was approached by development for 12 min. This investigation, however, is not restricted to this long development time alone, but was extended to include shorter development times. With such development times, a smaller number of developable grains will develop, producing photographic images of lower contrasts.

The curves of the silver halide grains, which were developed, have the same general form as those of the original emulsion. As the exposure proceeds, the breaks in the size-frequency curves of the developable silver halide grains, which develop, tend to disappear, and the curves assume a simple lognormal (1) form.

The sensitometric step exposures were the same for all film strips. The logarithmic exposures and the corresponding densities obtained after 3-, 5- and 12-min. development, respectively, are given in Table III, to be correlated with the curves of Figs. 6 and 7.

TABLE III.

Curve No.	Log E	$D_{dev.}$ in 3 min. (Fig. 6)	$D_{dev.}$ in 5 min.	$D_{dev.}$ in 12 min (Fig. 7)
1	$\bar{1}.39$	0.08	—	0.15
2	$\bar{1}.69$	0.16	0.18	0.25
3	$\bar{1}.99$	0.30	0.34	0.43
4	0.28	0.48	0.54	0.66
5	0.55	0.71	0.78	0.92
6	0.70	0.83	—	1.09
7	0.98	1.01	1.17	1.31
8	1.31	1.28	1.44	1.58
9	1.61	1.46	1.58	1.71
10	1.91	1.54	1.65	1.80

The data obtained with 3- and 12-min. development will be considered first and the data obtained with 5-min. development will be used later for grain analysis.

THE SIZE-FREQUENCY CURVES OF THE GRAINS LEFT UNDEVELOPED AT MAXIMUM DENSITIES.[3]

To facilitate the discussion in the subsequent section, the term "developed grains" will be used to designate those developable grains that developed at given times — 3, 5, or 12 min. — as shown by the control strips.

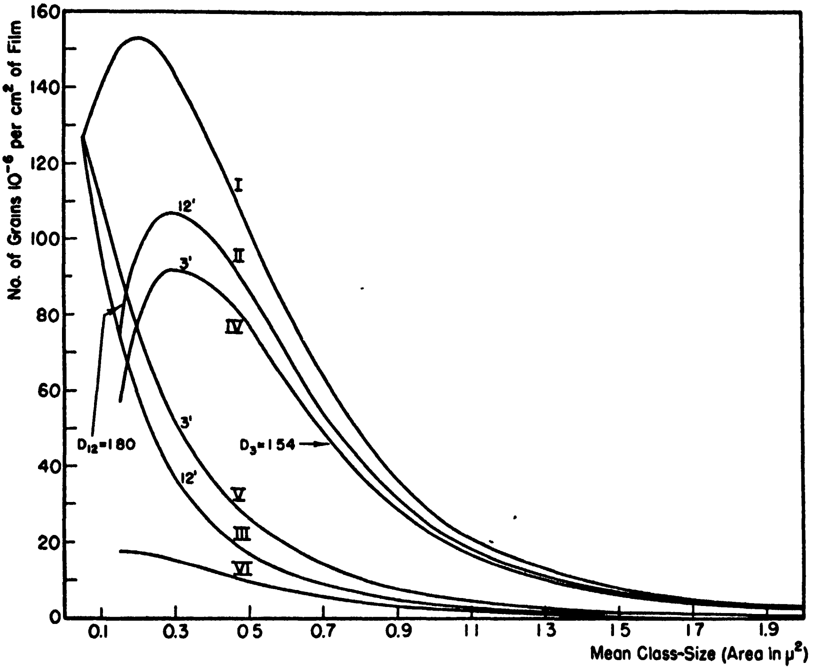


FIG. 8.

When the frequencies of the size distribution of the developed grains belonging to a maximum density for some development time are subtracted from the size frequencies of the silver halide grains of the original whole emulsion, the size-frequency distribution is obtained of the grains that have been left undeveloped. This is represented in Fig. 8, where Curves II and IV, which are the size-frequency curves of the developed grains for the maximum densities, $D_{12} = 1.80$ and $D_3 = 1.54$ for 12- and 3-min. development, respectively, are subtracted from Curve I, which is that of the original emulsion, to give Curve III (I–II) and Curve V (I–IV), which are the size-frequency curves of these undeveloped grains that remain substantially unaffected by prolonged exposure at the given development time. Obviously the average size of such grains that apparently cannot be exposed depends upon the development time. The following percentages represent the maximum number of grains that were found to develop at these successive development times:

---

[3] Owing to the findings of this investigation, the explanation for the occurrence of these grains, as given by Trivelli on page 8 of the JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 241, 1946, must be replaced by the explanation given in this section.

Development Time, min.	2	3	4	5	8	12
Percentage of Developed Grains	53	56	57	61	63	68

The size-distribution curves of grains corresponding to maximum densities, obtained by development to maximum gamma, are of special interest. Curve III of Fig. 8 represents the grain-size distribution of undeveloped grains which, so far as these experiments are concerned, do not become developable either at the long exposure applied or on prolonged development. These undeveloped grains are not solarized, because prolonged development does not increase the density materially as Trivelli (5) has shown should occur.

A comparison of Curve III with Curve I in Fig. 8 indicates that the smallest grains of the photographic emulsion are not affected by the exposure and development conditions chosen. Curve III shows that the number of undeveloped grains increases very rapidly towards the smaller grains, although it is not a simple exponential type of curve. The average size of these grains is $\bar{a} = 0.350\mu^2$, with a dispersion of grain sizes, $\sigma = 0.427\mu^2$, calculated from the available data, lying above the resolving power of the microscope. The occurrence of undeveloped grains at the maximum density is not new. It has been observed by several investigators. For instance, Trivelli (6) published recently four photomicrographs of such grains among other developed grains.

In this investigation 32 per cent of all the grains of the emulsion at 12-min. development remain undeveloped. These grains show very low sensitivities. They are distributed among the other 68 per cent of more highly sensitive grains over a range of different grain sizes, as shown in Table IV.[4]

TABLE IV.

Mean Class-Size in μ^2	Per Cent Undeveloped Grains
0.05	100
0.15	50
0.25	30
0.35	22
0.45	18
0.55	15
0.65	14
0.75	13
0.85	13
etc.	etc.

---

[4] For still larger grain sizes, the data become less reliable.

In a few cases it has been possible to show that the grains with very low sensitivities can be made developable by prolonged exposures. This is evident by the slow increase of the density beyond the shoulder of the fog-corrected characteristic curve over a great range of increasing exposures. Such a curve is given in Fig. 9, in which the increase in density is only 0.11 from Log $E = \bar{1}.60$ to Log $E = 1.80$. This curve was obtained from the film coating of a bromoiodide emulsion containing 0.77 mole per cent of silver iodide having an average grain size, $\bar{a} = 2.93\mu^2$, and a dispersion of grain sizes, $\sigma = 2.13\mu^2$. Development was accomplished in 12 min. in DK-50 developer at 18° C.

The occurrence of grains with low sensitivities in many sensitized photographic emulsions explains the problems encountered in the early investigations of sensitivity distributions among the grains of one-grain-layer coatings. Several times a distinct sensitivity difference has been observed between the smaller and larger grains of an emulsion, which could not be explained by grain-size differences alone. It was understandable, therefore, that a few investigators restricted themselves to the sensitivity distributions among the larger grains, where a smaller percentage of grains with very low sensitivities was present. This was done in anticipation of an extended investigation to solve the problem for all grain sizes of the emulsion. However, at that time it was found to be possible only with X-ray exposures on unsensitized silver bromide grains (7). If a sensitized emulsion contains grains with both low and high sensitivities, there is a complex sensitivity distribution among the grains of this emulsion. Several cases of such complex sensitivity distributions among the grains of an emulsion have been described previously by Trivelli (8). These grains of extremely low sensitivity, however, complicate the sensitivity problem even more.

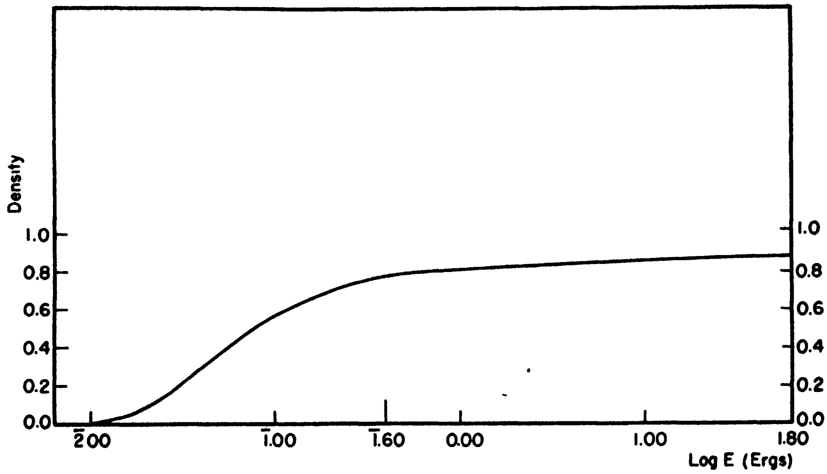


FIG. 9.

Returning to Fig. 8, Curve V = Curve I − Curve IV represents the size frequencies of the undeveloped grains at the maximum density, $D_8 = 1.54$, obtained upon 3-min. development. It is obvious that the frequencies of these grains become smaller with prolonged development until the size-distribution curve, III, is reached. Curve V represents the combined size distributions of the grains with very low sensitivities and the residue of the developable grains which have not yet been reduced by 3-min. development. By subtracting the frequencies of Curve III from those of Curve V, Curve VI is obtained, representing the size distributions of all developable grains left after 3-min. development.

SIZE-DISTRIBUTION RELATIONSHIPS OF THE DEVELOPED SILVER HALIDE GRAINS.

It has previously been considered (9) that the size distribution of the developed grains follows the same law as that of the grains of the original emulsion. The size-frequency distribution of the photographic emulsion that was used to demonstrate this was unusually simple in that it contained no breaks and was well fitted by the simple lognormal function:

$$y = A_0 e^{-k(\log x - \alpha_0)^2} \cdot dx. \tag{2}$$

This can also be expressed more appropriately, from our present viewpoint, merely by a change of constants (10):

$$y = \frac{A_{\bar{1}}}{x} \cdot e^{-k(\log x - \alpha_{\bar{1}})^2} \cdot dx. \tag{3}$$

The symbols in these equations are those generally used in particle-size investigations.

This conclusion was based on the fact that the curves on double logarithmic paper apparently retained the same fundamental shape and that the curves applied to the data by mere algebraic application of the formula fitted the data quite well. A break in the size-frequency distribution curve as found in this photographic emulsion is more usual, however, and this restricts the application of the simple formulas, 2 and 3, to one segment; the one containing the mode is usually chosen. The operation of fitting this segment to the size-frequency formula is a very helpful procedure. In the series of graphs of the size-frequency distributions of the developed grains of the exposure steps at each development time, it was found that the break in the distribution tended to disappear on prolonged exposure. This is illustrated in Fig. 10, which shows the size-frequency curves of three widely separated exposure steps from a strip developed for 5 min. The curves for other development times are similar.

If this lognormal type of formula applies and the distribution curve is affected as a whole, the parameters of these formulas are fundamentally and usually more simply applicable than the usual statistical constants obtained with the assumption that the Gauss-LaPlace "normal" distribution function is descriptive. For instance, it has been shown (10) that during ripening of a photographic emulsion, the width of the double logarithmic curves is constant, that is, the parameter, *k*, is an invariant in this case, although the average particle size and the standard deviation are changed.

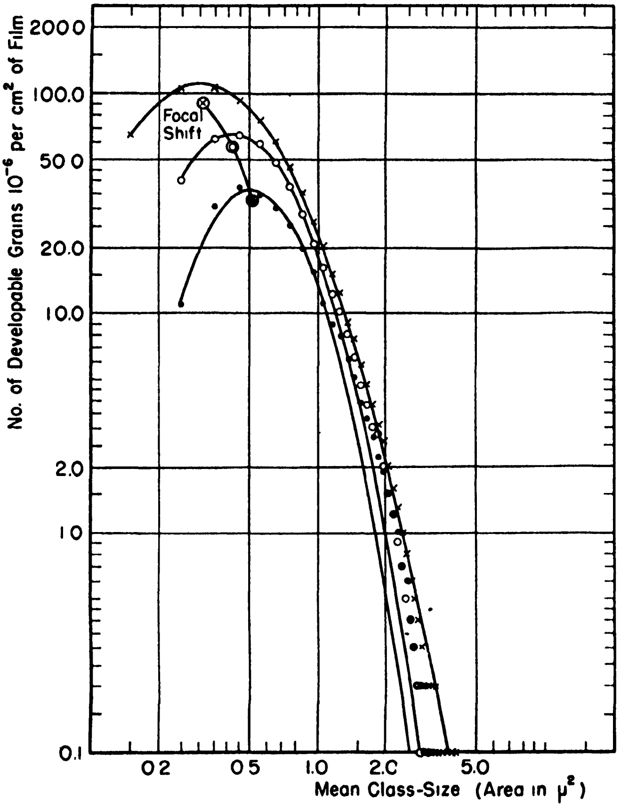


FIG. 10. Five-minute development.

Log E
● 1.69
○ 0.55
× 1.91

Although these parameters can be calculated from their relations with the normal Gaussian parameters, and *vice versa*, they can easily be determined graphically *for a segment or for a whole curve for which the formula applies:*

α_0 = logarithm of the modal size (vertex)—most frequently occurring size;

$\alpha_{\bar{1}}$ = logarithm of the median size = log $\bar{x}_{med}$;

Mk = the width of the log-log curve through the focus (latus rectum), where $M = 0.434$ is the modulus;

A_0 = the antilogarithm of the ordinate of the mode; and

k is a measure of the fundamental dispersion of the emulsion, varying inversely with it, so that $K = 1/k$ is a direct measure of the fundamental dispersion:

$$\log \bar{x}_a = \alpha_{\bar{1}} + 1/(4Mk),$$
$$(C_v)^2 = (\sigma/\bar{x}_a)^2 = e^{K/2M^2} - 1,$$

where $\bar{x}_a$ = the arithmetic mean size and σ = the standard deviation.

A simpler measure of dispersion when the distribution is lognormal but has the usual statistical constants is the ratio:

$$\bar{x}_a/\bar{x}_{\text{med}} = e^{K/4M^2},$$

or

$$\log \bar{x}_a - \log \bar{x}_{\text{med}} = K/4M = 0.5757K.$$

If natural logarithms were used in the lognormal formula, the modulus, M, would, of course, disappear.

The values of $\alpha_{\bar{1}}$ and K are given for the exposure steps of the 3-, 5-, and 12-min. development strips, respectively (Table V).

TABLE V.

Log E	$\alpha_{\bar{1}}$			K		
Development Time	3 min.	5 min.	12 min.	3 min	5 min.	12 min.
$\bar{1}.69$	−0.193	−0.199	−0.225	0.083	0.080	0.097
0.55	−0.231	−0.249	−0.280	0.115	0.111	0.127
1.91	−0.306	−0.325	−0.321	0.184	0.168	0.191
	$\bar{x}_{\text{med}}$ = antilog $\alpha_{\bar{1}}$ in μ^2 (projective area)					
$\bar{1}.69$	0.64	0.63	0.60			
0.55	0.59	0.56	0.53			
1.91	0.50	0.47	0.48			

The trend here is the opposite of that observed with the residual grains. There is a definite but slight decrease in average size of the developed grains with increase of exposure and an increase in the dispersion range of sizes. Figure 11 shows a plot of the change in the average grain size for 5-min. development with increasing exposure, both as the arithmetic average and as the median. The shift is linear with log E but by the relationships shown in the previous pages the two cannot be quite parallel.

It should be noted that the slope of the line exhibiting the relationship between the *average* particle size of a distribution of grains is necessarily less than the direct effect of the particle size, since each change of particle size with each exposure step principally exhibits the *relative* change of a fraction of grains at the larger end of the distribution that are exhausted and the addition of a fraction of smaller grains entering the affected group.

SENSITIVITIES OF GRAINS OF DIFFERENT SIZES.

Let N be the number of developed grains per square centimeter of emulsion layer of average class-size, $\bar{a}$, for a given exposure and development time. Then the $N/\log E$ curves for such a grain size can be derived from the size-frequency data shown in Figs. 6 and 7 for 3- and 12-min. developments. The data for 5-min. development will be used in this section. They have not been used in this paper up to this point because of their similarity to the data for the 3-min. development.

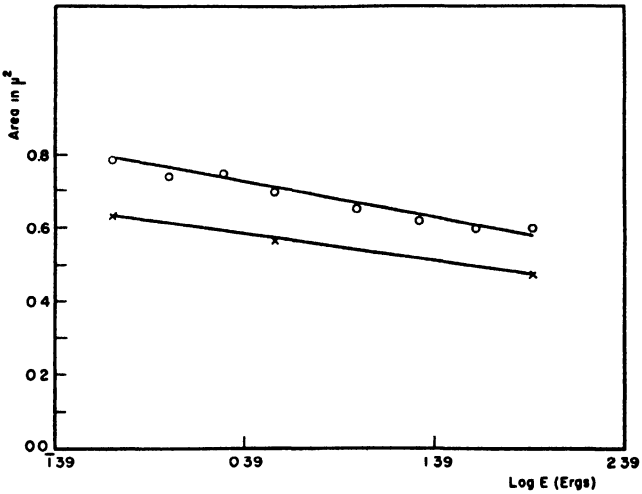


FIG. 11. Five-minute development.
Developable grains
⊙ Arithmetic average grain size
× Median grain size

A picture of the $N/\log E$ curves for the different mean class-sizes, ranging from $0.25\mu^2$ to $1.85\mu^2$, is shown in Fig. 12. All the curves have been smoothed in the same manner as the curve for $\bar{a}_3 = 0.25\mu^2$, for which the observed data are also given. The graph shows that at the lowest exposures nearly all the largest grains are already exposed and developed effectively.

THE AVERAGE SIZES OF THE DEVELOPED GRAINS OF THE DIFFERENT DENSITIES OF THE CHARACTERISTIC CURVE.

The average size of the developed grains of each density of the emulsion layer can be derived from the size-frequency curves of the developed grains for 3-, 5-, and 12-min. development. The results

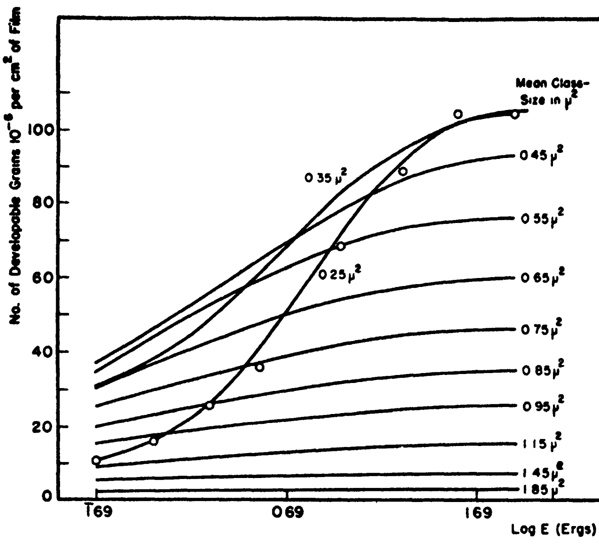


FIG. 12. Five-minute development.
○ N_{obs} for $\bar{a}_s = 0.25\mu^2$

are collected in Table VI, in which are given the Log *E* exposures, the densities obtained for different development times, and the average developed grain sizes, $\bar{a}$, in μ^2.

TABLE VI.

3-min. Development

Log *E*	$\bar{1}.39$	$\bar{1}.69$	$\bar{1}.99$	0.28	0.55	0.70	0.98	1.31	1.61	1.91
D	0.08	0.16	0.30	0.48	0.71	0.83	1.01	1.28	1.46	1.54
$\bar{a}$	0.740	0.761	0.778	0.725	0.713	0.702	0.680	0.641	0.620	0.609

5-min. Development

Log *E*	—	$\bar{1}.69$	$\bar{1}.99$	0.28	0.55	—	0.98	1.31	1.61	1.91
D	—	0.18	0.34	0.54	0.78	—	1.17	1.44	1.58	1.65
$\bar{a}$	—	0.781	0.738	0.745	0.692	—	0.650	0.619	0.597	0.597

12-min. Development

Log *E*	$\bar{1}.39$	$\bar{1}.69$	$\bar{1}.99$	0.28	0.55	0.70	0.98	1.31	1 61	1.91
D	0.15	0.25	0.43	0.66	0.92	1.09	1.31	1.58	1.71	1.80
$\bar{a}$	0.723	0.722	0.722	0.690	0.682	0.672	0.637	0.614	0.590	0.587

If the values of $\bar{a}$ are plotted against the corresponding densities, straight-line relations are obtained which, by the method of least squares, give the following simple equations for the different development times:

$$\text{For 3 min.}: \bar{a} = 0.790 - 0.115\,D. \qquad (2a)$$

$$\text{For 5 min.}: \bar{a} = 0.800 - 0.127\,D. \qquad (2b)$$

$$\text{For 12 min.}: \bar{a} = 0.740 - 0.080\,D. \qquad (2c)$$

The $\bar{a}/D$ relations are shown in Fig. 13. It is to be noted that the differences between the average size of the developed grains of the low and high densities are relatively small. They are nearly the same for different development times. This result need not be the same for other emulsions having a different sensitivity distribution among the grains.

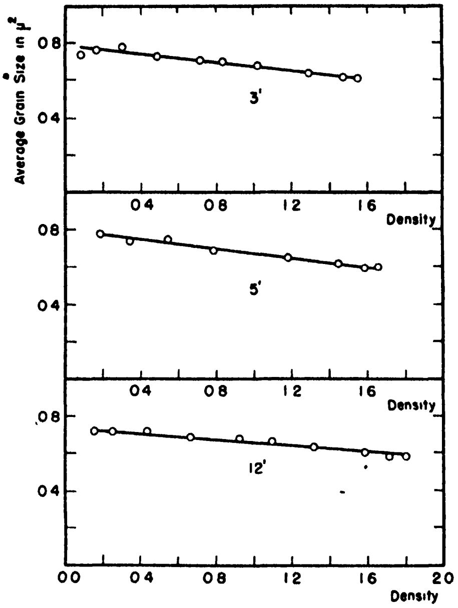


FIG. 13.

THE RELATION BETWEEN DENSITY AND NUMBER OF DEVELOPED GRAINS: RECONSIDERATION.

It will be noted that the graphs of Figs. 3 and 13, as well as the data which they represent, are apparently incompatible. That is, if the size of the average developable grain varies directly with the increase of density obtained with increased photographic exposure, then simple theory would not lead one to expect that the number of such grains would increase linearly with increasing density, since the higher densities must contain a greater proportion of small grains, each contributing less silver. The size-frequency curve is, of course, nonlinear.

In explaining the relation of the observed density-number ratio, (D/N_d), of Fig. 3 and the density-average grain size relation of Fig. 3, at least two factors are involved.

In the first place, the variation of *average* grain size with density is relatively small, as has already been discussed. Moreover, as already stated, the validity of the statistical size-frequency data is much less at the low densities, so that small departures from linearity could not be established.

However, the constancy of the D/N_d ratio seems to be established by Fig. 3, especially by the curve for 4-min. development, within an accuracy sufficient for most purposes.

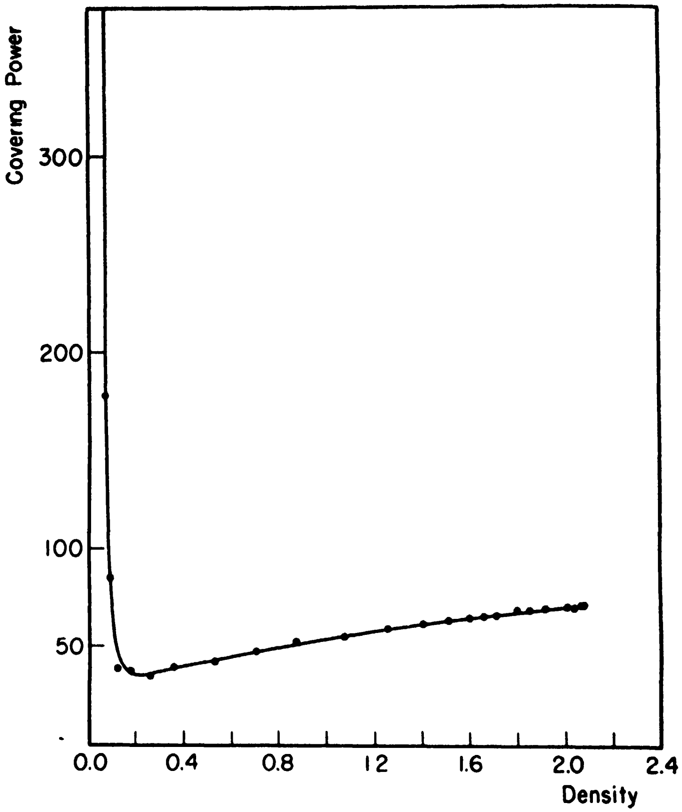


FIG. 14. Twelve-minute development.

Since, according to the above reasoning, only a relatively small nonlinear component is to be expected, this is probably well offset by another one, that is, the greater covering power obtained with decreasing grain size. This factor was studied for this case by means of the analysis of the silver content per unit film area for each density step of the film strips developed for 12 min., which was made for us by Mr. A. Ballard, of these Laboratories. The covering power, which is the ratio,

$$\frac{\text{Photographic Density}}{\text{Grams of Silver per Unit Area}},$$

and which is the reciprocal of the photometric equivalent, is plotted in Fig. 14 against the densities of the sensitometric strip. Considering, for the moment only, the curve above a density 0.2, we know from previous work (11) that the covering power-density curve for shorter times of development, such as 4 min., would be a little steeper (would show somewhat greater gain in covering power with density) and would even become slightly concave upward. That is, the covering power of the silver grains increases for the smaller grains, offsetting their smaller contribution of silver. Specifically, the effect of this covering power curve of Fig. 14 offsets the effect of the size-frequency curves of the developable grains of Fig. 8.

As has been pointed out, the statistical fluctuation of the data at the very low densities was so great that such data were considered too unreliable to report. The other covering power-density curves (11), however, have all shown the same fundamental curve shape down to a density of 0.2.

SUMMARY.

1. The concentration of silver halide grains (number per square centimeter) in the multilayer photographic film coating and their size distribution were determined for the emulsion under investigation. These two analyses were also made for the halide grains unaffected by exposure and development for the steps of an unfixed sensitometric step wedge; a corresponding normally fixed density step wedge was used to determine the densities. The concentration and size distribution of the *developable grains* that, upon exposure and development, produce the densities of the step wedge were obtained by subtraction of the data of the residual grains from the corresponding data of the original total number of emulsion grains.

2. The grains of this emulsion were formed during precipitation principally on two types of nuclei, as evidenced by the break in the normal function.

3. The ratio of the number of grains, N_d, to the optical density, D, is a constant,

$$N_d = 4.0 \times 10^8 \cdot D,$$

which is substantially unchanged between 3- and 12-min. development.

4. The relation between the average size (as projective area) of the developable grains and the density is practically a linear one.

5. The size distribution of the undeveloped grains in the maximum densities, D_{max}, by exposure, with increasing development times, is shown graphically and discussed. About 32 per cent of the grains are still undeveloped as the development approaches maximum development. The size distribution resembles that of a simple negative expo-

nential form, with the smallest grains being predominant in number. These fine grains have a very low sensitivity.

6. A case is cited in which a very slow but definite increase of density occurred upon prolonged exposure after the usual shoulder was reached.

7. Since the size distribution of the residual undeveloped grains of each exposure step resembles that of the undeveloped emulsion in type, the effect of exposure on the statistical size-distribution parameters is presented. For a lognormal distribution, the ratio of the arithmetic average to the median size, $\bar{x}_a/\bar{x}_{med}$, or the difference of their logarithms, is simply related to the *fundamental dispersion*, *K*, of the emulsion precipitate and is a good measure of it.

8. The arithmetic average size and also the median size of the *developable grains* decrease linearly with increase of the logarithm of the exposure for the range studied. The fundamental dispersion of the developable grains, however, increases with exposure.

REFERENCES.

(1) R. P. Loveland and A. P. H. Trivelli, *J. Phys. Chem.*, Vol. 51, pp. 1004–1027 (1927).

(2) S. E. Sheppard and C. E. K. Mees, "Investigations on the Theory of the Photographic Process," Longmans, Green and Co., London, 1907, p. 79.

(3) P. G. Nutting, *Phil. Mag.*, Vol. 26, p. 423 (1913).

(4) L. Silberstein and A. P. H. Trivelli, *J. Opt. Soc. Am.*, Vol. 28, p. 441 (1938).

(5) R. B. Wilsey and H. A. Pritchard, *J. Opt. Soc. Am.*, Vol. 12, p. 161 (1926); and A. P. H. Trivelli, *Zeitschr. f. wiss. Phot.*, Vol. 37, p. 98 (1938).

(6) A. P. H. Trivelli, Journal Franklin Inst., Vol. 241, p. 12 (1946).

(7) A. P. H. Trivelli and L. Silberstein, *Phil. Mag.*, Vol. 9, p. 787 (1930).

(8) A. P. H. Trivelli, Journal Franklin Inst., Vol. 239, p. 101 (1945); *ibid.*, Vol. 241, p. 16 (1946).

(9) E. C. Jensen and A. P. H. Trivelli, Journal Franklin Inst., Vol. 211, p. 489 (1931).

(10) R. P. Loveland and A. P. H. Trivelli, Journal Franklin Inst., Vol. 204, pp. 193, 377 (1927).

(11) S. E. Sheppard and A. Ballard, Journal Franklin Inst., Vol. 206, p. 659 (1928).

Microfilm Camera for Unusually Large Copy.—A new revolutionary triple-purpose microfilm camera capable of photographing unusually large copy up to 42 in. in width by any length in one continuous exposure was exhibited recently at the showrooms of Diebold Incorporated, New York City.

The camera, which also serves as a film printer and enlarger, is the result of years of experiment by Diebold Laboratories to broaden the application of the automatic flow principal of microphotography to include records and documents wider than the present limitation of 14 in. Its principal applications are in the microreproduction of engineering drawings, newspapers, blueprints, maps, surveys, and large production control and accounting records.

Development of the new Flofilm 42-in. camera, a compact unit which resembles a home laundry mangle and is fed much the same as a blueprint printer, necessitated new engineering designs which would make possible satisfactory photographic results at higher reduction ratios. A reduction ratio of 34 times which reduces the microfilm image to less than 1/1000 the area of the original copy is made possible by inverting the camera so that copy passes over the lens instead of underneath it.

The flow-type camera is capable of microfilming the longest copy in existence in one operation. A 100-ft. roll of film can take copy up to $\frac{5}{8}$ of a mile in length. Through use of the enlarging feature of the camera, images can be "blown back" at reduced size, original size or larger on a single sheet or roll of paper in one printing operation.

Any desired number of inexpensive contact prints can be made from the original roll of negative film. The camera when converted to a flow enlarger will print blow-ups of any desired image on photosensitive paper or cloth from the roll of negative film.

Copy can be moved through the camera at two speeds of 30 or 60 ft. per minute. At the speed of 60 ft. per minute, a drawing 10 ft. long can be microfilmed at 10 sec. Through a series of five buttons on the control panel, proper light intensity is controlled by the operator by merely touching the button offering the best lighting for any type of copy.

Du Pont Announces New Synthetic—Orlon (*Textile World*, Vol. 98, No. 9).—Orlon, formerly called Fiber A, is the name given by Du Pont to the acrylic yarn it is producing in small quantities at its Waynesboro, Va., plant. Large-scale production is rumored as planned for Du Pont's projected plant at Camden, S. C.

Orlon is technically a polyacrylontrile fiber, commonly called acrylic. Its properties place it between nylon and rayon. Work on it was begun in 1940, and for several years laboratory and pilot-plant production has been used to explore the possibilities of the new fiber in continuous-filament form. Yarn sizes from 30 to 600 den. have been produced. Filaments range from 1 to 5 den.

The new fiber can be used on conventional equipment for winding, twisting, knitting, slashing, weaving, braiding, dyeing, and finishing. Some ultimate uses for Orlon include: automobile tops, cordage, fish nets, laundry bags, electrical insulation, rainwear, bathing suits, curtains, window shades and awnings.

GENERALIZED MAGNETO-IONIC THEORY.

BY

N. C. GERSON [1] AND S. L. SEATON.[2]

ABSTRACT.

A vector equation relating the polarization of a homogeneous isotropic medium to an external impressed magnetic field was derived under the condition that the medium is penetrated by an electromagnetic wave. From the three implicit scalar relationships, a final equation involving the fourth degree of the refractive index was obtained. The identity of the quartic equation with the corresponding relationships of Appleton and Booker under the appropriate restrictive conditions is shown.

INTRODUCTION.

The theory of magneto-ionic refraction in the ionosphere as propounded by Appleton (**1**, **2**),[3] treated what may be considered a one-dimensional propagational case: an electromagnetic wave propagating solely along the Z axis and penetrating a dielectric normal to its surface. Two conditions were stipulated: (1) that the lapse rate of refractive index along Z within the dielectric was so small that for several wave lengths the medium could be considered homogeneous, and (2) that an external static or very slowly varying magnetic field existed which could be resolved into two components, longitudinal (along the direction of propagation of the electromagnetic wave) and transverse (perpendicular to the direction of wave propagation). It was assumed that the transverse component existed along only one of the remaining two coordinate axes. The ionosphere was taken as parallel to and separated from a horizontal flat earth.

Similar treatments were later applied by Nichols and Schelleng (**3**), Breit (**4**), Goldstein (**5**), and Hartree (**6**). Booker (**7**) in 1938 advanced the problem by discussing the two-dimensional phase propagational case: an electromagnetic wave propagating linearly in the Y–Z plane and penetrating a plane stratified dielectric obliquely to its surface. He also considered the external magnetic field as possessing two mutually perpendicular transverse components, in distinction to only one assumed by Appleton. The remaining factors were the same as those prescribed above. Thus, under the condition that the angle of incidence of the wave was 90°, and that the transverse component of the magnetic field extended along only one of the coordinate axes, Booker's more general exposition degenerated into that of Appleton's.

---

[1] Chief, Electromagnetic Wave Propagation Section, Watson Laboratories, Red Bank, N. J.

[2] Director, Geophysical Observatory, University of Alaska, College, Alaska.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

A logical extension of the above treatments is the complete generalization into the three-dimensional propagational case: an electromagnetic wave in space penetrating a dielectric when the velocity vector is distinct from each of the three coordinate axes. A formulation and development of the problem on these lines is presented for a homogeneous isotropic medium.

In general, the final quartic equation which is obtained does not admit of a ready analytical solution through any of the various devised methods. Because of its complexity, the equation will be solved mainly by recourse to one of the standard numerical methods (for example, Newton's or Horner's method) for the solution of algebraic quartic equations.

SYMBOLS.

$\mathbf{c}$ = velocity of electromagnetic propagation in vacuum,
e = charge of electron,
$\mathbf{E}$ = mean electric intensity of the electromagnetic wave,
$\mathbf{H}$ = mean magnetic intensity of the electromagnetic wave,
$\mathbf{H}^o$ = mean impressed external magnetic field,
$i = \sqrt{-1}$,
m = mass of electron,
$n = \mathbf{S}\cdot\mathbf{c}$ = refractive index of medium,
N = number of bound electrons in the medium,
$\mathbf{P}$ = electric polarization of the medium,
$\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$,
$\mathbf{S}$ = reciprocal of actual velocity of wave in the medium,
t = time in seconds,
$\omega = 2\pi\nu$ = angular velocity,
ν = frequency of electromagnetic wave.

Note that $\mathbf{S} \times \mathbf{c} = 0$.

Components of each of the vectors along the x, y, z axes will be indicated by the subscripts 1, 2, 3, respectively.

Vectors are in boldface type.

FUNDAMENTAL ELECTROMAGNETIC RELATIONSHIPS.

Maxwell's equations for the propagation of electromagnetic waves are, in Heavyside-Lorentz Units,

$$\nabla \times \mathbf{E} = -\dot{\mathbf{H}}/c, \tag{1}$$

$$\nabla \times \mathbf{H} = (\dot{\mathbf{E}} + \dot{\mathbf{P}})/c. \tag{2}$$

The magnetic field $\mathbf{H}$ may be eliminated by taking the cross product of ∇ with (1), the time derivative of (2), and combining equations.

$$\nabla \times \nabla \times \mathbf{E} = (-1/c)\nabla \times \dot{\mathbf{H}} = -(1/c^2)(\ddot{\mathbf{E}} + \ddot{\mathbf{P}}). \tag{3}$$

Expanding the triple vector product,

$$\nabla\nabla\cdot\mathbf{E} - \nabla\cdot\nabla\mathbf{E} + (1/c^2)(\ddot{\mathbf{E}} + \ddot{\mathbf{P}}) = 0. \tag{4}$$

Expression (4) provides the relationship between the electric field of the propagating wave and the polarization of the medium through which the wave passes. As a solution of (4) is desired explicitly in terms of **P**, a second independent relationship involving **P** and **E** under the required conditions is sought. The relationship may be established by examining the dependence of **E** and **P** in a homogeneous isotropic medium which lies in an external magnetic field.

As an aid to the subsequent analysis, a wave solution of the type

$$\mathbf{E}, \mathbf{H}, \mathbf{P}, \text{etc.}, = \mathbf{A}e^{i\omega(\mathbf{S}\cdot\mathbf{r}-t)}, \tag{5}$$

is postulated for all fields associated with the electromagnetic wave where $\mathbf{A}$ = constant vector amplitude.

By means of (5) the several differential operators may be replaced by the following algebraic equivalents:

$$\left.\begin{aligned} \frac{\partial}{\partial t} &= -i\omega, \qquad & \nabla &= i\omega(\mathbf{i}S_1 + \mathbf{j}S_2 + \mathbf{k}S_3), \\ \frac{\partial^2}{\partial t^2} &= -\omega^2, \qquad & \nabla\cdot\nabla &= -\omega^2 S^2, \end{aligned}\right\} \tag{6}$$

etc., provided $\mathbf{S} \neq f(x, y, z, t)$.

POLARIZATION IN A HOMOGENEOUS ISOTROPIC MEDIUM.

The medium under consideration is visualized as being composed of an almost infinite number of regularly spaced atoms, each containing close to its assigned number of asymmetrically concentrated planetary electrons. In penetrating the medium an electromagnetic wave of frequency less than the ionization frequency reacts with some or all of the electrons in its path and sets them in vibration. The relatively massive nucleii, however, remain unaffected by the wave. From a microscopic outlook, the resultant field within the body could be obtained on a statistical basis by integrating, over the entire volume, the combined fields of each electron and each nucleus and the field of the electromagnetic wave. Employing the integrated field equations thus obtained, the interactions of the field on the electrons and the equations of motion for the electrons could be derived. However, the method outlined is subject to great mathematical difficulties unless rather broad assumptions be made regarding the existent highly irregular distribution in time and space of each electron.

A rigorous treatment, nevertheless, may be formulated by classical procedures. In this method the problem is considered macroscopically and mean values of the electromagnetic field are utilized. The assumption is implicitly made that microscopic irregularities, mutual interactions, etc., of the individual nucleii and electrons effectively cancel, leaving only a net gross effect. As the latter effect is the one with which we are concerned, the course outlined immediately above will be followed.

Because of the relative magnitude of the nuclear and electronic masses, each term of the momental dyadic, that is, the moments and products of inertia, for the permanent dipoles will be very much greater than those for the electrons. The rapidly fluctuating electric and magnetic intensity of an electromagnetic wave consequently will affect the lighter electrons to a far greater extent than the massive permanent dipoles. At moderate and high frequencies, the total reaction of the wave with the medium may be regarded as confined solely to producing an oscillatory motion of the electrons. Similar considerations apply to ions in the electromagnetic field, positive ions remaining unaffected.

A mobile electron (in a homogeneous isotropic dielectric) whose motion has been induced by interaction with an electromagnetic field is subject to two forces: (*a*) a force of restitution, proportional to the displacement of the electron, which acts to restore the electron to its equilibrium position, and (*b*) a damping force, proportional to the electron velocity, which acts to resist motion. When an external magnetic field is impressed upon the medium, a third force exists—a magnetic force, proportional to the external magnetic field—which acts to change the velocity of the moving electron.

It should be noted that the model of a homogeneous isotropic dielectric was taken to provide generality of treatment. In the ionosphere there exist free electrons and ions, much in the manner of a spherical shell of somewhat rarified electron gas. The distinction between the model of a dielectric and that of an electron gas becomes evident in the restitutive force term. With a dielectric this term exists, but with an electron gas no such restoring force arises. Thus, when the discussion later is restricted to the ionosphere, this term vanishes. Also, the damping force term, due to irreversible energy absorption from the electromagnetic field, proceeds from electron collisional and recombinational activity.

In the development which follows, a steady state condition, for example, one which exists after unlimited wave trains have traversed the medium for a time long in comparison with the vibratory period, will be considered. The discussion is restricted to low electron velocities ($c \gg \dot{\mathbf{r}}$), thereby making it possible to neglect the relativistic variations of mass with velocity. Under these circumstances, the motion of an electron subject to the simultaneous action of an external electromagnetic and impressed magnetic field is given by:

$$m\ddot{\mathbf{r}} + ml\dot{\mathbf{r}} + mk^2\mathbf{r} = e[\mathbf{E}' + (1/c)\dot{\mathbf{r}} \times \mathbf{H}'], \qquad (7)$$

where

$$\mathbf{E}' = \mathbf{E} + (1/3)\mathbf{P} = \text{total mean electric field,}$$
$$\mathbf{H}' = \mathbf{H} + \mathbf{H}^o = \text{total mean magnetic field,}$$
$$m\ddot{\mathbf{r}} = \text{accelerating force on electron,}$$
$$-ml\dot{\mathbf{r}} = \text{damping force on electron, and}$$
$$-mk^2\mathbf{r} = \text{restorative force on bound electron.}$$

Since $\mathbf{E}$ and $\mathbf{H}$ are of the same order of magnitude, and since $|\dot{\mathbf{r}}| \ll |\mathbf{c}|$, the term $(\dot{\mathbf{r}} \times \mathbf{H})/c$ is negligibly small in comparison with $\mathbf{E}$ and may be neglected. Discarding the term involving $\mathbf{H}$, (7) may be rewritten as

$$m\ddot{\mathbf{r}} + ml\dot{\mathbf{r}} + mk\mathbf{r} = e[\mathbf{E} + (1/3)\mathbf{P} + (1/c)\dot{\mathbf{r}} \times \mathbf{H}^o]. \tag{8}$$

To simplify (8) note that the periodic oscillation of the electron permits the utilization of the relationships (6), whence

$$\ddot{\mathbf{r}} = -\omega^2\mathbf{r}, \qquad \dot{\mathbf{r}} = -i\omega\mathbf{r}. \tag{9}$$

Further simplifications in (8) may be made as follows: by defining the dyadic

$$\mathbf{\Psi} = \mathbf{ii}0 + \mathbf{ij}H^o{}_3 - \mathbf{ik}H^o{}_2 - \mathbf{ji}H^o{}_3 + \mathbf{jj}0 + \mathbf{jk}H^o{}_1 + \mathbf{ki}H^o{}_2 - \mathbf{kj}H^o{}_1 + \mathbf{kk}0, \tag{10}$$

the cross product $\dot{\mathbf{r}} \times \mathbf{H}^o$ may be replaced by

$$\dot{\mathbf{r}} \times \mathbf{H}^o = -i\omega\mathbf{r} \times \mathbf{H}^o = -i\omega\mathbf{\Psi} \cdot \mathbf{r}. \tag{11}$$

Use will also be made of the idemfactor

$$\mathbf{I} = \mathbf{ii} + \mathbf{jj} + \mathbf{kk}. \tag{12}$$

Employing Eqs. 8 to 12, (7) becomes

$$(k^2 - \omega^2 - i\omega l)\mathbf{I}\cdot\mathbf{r} = (e/m)[\mathbf{E} + (1/3)\mathbf{P} - i(\omega/c)\mathbf{\Psi}\cdot\mathbf{r}]. \tag{13}$$

Elimination of $\mathbf{r}$ may be accomplished through the elementary electrostatic formula

$$\mathbf{P} = Ne\mathbf{r}. \tag{14}$$

Thus

$$(m/Ne^2)[(k^2 - \omega^2 - i\omega l - Ne^2/3m)\mathbf{I} + i(e\omega/cm)\mathbf{\Psi}]\cdot\mathbf{P} = \mathbf{E} \tag{15}$$

or

$$a\mathbf{P} + b\mathbf{\Psi}\cdot\mathbf{P} = \mathbf{E}, \tag{16}$$

if the complex constants a and b are defined by

$$\left.\begin{aligned} a &\equiv (m/Ne^2)(k^2 - \omega^2 - Ne^2/3m - i\omega l), \\ b &\equiv (i\omega)/Nec. \end{aligned}\right\} \tag{17}$$

Equation 17 provides the second relationship involving $\mathbf{P}$ and $\mathbf{E}$ which was necessary for the solution of (4). Solving (17) and (4) simultaneously for $\mathbf{P}$

$$[a(\nabla\nabla - \nabla\cdot\nabla\mathbf{I}) + b(\nabla\nabla\cdot\boldsymbol{\Psi} - \nabla\cdot\nabla\boldsymbol{\Psi})]\cdot\mathbf{P} + (1/c^2)(1 + a)\ddot{\mathbf{P}} + (b/c^2)\frac{\partial^2}{\partial t^2}(\boldsymbol{\Psi}\cdot\mathbf{P}) = 0. \qquad (18)$$

Inasmuch as $\boldsymbol{\Psi} \neq F(t)$

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\Psi}\cdot\mathbf{P} = \boldsymbol{\Psi}\cdot\ddot{\mathbf{P}} = -\omega^2\boldsymbol{\Psi}\cdot\mathbf{P} \qquad (19)$$

and (18) becomes

$$\boldsymbol{\Theta}\cdot\mathbf{P} = 0, \qquad (20)$$

where

$$a(\nabla\nabla - \nabla\cdot\nabla\mathbf{I}) + b[\nabla\nabla\cdot\boldsymbol{\Psi} - \nabla\cdot\nabla\boldsymbol{\Psi} - (\omega^2/c^2)\boldsymbol{\Psi}] - (\omega^2/c^2)(1 + a)\mathbf{I} \equiv \boldsymbol{\Theta}. \qquad (20a)$$

Equation 20 provides the relationship between the polarization of a homogeneous isotropic medium and an impressed magnetostatic field under the condition that the former is penetrated by an electromagnetic wave and that **S** is not a function of either time or space. Under the same circumstances, (20) is also valid for an electron gas if the term containing k is eliminated.

The rather powerful vector equation (20) may be made more amenable to solution when written in terms of its three scalar relationships. The algebra is straightforward but cumbersome.

Note that the various terms may be expanded by the use of:

$$\begin{aligned}\nabla\nabla\cdot\mathbf{P} = -\omega^2[&\mathbf{i}(S_1^2P_1 + S_1S_2P_2 + S_1S_3P_3)\\ &+ \mathbf{j}(S_1S_2P_1 + S_2^2P_2 + S_2S_3P_3)\\ &+ \mathbf{k}(S_1S_3P_1 + S_2S_3P_2 + S_3^2P_3)]. \qquad (21)\end{aligned}$$

$$\begin{aligned}-\nabla\cdot\nabla\mathbf{P} = +\omega^2[&\mathbf{i}(S_1^2 + S_2^2 + S_3^2)P_1 + \mathbf{j}(S_1^2 + S_2^2 + S_3^2)P_2\\ &+ \mathbf{k}(S_1^2 + S_2^2 + S_3^2)P_3]. \qquad (22)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Psi}\cdot\mathbf{P} = &\mathbf{i}(P_2H^\circ_3 - P_3H^\circ_2) + \mathbf{j}(P_3H^\circ_1 - P_1H^\circ_3)\\ &+ \mathbf{k}(P_1H^\circ_2 - P_2H^\circ_1). \qquad (23)\end{aligned}$$

$$\begin{aligned}\nabla\nabla\cdot(\boldsymbol{\Psi}\cdot\mathbf{P}) = &\\ -\omega^2\{&\mathbf{i}[S_1^2(P_2H^\circ_3 - P_3H^\circ_2) + S_1S_2(P_3H^\circ_1 - P_1H^\circ_3)\\ &\quad + S_1S_3(P_1H^\circ_2 - P_2H^\circ_1)]\\ &+ \mathbf{j}[S_1S_2(P_2H^\circ_3 - P_3H^\circ_2) + S_2^2(P_3H^\circ_1 - P_1H^\circ_3)\\ &\quad + S_2S_3(P_1H^\circ_2 - P_2H^\circ_1)]\\ &+ \mathbf{k}[S_1S_3(P_2H^\circ_3 - P_3H^\circ_2) + S_2S_3(P_3H^\circ_1 - P_1H^\circ_3)\\ &\quad + S_3^2(P_1H^\circ_2 - P_2H^\circ_1)]\}. \qquad (24)\end{aligned}$$

$$\begin{aligned}-\nabla\cdot\nabla(\boldsymbol{\Psi}\cdot\mathbf{P}) = &\\ +\omega^2\{&\mathbf{i}[(S_1^2+S_2^2+S_3^2)P_2H^\circ_3 - (S_1^2+S_2^2+S_3^2)P_3H^\circ_2]\\ &+\mathbf{j}[(S_1^2+S_2^2+S_3^2)P_3H^\circ_1 - (S_1^2+S_2^2+S_3^2)P_1H^\circ_3]\\ &+\mathbf{k}[(S_1^2+S_2^2+S_3^2)P_1H^\circ_2 - (S_1^2+S_2^2+S_3^2)P_2H^\circ_1]\}. \qquad (25)\end{aligned}$$

Through the utilization of the above expressions, three scalar equations, each of the type (26) are obtained. The running index q goes cyclically 1, 2, 3. Thus the subscripts q, $q+1$, $q+2$ become

equation	q	$q+1$	$q+2$
1st	1	2	3
2nd	2	3	1
3rd	3	1	2.

$$\begin{aligned}P_q[a(S_{q+1}{}^2+S_{q+2}{}^2-1/c^2)+b(S_qS_{q+1}H^o{}_{q+2}-S_qS_{q+2}H^o{}_{q+1})-1/c^2]\\ +P_{q+1}[bH^o{}_{q+2}(S_{q+1}{}^2+S_{q+2}{}^2-1/c^2)-aS_qS_{q+1}+bS_qS_{q+2}H^o{}_q]\\ +P_{q+2}[-bH^o{}_{q+1}(S_{q+1}{}^2+S_{q+2}{}^2-1/c^2)-aS_qS_{q+2}-bS_qS_{q+1}H^o{}_q]=0.\end{aligned}\tag{26}$$

Use will also be made of the definitive relationships

$$\left\{\begin{aligned}S'^2 &\equiv S_1{}^2+S_2{}^2+S_3{}^2-1/c^2\equiv S^2-1/c^2,\\ A_q &\equiv a(S'^2-S_q{}^2)+bS_q(S_{q+1}H^o{}_{q+2}-S_{q+2}H^o{}_{q+1})-1/c^2,\\ B_q &\equiv bH^o{}_{q+2}(S'^2-S_q{}^2)+S_q(-aS_{q+1}+bS_{q+2}H^o{}_q),\\ C_q &\equiv bH^o{}_{q+1}(S'^2-S_q{}^2)+S_q(aS_{q+2}+bS_{q+1}H^o{}_q).\end{aligned}\right.\tag{27}$$

Each equation containing the running index q consists of three equations as indicated above. Employing (26) and (27) together with the preceding identities, (20) simplifies to

$$\left.\begin{aligned}A_1P_1+B_1P_2-C_1P_3&=0,\\ -C_2P_1+A_2P_2+B_2P_3&=0,\\ B_3P_1-C_3P_2+A_3P_3&=0.\end{aligned}\right\}\tag{28}$$

A non-trivial solution of (28) is obtained only when its determinant $=0$. Thus

$$\left.\begin{aligned}P_2&=P_1(C_1C_2-A_1B_2)/(B_1B_2+A_2C_1),\\ P_3&=P_1(A_1A_2+B_1C_2)/(B_1B_2+A_2C_1)\end{aligned}\right\}\tag{29}$$

and

$$A_1A_2A_3+B_1B_2B_3-C_1C_2C_3+A_1B_2C_3+A_2B_3C_1+A_3B_1C_2=0.\tag{30}$$

It is readily seen that, after a few manipulations, (30) reduces to

$$\begin{aligned}J(S'^2-S_1{}^2)(S'^2-S_2{}^2)(S'^2-S_3{}^2)+K(S'^2-S_1{}^2)(S'^2-S_3{}^2)\\ +L(S'^2-S_1{}^2)(S'^2-S_2{}^2)+M(S'^2-S_2{}^2)(S'^2-S_3{}^2)\\ +N(S'^2-S_1{}^2)+P(S'^2-S_2{}^2)+Q(S'^2-S_3{}^2)+R=0,\end{aligned}\tag{31}$$

where the coefficients of the terms involving S, that is, J, K, L, M, N,

P, Q, R are given by

$$J = a^3 + ab^2H^{o2}, \tag{32}$$

$$\left.\begin{aligned} K &= -(1/c^2)(a^2 + b^2H^o{}_2{}^2),\\ L &= -(1/c^2)(a^2 + b^2H^o{}_3{}^2),\\ M &= -(1/c^2)(a^2 + b^2H^o{}_1{}^2),\end{aligned}\right\} \tag{33}$$

$$\left.\begin{aligned} N &= -aS_2{}^2S_3{}^2(a^2 + b^2H^{o2}) - (1/c^2)(2b^2S_2S_3H^o{}_2H^o{}_3) + a/c^4,\\ P &= -aS_1{}^2S_3{}^2(a^2 + b^2H^{o2}) - (1/c^2)(2b^2S_1S_3H^o{}_1H^o{}_3) + a/c^4,\\ Q &= -aS_1{}^2S_2{}^2(a^2 + b^2H^{o2}) - (1/c^2)(2b^2S_1S_2H^o{}_1H^o{}_2) + a/c^4,\end{aligned}\right\} \tag{34}$$

$$\begin{aligned} R = &-2aS_1{}^2S_2{}^2S_3{}^2(a^2 + b^2H^{o2})\\ &-(1/c^2)[2b^2S_1S_2S_3(S_3H^o{}_1H^o{}_2 + S_2H^o{}_1H^o{}_3 + S_1H^o{}_2H^o{}_3)\\ &\quad - b^2(S_2{}^2S_3{}^2H^o{}_1{}^2 + S_1{}^2S_3{}^2H^o{}_2{}^2 + S_1{}^2S_2{}^2H^o{}_3{}^2)\\ &\qquad - a^2(S_1{}^2S_2{}^2 + S_1{}^2S_3{}^2 + S_2{}^2S_3{}^2)] - (1/c^6).\end{aligned} \tag{35}$$

At this point the various factors involving S_q may be expanded and the resulting equation then reduced to a quartic in $(cS_q)^4$. Considering the case where $q = 1$,

$$\alpha(cS_1)^4 + \beta(cS_1)^3 + \gamma(cS_1)^2 + \delta(cS_1) + \epsilon = 0, \tag{36}$$

where

$$\alpha = a^2(a+1) + ab^2H^{o2} + b^2H^o{}_1{}^2, \tag{37}$$

$$\beta = 2b^2c(S_2H^o{}_2 + S_3H^o{}_3)H^o{}_1, \tag{38}$$

$$\begin{aligned}\gamma = c^2(S_2{}^2+S_3{}^2)[2a(1+a)+2ab^2H^{o2}+b^2H^o{}_1{}^2]+b^2c^2(S_2H^o{}_2+S_3H^o{}_3)^2\\ -[2a(1+a)^2+(1+2a)b^2H^{o2}+b^2H^o{}_1{}^2],\end{aligned} \tag{39}$$

$$\delta = 2b^2cH^o{}_1(S_2H^o{}_2+S_3H^o{}_3)[c^2(S_2{}^2+S_3{}^2)-1], \tag{40}$$

$$\begin{aligned}\epsilon = c^4\{(S_2{}^2+S_3{}^2)^2[a^2(1+a)+ab^2H^{o2}]+b^2(S_2{}^2+S_3{}^2)(S_2H^o{}_2+S_3H^o{}_3)^2\}\\ -c^2\{(S_2{}^2+S_3{}^2)[2a(1+a)^2+(1+2a)b^2H^{o2}]+b^2(S_2H^o{}_2+S_3H^o{}_3)^2\}\\ +(1+a)^3+(1+a)b^2H^{o2}.\end{aligned} \tag{41}$$

COMPARISON OF RESULTS WITH APPLETON'S AND BOOKER'S RELATIONSHIPS.

To establish the identity of (36) with the corresponding equation of Appleton (2), assume that the restitutive term in (7) is non-existent, that is, that we deal with an electron gas, and that the damping term arises solely from collisional friction between the electrons and other particles. Further, let $S_3 = S_2 = 0$, whence (36) assumes the form

$$A(cS_1)^4 + B(cS_1)^2 + C = 0 \tag{42}$$

or in Appleton's notation

$$A'(c^2S_1^2 - 1)^2 + B'(c^2S_1^2 - 1) + C' = 0, \tag{43}$$

where

$$\begin{aligned} A' &= A, \\ B' &= 2A + B, \\ C' &= A + B + C \end{aligned}$$

and

$$\left.\begin{aligned} A &= a^2(1 + a) + ab^2H^{o2} + b^2H^o{}_1{}^2, \\ B &= -\left[2a(1 + a)^2 + b^2H^o{}_1{}^2 + b^2H^{o2}(1 + 2a)\right], \\ C &= (1 + a)^3 + (1 + a)b^2H^{o2}. \end{aligned}\right\} \tag{44}$$

When $H^o{}_2 = 0$, (43) becomes

$$(cS_1)^2 - 1 = 2\left[2a + (b^2H^o{}_3{}^2)/(1 + a) \pm \sqrt{(bH^o{}_3)^4/(1 + a)^2 - 4b^2H^o{}_1{}^2}\right]^{-1}. \tag{45}$$

Equation 45 is identical with that deduced by Appleton except for one trivial respect. Following Lorentz (8), Appleton employed a wave solution of the type $\mathbf{A}e^{i\omega(t - \mathbf{S}\cdot\mathbf{r})}$ whereas in this presentation a wave solution $\mathbf{A}e^{i\omega(\mathbf{S}\cdot\mathbf{r} - t)}$ was utilized (9). Obviously the choice between the two is wholly arbitrary. The results differ only in the algebraic sign of the imaginary terms.

The reduction of (36) to Booker's equation is similarly obtained. When $S_3 = 0$, (36) becomes

$$A(cS_1)^4 + B(cS_1)^3 + C(cS_1)^2 + D(cS_1) + E = 0, \tag{46}$$

where

$$\begin{aligned} A &= a^2(1 + a) + ab^2H^{o2} + b^2H^o{}_1{}^2, \\ B &= 2b^2cS_2H^o{}_1H^o{}_2, \\ C &= c^2S_2{}^2[2a^2(1 + a) + 2ab^2H^{o2} + b^2H^o{}_2{}^2 + b^2H^o{}_1{}^2] \\ &\qquad - [2a(1 + a)^2 + (1 + 2a)b^2H^{o2} + b^2H^o{}_1{}^2], \\ D &= 2b^2cS_2H^o{}_1H^o{}_2(c^2S_2{}^2 - 1), \\ E &= c^4S_2{}^4[a^2(1 + a) + ab^2H^{o2} + b^2H^o{}_2{}^2] \\ &\qquad - c^2S_2{}^2[2a(1 + a)^2 + (1 + 2a)b^2H^{o2} + b^2H^o{}_2{}^2] \\ &\qquad + (1 + a)^3 + (1 + a)b^2H^{o2}. \end{aligned}$$

Multiply (46) by the factor $-(N^3e^6/m^3\omega^6)$. Neglect the polarization term $(Ne^2/3m)$ and the restitutive force term (involving k).

Define the following equalities

$$\begin{aligned} u &= 1 - il/\omega, \\ x &= Ne^2/m\omega^2, \\ y &= e\mathbf{H}^o/mc\omega, \\ cS_2 &= \sin\theta. \end{aligned}$$

Employing these relationships, the coefficients of (cS_1) in (46) become

$$\left.\begin{aligned} A &= u^3 - xu^2 - uy^2 + xy_1^2, \\ B &= 2xy_1y_2\,(\sin\theta), \\ C &= 2u(y^2\cos^2\theta - u^2\cos^2\theta + xu\cos^2\theta + xu - x^2) \\ &\qquad - xy_1^2(\cos^2\theta + 1) - xy_2^2\cos^2\theta + xy_3^2, \\ D &= -\,2xy_1y_2\sin\theta\cos^2\theta, \\ E &= (u - x)[(u\cos^2\theta - x)^2 - y_2^2\cos^4\theta] \\ &\qquad - \cos^2\theta(y_1^2 + y_3^2)(u\cos^2\theta - x). \end{aligned}\right\} \quad (49)$$

Equation 46 with the coefficients (49) is Booker's relationship for the index of refraction when wave packets impinge obliquely upon a stratified doubly refracting ionosphere. Note that he also employed a wave solution $Ae^{i\omega(t-\mathbf{S}\cdot\mathbf{r})}$ and that therefore the signs of the imaginaries in (49) are negative with respect to those which he obtained.

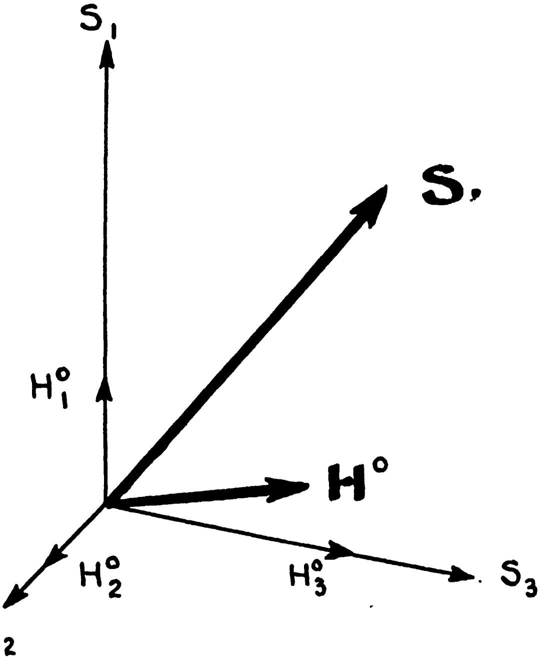


FIG. 1.

From the above comparison, it would appear that both Appleton, Booker, and others considered an ionosphere which was essentially homogeneous and isotropic. Components of the vectors **S** and $\mathbf{H}^0$ are depicted in Fig. 1.

The preceding development is perfectly general. The results may be applied to the ionosphere under the assumption that this region is homogeneous and isotropic and that no forces of interaction other than restitutive, damping or accelerating forces on the electrons appear.

Some workers have applied Eq. 36 to the atmosphere's ionized strata under the restrictions that S_2, S_3, $H^{\circ}{}_2$, etc. $= 0$.

The significance of the components S_2 and S_3 may be readily seen from the following brief exposition.

To a first approximation the earth and its ionosphere will be considered as a sphere and its concentric spherical shell, respectively. Under this simplification, refraction in the ionosphere occurs in accordance with the spherical rather than with the plane form of Snell's Law. Snell's Law for spherical surfaces is (see Fig. 2)

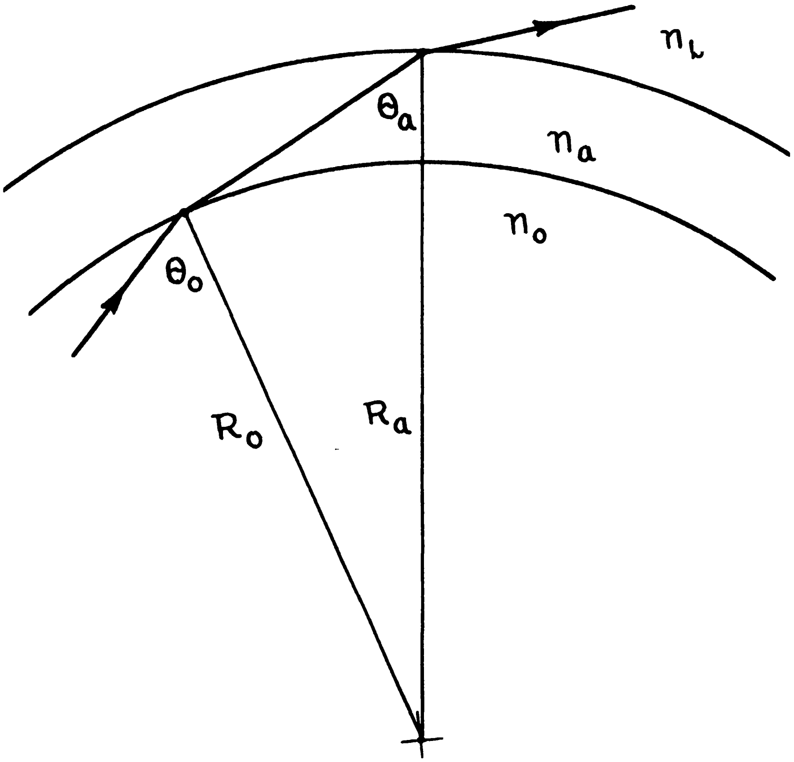


FIG. 2.

$$n_o R_o \sin\theta_o = n_a R_a \sin\theta_a = \cdots = n_r R_r \sin\theta_r. \qquad (50)$$

In ionospheric studies an important factor is the maximum height of penetration, that is, the height at which R_r of Eq. 50 is the hypotenuse of a right triangle. At this height, with little error, $\theta_r = \pi/2$; a reasonable assumption since $R_{r-1} \cong R_r$. Assume also that n_o = unity immediately below the base of the ionosphere. Thus, at the maximum height

$$\left.\begin{aligned}(R_o/R_2)\sin\theta_2 &= n_2 = cS_2,\\ (R_o/R_3)\sin\theta_3 &= n_3 = cS_3,\end{aligned}\right\} \qquad (51)$$

where

R_o = distance from center of spherical earth to the base of the ionosphere,

$R_2(R_3)$ = distance from center of spherical earth to maximum height of the components $S_2(S_3)$,

$\theta_2(\theta_3)$ = angle of incidence on ionosphere of wave component having reciprocal velocity $S_2(S_3)$.

Note that in general $R_2 \neq R_3$.

This last consideration appears an important one in any application of the general equations to experimental studies designed to verify the theory. While the additional accuracy to be obtained by use of Eq. 51 may be small for the E-layer it is by no means negligible for the F-layers.

REFERENCES.

(1) E. V. APPLETON, *Proc. Phys. Soc.*, Vol. 37, p. 16 D (1924).
(2) E. V. APPLETON, *Jour. Inst. Elec. Engrs.*, Vol. 71, p. 642 (1932).
(3) H. W. NICHOLS AND J. C. SCHELLENG, *Bell Syst. Tech. Jour.*, Vol. 4, p. 215 (1925).
(4) G. BREIT, *Proc. Inst. Radio Engrs. N. Y.*, Vol. 15, p. 709 (1927).
(5) S. GOLDSTEIN, *Proc. Roy. Soc.*, Vol. A 121, p. 260 (1928).
(6) D. R. HARTREE, *Proc. Camb. Phil. Soc.*, Vol. 25, p. 97 (1929); Vol. 27, p. 143 (1931a); *Proc. Roy. Soc.*, Vol. A 131, p. 428 (1931b).
(7) H. G. BOOKER, *Trans. Roy. Soc.*, Vol. A 237, pp. 411–451 (1938).
(8) H. A. LORENTZ, "Theory of Electrons," Teubner, Leipzig, p. 142 (1907).
(9) J. JEANS, "The Mathematical Theory of Electricity and Magnetism," 5th Ed., Cambridge, p. 534 (1941).

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.[1]

LABORATORY FOR STUDY OF CYLINDER WEAR.

A new laboratory for the study of automotive engine performance has been installed at the National Bureau of Standards under the direction of A. R. Pierce. Equipped with completely automatic controls, this laboratory offers a means of closely simulating actual operating conditions in such a way that tests can be reliably duplicated. While designed primarily for investigations of cylinder wear, the new equipment may also be used in studying the effects of various types of fuels and lubricating oils on pistons, piston rings, bearings, and carburetors. A problem under investigation at the present time is the determination of the amount of sulfur that can be tolerated in gasolines without appreciably increasing wear.

Cylinder wear is an important factor limiting the useful life of automotive engines. In general, reliable data on the process can be obtained only by actual operation of a test engine under conditions of frequent starting, stopping, accelerating, and cooling, such as occur in normal driving. Road testing has not given reliable results in this work because it is impossible to control properly all conditions of operation. In connection with the Bureau's study of cylinder wear, it was therefore necessary to develop a laboratory arrangement for testing automotive engines under conditions such that any run could be exactly duplicated.

The automatic test equipment utilizes a battery of five automotive engines from three popular makes of automobiles. Each engine is complete except for fans, radiators, and mufflers, and is coupled to a "dynamometer' —an electric generator which provides a uniform load. By varying the electrical output of the dynamometer, and by regulating the amount of throttle opening, any speed or power within the range of the engine may be obtained. The extent of cylinder wear is determined by use of the McKee Wear Gage,[2] which employs a sensitive indentation method developed at the National Bureau of Standards for investigations of the wear of walls and pistons of internal combustion engines.

To obtain data that are valid for actual service, the engines, under the control of a master clock, are put through an operating cycle which provides conditions corresponding to cold starting, warm-up, speed-up, idling, stopping, and cooling. The half-hour cycle that is used begins with cold starting, after which the engines are first idled for 1½ min. at 500 rpm., then accelerated for 3½ min. at a speed equivalent to 40 miles per hour on the road and a horsepower output equal to "road load." The idle and speed-up periods are then repeated three times. At the end of the fourth speed-up period the engines are stopped, and during the remaining 10 min. of the cycle they are artificially cooled to the original starting temperature in preparation for another cold start. The cycle can be repeated automatically as many times as desired.

---

[1] Communicated by the Director.

[2] *NBS Technical News Bulletin*, Vol. 31, No. 7 (July, 1947).

A wear test requires many comparison runs. In order that data from successive runs may be comparable, the engine temperature, load, and speed must be accurately controlled. Studies at the National Bureau of Standards have shown that both starting and running temperatures have an important effect on cylinder wear. For example, it was determined that a greater amount of wear occurs when the engine is operated at temperatures lower than normal. The temperature reached during the cooling period was found to be most important, as wear was doubled when shutdown temperature was dropped 10° F. in the region near freezing. Temperature must therefore be carefully regulated in order that results obtained with different fuels may be comparable.

Maintenance of standard conditions also requires proper control of load. If the power output were not constant, the fuel consumption would vary, causing a changing temperature within the combustion chamber. Obviously, speed must also be controlled in wear tests in order that each engine may run approximately the same number of revolutions and thus have the same amount of piston travel.

As the engines are not equipped with radiators and fans, proper cooling is obtained during normal operation by means of the engine water pump, which circulates water through the water jacket and into a stand pipe, from which it is returned to the pump. Whenever the temperature of the water exceeds the normal operating temperature, an adjustable thermostat in the pipe line automatically opens a hydraulically operated valve, admitting sufficient tap water to maintain the desired temperature, while the excess passes into the drain.

At the end of the operating period, tap water is first circulated through each engine and through a coil in the oil pan to remove most of the heat. Further cooling to the cold-starting temperature is then accomplished with water artificially cooled to 50° F. Two cold-water tanks are used for this purpose. One is an ice-water tank containing water a few degrees above freezing. It is equipped with a refrigeration coil, and a circulating pump to prevent excessive icing around the coils. The second tank contains a supply of cold water for circulation. Ice water from the first tank is pumped into this tank as needed to maintain the circulating water at the required temperature. The temperature of the cooling water at all times is shown on a 24-hour recording thermograph.

AUTOMATIC CONTROLS.

To permit simultaneous cyclical operation of the five test engines, a central panel for automatic timing and control has been installed. The timing device is basically an electric clock which makes contacts at five-minute intervals, opening and closing circuits to the relay panel at the proper time. The relay panel, in turn, opens, closes, or holds the circuits that operate the throttle, the cooling system, the starter, and the stopping mechanism. Five indicating lights on the panel show what part of the cycle is in progress, thus permitting a ready check on the compliance of the engines with the controls. From this central panel any cycle of operations may be arranged.

In addition, each engine has an individual control panel which contains a recording vacuum gage, an oil-pressure gage, temperature indicators for water and oil, tachometers to show the speed of the engines, and switches for each type of operation. Any engine may thus be cut out of the test cycle and controlled manually during adjustment or repair. This arrangement also permits an engine to be stopped, if necessary, for observation without interfering with the others that are running on the cycle.

In order that corresponding valves for all engines may be operated simultaneously, electric solenoid valves are used. For similar reasons, the throttles are provided with electric solenoids. During the speed run the throttles are held by a spring against a stop adjusted to a definite setting; during the idling period the solenoid closes the throttle and holds it in this position.

Each engine is equipped with an automatic choke to insure starting after the cooling period, and also with a device which automatically engages the starter at the beginning of the cycle, or whenever the engine may stop running. Intake manifold pressures of each engine are recorded over a 24-hr. period by a recording gage. Since the manifold vacuum at a given speed is a function of throttle opening, the operation of the engine is thus charted during the cycle, showing periods of starting, stopping, idling, and speeding.

Installation of an automatic multiple thermograph to record continuously the temperatures of all engines is planned. When this has been done, tests will proceed without the intervention of an operator. After the engines, the refrigeration, and the clock have been turned on, the central control panel will take over the operation of the test cycle, repeating the cycle until the test is stopped. At the conclusion of each 24-hr. period, the temperature chart will show the temperatures of the engines and cooling water, and the charts from the manifold gage will show engine performance.

NEW MOTION PICTURE ON DENTAL AMALGAM RELEASED.

A new motion picture in color, entitled "Dental Amalgam—Failures Caused by Moisture Contamination," has just been completed at the National Bureau of Standards. This film, prepared under the joint auspices of the Bureau and the Research Commission of the American Dental Association, is the second of a series of technical films prepared for dental and allied professions.

The dental amalgam film demonstrates one of the most common causes of amalgam failure—namely, contamination with moisture. This contamination with moisture is usually produced by mixing amalgam in the palm of the hand and by condensing amalgam in a wet field. The moisture introduced into the amalgam reacts with the alloy to produce hydrogen gas which causes excessive expansion, lowers compressive strength, and forms blisters. Clinically, contamination of amalgam with moisture is evidenced by expansion causing post operative pain, extrusion of the alloy from the cavity, formation of blisters, faulty margins, and fractured restorations. The film describes a satisfactory technique for the prevention of contamination.

Since 1928 the National Bureau of Standards and the American Dental Association have conducted cooperative research on the physical and chemical properties of dental materials and the proper techniques involved in their use. The present film, the second of a series that will stress the clinical significance of the physical and chemical properties of dental materials and the importance of technique, is a result of this research. The first film, "Silicate Cement," was made available to the dental profession last fall.

The new motion picture on amalgam, a 16-mm. sound film photographed in color, has a running time of 15 min. The film is available from the NBC: loan or purchase information can be obtained by writing to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C. The heavy demands for the silicate cement film suggest that the amalgam film will also be heavily booked: it is therefore advisable that loan requests be made at least 30 days in advance of any projected showing.

A RADIO-FREQUENCY MASS SPECTROMETER.

A comprehensive investigation of the conditions for the formation of negative atomic ions of the heavier metallic elements has led to the development, by Dr. Willard H. Bennett of the National Bureau of Standards, of an extremely valuable method for detecting, separating, identifying, and measuring such ions—one of the most neglected fields of research in pure physics. Since negative atomic ions, consisting of atoms with extra electrons, have very low energies of formation, their study has been difficult because of the rapid dissociation of the ions in very short distances of travel; that is, before they can reach a measuring electrode. The exacting requirements of this field of research have now been met by means of a specially designed vacuum tube which provides a greatly simplified and flexible radio-frequency mass spectrometer. This spectrometer not only opens up a new means of exploring the little-known fundamentals of negative atomic ions but its special characteristics make it widely applicable for other uses, including the mass spectrometry of positive ions.

Negative atomic ions have been detected for only a few elements since they were first discovered about 40 years ago, although accepted theory has indicated the possibility of producing such ions from many of the elements. Experiments begun at the National Bureau of Standards by Dr. Bennett in 1946 indicated that negative atomic ions might well exist in the many familiar forms of electrical discharge in vacuum tubes, but that they would not be detected if the distance through the tube between the discharge and the electrode was large. It was necessary, therefore, to devise an experimental method for separating and identifying such ions within distances of only a few centimeters. This was accomplished through the new mass spectrometer tube.

In its more advanced form, this two-stage spectrometer may be used for positive ions as well as negative ions. The equipment consists essentially of a multigrid tube in which an adjustable radio frequency is applied to two grids while all other electrodes are held at the proper direct-current potentials, and the ion current is measured at the plate. The more exacting requirements of negative-ion separation require the use of a small magnetic field produced with coils, but if positive ions are being separated, no magnetic field is needed. Because large electromagnets or tubes containing electrodes with elaborate slit systems are not required in this method, the radio-frequency mass spectrometer should find wide application where its resolution is sufficient and where the expense of the more elaborate, conventional beam-deflection equipment is not justified.

One of the principal limitations upon the resolution possible with the ordinary mass spectrometers using magnetic deflection of beams has been the spread in energies of the ions at the ion source. The percentage spread occasioned by this factor can be reduced by increasing the voltage applied to the ions before they are magnetically resolved. The extent to which this can be done is limited, however, by the magnetic field that can be obtained in a space sufficient to contain the tube.

In the radio-frequency mass spectrometer, this difficulty is eliminated, and the voltage of the ions can easily be pushed up at least an additional order of magnitude to any value for which insulation can be provided. The frequencies required are then increased by an amount equal to the square root of the factor by which the voltage is increased.

Raising the voltage from 100 to 10,000 v., for example, increases the frequencies ten-fold, and reduces the percentage spread of mass line due to velocity spread in the ion sources by a factor of one one-hundredth.

The first successful radio-frequency mass spectrometer, a single-stage six-electrode tube, consists of a cathode about which are arranged four co-axial cylindrical grids and an ion-collecting plate. Helmholtz coils surrounding the tube provide a 100-gauss magnetic field in a direction parallel with the axis of the tube. Such a field is required in negative-ion work to confine the electrons to the space inside the first grid and to avoid the formation of positive ions in parts of the tube where neutralization of the negative ions may occur before they reach the detecting electrode.

In the operation of the single-stage negative-ion spectrometer tube, ions arising at the cathode are accelerated through the first grid at a velocity corresponding to the d-c. potential of that grid (say 15 v.). They are further accelerated by the field due to the potential on the second grid which is an alternating radio frequency (say 5 v.) plus a d-c. potential (reduced to 10 v.). The third grid is held at a d-c. potential (reduced to 5 v.). Those ions which pass the first grid at the proper phase of the r-f. field, and which have a mass and related velocity such that they pass through the second grid while the r-f. potential changes phase, receive an additional acceleration due to the r-f. field while travelling to the third grid. It was found experimentally, verifying theoretical calculation, that ions passing the first grid at 45°27.5′ of phase angle of the alternating current on the second grid, and having just the right mass (and corresponding velocity) to pass the second grid at 180° of phase angle of the current on that grid, will pass each of the first three grids with the same velocity (corresponding to 15 v.), although the d-c. potentials on those grids are successively reduced. This is caused by the ion picking up energy from the field of the alternating current on the second grid since the change in phase occurs as the ion passes through this grid.

A blocking potential difference, nearly equal to the maximum energy which an ion with the proper mass (corresponding to the frequency) could pick up from the alternating current, is applied between the cathode and the fourth grid. The blocking potential may be, for example, 9.6 v. negative with respect to the cathode. The collecting electrode, or ion-collection plate, is held at 40 v. positive with respect to the cathode to insure that no positive ions will reach the anode. An electrometer tube circuit and galvanometer are used for measuring the selected ion currents to the anode.

By observing the negative atomic ions produced at an oxide-coated cathode in a water-vapor atmosphere at less than 10^{-6} mm. of mercury pressure, typical mass-resolution data may be obtained for plotting resolution curves. The data are taken while holding all potentials constant and varying only the applied radio frequency. For example, an a-c. potential of 5 v. may be used while d-c. potentials of less than 100 v. are applied to the other electrodes. The required frequency range extends from 3300 kilocycles for a mass value of ½, to 104 kilocycles for mass value 250. This range of readily producible radio frequencies more than covers the full mass range of all elements.

By extending the method to two stages in a nine-electrode tube, either with cylindrical or parallel-plane electrodes, and applying the radio frequency to two of the grids, a high order of mass resolution is obtained. Such a large number of grids would not be practical in a tube if the usual woven-wire screens were used. There is available now, however, a knitted wire fabric which has more than 95 per cent open area and with which this kind of tube can readily be constructed. This apparatus may be used in the mass spectrometry of positive ions and in ionizaton studies, as well as in the present negative-ion investigations. The magnetic field is again applied in a direction parallel with the planes of the grids for negative-ion investigations. A collimating electrode containing the cathode is used to direct the ions in a direction approximately normal to the planes of the grids.

In the positive-ion mass-spectrometer tube, the ions, accelerated toward the second grid, are resolved by the application of a radio frequency on the third grid, just as in the single-stage tube. The distance between the fourth and fifth grids is selected to assure that those ions which acquire the maximum energy from the alternating-current fields at the first stage (grids 2, 3, and 4) will travel from grid 3 to grid 6 in the time that the radio frequency takes to traverse an exactly integral number of cycles. The same radio frequency is applied to the sixth grid as to the third grid and the other potentials on the second stage (grids 5, 6, and 7) are selected in a similar manner to those for the first stage. The blocking potential is applied to the anode, and the selected ions are collected there.

The simplicity and low cost of the radio-frequency mass spectrometer should make it attractive not only in those applications in which its special characteristics make it superior to any other kind of mass spectrometer, but also in those laboratories where the expense of other types of equipment is prohibitive. Though the possibilities of the new apparatus have not yet been adequately explored, preliminary experimentation at the National Bureau of Standards indicated that many specilized uses for this kind of tube will come to light as it becomes better known.

Frost Control with Air Circulators (*Weatherwise,* Vol. 1, No. 5).—Wind machines have been used in California for the past ten years to combat frost. On a clear, cool night with active radiation, the temperature at a height of 45 ft. may be 10 degrees warmer than the air 5 ft. above the ground. A wind machine breaks up this stratification of the air and causes vertical mixing of the warm air with the cooler air at fruit level.

The machine consists of a tubular column 30 ft. high, 24 in. in diameter, on top of which are mounted two 100-hp. engines. The propellers, each 11½ ft. long and of solid aluminum alloy, are connected to the engines through a transmission. Each propeller turns at 900 revolutions per minute and moves approximately 300,000 cu. ft. of air per minute at a low velocity, since the blades are set at a very flat pitch.

The tubular column is mounted in a base block of concrete 8 ft. square weighing 8½ tons, reinforced with 350 lb. of steel. The machine is entirely controlled from below, with visual temperature, ammeter, and oil gages. The center column holds 600 gal. of gasoline, which is sufficient for 40 hr. of operation, at a cost of approximately $2.50 per hour.

The distributors state that one machine will protect up to 40 acres of deciduous fruits and up to 20 acres of citrus, depending on the heights of the trees and the density of the foliage. However, the wind machines are of little value in combating a "deep-freeze condition," when polar continental air spills over into the Pacific Coast area, bringing a constant flow of below-freezing air.

Puri-Pulley Magnetic Pulley (*Rubber Age,* Vol. 63, No. 6).—A completely new type of magnetic pulley, known as the "Puri-Pulley," has been produced by the Eriez Mfg. Co., of Erie, Penna. The unit is non-electric and was developed solely to remove fine iron or less magnetic particles and afford automatic separation of ferrous from non-ferrous material ranging in size from ten to two hundred mesh.

While present magnetic pulleys are built to throw a magnetic field out several inches (for the removal of ordinary tramp iron), the Puri-Pulley's strength is concentrated in close to the face plate. At one inch from the pulley's surface, the magnetic field is approximately three times as strong as a standard pulley. Design characteristics, as well as new alnico material, account for additional strength.

The effective cleaning surface has been increased substantially by narrowing the air-gaps and increasing the number of pole plates. This provides more major areas of magnetic concentration, hence better magnetic separation. These features make the unit highly adaptable for removing fines from such materials as talcs, powders, chemicals, salts, glass sands, ball clays, granulated slag, rubber and any other substances requiring a high degree of purification. The effective cleaning range varies depending on the size of material being processed and the type of separation desired.

Designed for use as a head pulley in belt conveying systems, the unit is available in 18- and 24-in. diameters and in belt lengths ranging from 12 to 60 in.

THE FRANKLIN INSTITUTE.

MEDAL DAY PROCEEDINGS.

The annual reception, dinner and presentation of awards, known as "Medal Day," took place at The Franklin Institute on Wednesday, October 20, 1948. The reception began at 6:30 P.M. and dinner was served at 7:15 P.M. Mr. Richard T. Nalle, President of the Institute, presided at the dinner and presented the subsequent awards.

RECEPTION.

The reception preceding the dinner was given by the Hostess Committee of The Franklin Institute, and was attended by 370 persons. It was held in rooms adjoining the Chemistry Section. Dinner was served in Franklin Hall.

Immediately before dinner was served, the National Anthem was played by Mr. Guy Marriner.

REMARKS OF THE PRESIDENT.

MR. NALLE: "Ladies and Gentlemen.

"The Franklin Institute was founded 34 years after Benjamin Franklin's death to perpetuate one of his important interests.

"This was the promotion of the mechanic arts, now broadly manufacture, and the encouragement and education of young engineers and mechanics.

"It is for this reason that it is a part of these Medal Day ceremonies to do him honor. Accordingly, Ladies and Gentlemen, I propose a toast to the memory of Benjamin Franklin."

* * *

MR. NALLE (*continued*): "On behalf of the Board of Managers and the Officers, I welcome you to The Franklin Institute. We are assembled tonight to honor fourteen gentlemen for their achievements in the world of Science and the Arts.

"The Hostess Committee, of which Mrs. John White Geary is Honorary Chairman and Mrs. Pennypacker Chairman, is responsible for the flowers and has helped with the reception. I give them our thanks."

* * *

MR. NALLE (*continued*): "Will the sponsors for the 1948 Recipients of Awards kindly take their places?"

STATED MONTHLY MEETING.

MR. NALLE (*continued*): "This is also the regular stated October meeting of the Institute.

"The minutes of the last regular meeting in May have been published in the June issue of the JOURNAL and, if there are no corrections or additions, I declare them approved as printed.

"It is the custom to elect our Franklin Medalists to Honorary Membership in the Institute, so at the September meeting of the Board of Managers nominations were made to present to the membership at this meeting with the recommendation that they consider favorably the election of Dr. Wendell Meredith Stanley and Dr. Theodor von Kármán to honorary membership in the Institute. Accordingly, will all the members present in favor of such election please say 'Aye'. Contrary minded—.

"Carried. I declare them elected."

* * *

MR. NALLE (*continued*): "It is customary to invite to our Medal Day proceedings the recipients of Awards made in previous years. There are twenty such here tonight and we welcome them. I shall read their names, city, and the Award received and ask that each rise as his name is called and remain standing until the roll is completed. To save time I suggest that you withhold your applause until all are standing.

FORMER MEDALISTS ATTENDING MEDAL DAY—1948.

Year	Medalist	City	Medal
1904	Clamer, Dr. G. H.	Philadelphia, Pa.	Cresson
1915	Young, General Charles D.	Chester County, Pa.	Longstreth
1918	Creighton, Dr. H. Jermain	Swarthmore, Pa.	Longstreth
1939	Creighton, Dr. H. Jermain	Swarthmore, Pa.	Potts
1920	Leeds, Mr. Morris E.	Philadelphia, Pa.	Longstreth
1921	Adams, Dr. L. H.	Washington, D. C.	Longstreth
1923	Parks, Mr. Harry S.	Pottstown, Pa.	Longstreth
1924	McBride, Mr. Thomas C.	Philadelphia, Pa.	Longstreth
1926	Smathers, Mr. James F.	Poughkeepsie, N. Y.	Longstreth
1930	Moultrop, Mr. Irving E.	Belmont, Mass.	Cresson
1932	Wenner, Dr. Frank	Washington, D. C.	Wetherill
1934	Shryock, Mr. Joseph G.	Philadelphia, Pa.	Certificate of Merit
1937	Eksergian, Dr. Rupen	Philadelphia, Pa.	Henderson
1945	Eksergian, Dr. Rupen	Philadelphia, Pa.	Levy
1939	DeJuhasz, Prof. Kalman J.	State College, Pa.	Levy
1939	Wilford, Mr. E. Burke	Philadelphia, Pa.	Certificate of Merit
1942	Larkin, Mr. Walter	Norristown, Pa.	Certificate of Merit
1944	Ennis, Mr. Joseph B.	New York, N. Y.	Henderson
1945	Ellis, Mr. Greer	New York, N. Y.	Certificate of Merit
1946	Southworth, Dr. George	New York, N. Y.	Levy
1947	Southworth, Dr. George	New York, N. Y.	Ballantine
1946	Barnes, Maj. Gen. G. M.	Philadelphia, Pa.	Cresson
1947	Billner, Mr. Karl P.	Philadelphia, Pa.	Brown

MR. NALLE (*continued*): "The Order of Business calls for a report from the Executive Vice President and Secretary of the Institute, Dr. Henry Butler Allen. Dr. Allen, will you present your report, please?"

REPORT OF THE SECRETARY.

"The Institute in 1948."

DR. ALLEN: "Mr. President, Honored Guests, Friends of the Institute, Again this year I am briefly going to run through the several activities of our Franklin Institute. We have some new faces here tonight and we want them to know what the Institute does, and our Medalists constitute a part of our alumni, even though they did not go to school here. We really did have school, by the way, which ran for 99 years.

"One of the Institute's strong points through the years has been its willingness to relinquish activities and to undertake new forms of service as times change; but always in its chartered field—physical science.

"Following this policy our former evening school enterprise was replaced by the establishment, 15 years ago, of a modern science Museum—visual education for the old and young. Incidentally, our Museum opens to the public on the twenty-ninth of this month for a three months' stay, an extensive Atomic Energy Exhibit from the Brookhaven Atomic Energy Laboratories—a FIRST for Philadelphia.

"Our technical and patent libraries continue to serve the manufacturer, research worker and student, as do our Lectures and our Journal, all of which activities were started *circa* 1824.

"Research is growing in The Franklin Institute to the extent of having today the largest staff of any department in the Institute. So-called pure research centers primarily in our Bartol Research Foundation started 34 years ago, where a force of fifty-five people contribute to the increase of knowledge about cosmic rays and other atomic particles, as well as other physical phenomena. Our more recent laboratories for industrial research now employ a staff of some three hundred persons. Manufacturers are finding our services increasingly useful. We have a particularly powerful tool for their problems through the combination of the brains of a pure research foundation and those of an applied science laboratory, superimposed upon the background of the Institute.

"One of our newest babies was born with the unveiling of our statue in this Hall ten years ago, namely, the reincarnation of Benjamin Franklin with the dissemination of his sound teachings about citizenship, science, and service to his fellow man. We are called upon from all over the country, and overseas, for information about him. We have evidence that this service has perhaps been not unhelpful in giving some a better understanding of our form of government, as well as what constitutes a useful citizen of our country and the world. This activity under our National Franklin Committee has shown within the last year a curiously tangible consequence that I know would intrigue our Patron Saint. It certainly pleased us. What happened was this: We had a call from a gentleman from a mid-west city, who had seen some of our Franklin pamphlets. He was coming east, and wanted to stop in for more information. He was giving a talk to a group of young people, about Franklin. He came, and in conversation learned that we had research laboratories. He suggested that we get in touch with a Foundation in his home city, the benefactor of which was a personal friend of his. We wrote this Foundation that we needed support on a certain research project, not of interest to any one industry. To make a long story short, the Foundation gave us very substantial financial aid.

"President Nalle has already spoken about our other century-old activity—Medal Awards. He did not tell you, however, that this year marks the 100th anniversary of one of our medals, the Elliott Cresson, and that his great uncle was among the first group of individuals to receive this award for development of the "Sand Blast"—Benjamin C. Tilghman.

"That, ladies and gentlemen, is a brief outline of The Franklin Institute's present-day activities. We hope that you will visit us often, learn more about us, and perhaps help us become even more useful to our public."

MR. NALLE: "Over the years, and next year will be our 125th Anniversary, fourteen medals have been established through the generosity of members. These medals are awarded for outstanding accomplishment in various scientific fields in accordance with the terms of the gift.

"Some 1050 medals have been awarded to date. The recipients have been of many nationalities, the coverage has been world-wide. In the last 20 years, twenty-one medals have gone to England, ten to Germany, six to Sweden, five to France, and thirteen to ten other countries.

"A Committee of the Institute, the Committee on Science and the Arts, is charged with the selection of the candidates. It is their responsibility to search out the worthwhile developments and discoveries which occur in the various sciences, make a thorough investigation of the case and satisfy themselves that the individual concerned has made the requisite personal contribution.

"When the Sub-committee in charge decides that a case is worthy, its findings are reviewed by the whole Committee and, if approved, the recommendation is submitted to the Board of Managers for final approval. If all goes well, the individual joins in our Medal Day exercises and receives the particular Medal applicable to his work.

"Each Medalist is sponsored by the Chairman of the Sub-committee that recommends him for the Award. The Sponsor makes a brief statement as to the nature of the Medalist's work and the citation.

"We regret that two of our Medalists could not be with us tonight. Dr. von Kármán is detained in Europe and Dr. Colpitts is in the hospital with an operation that could not be delayed. Their Medals will be sent to them by the Secretary.

"We will now proceed to the presentation of the Awards."

Mr. Nalle then called upon the sponsors to present their candidates for the awards.

PRESENTATION OF THE ELLIOTT CRESSON MEDAL.

(To Edwin H. Colpitts, Orange, New Jersey, Director, The Engineering Foundation, New York 18, N. Y.)

MR. NALLE: "I recognize Dr. Winthrop R. Wright, the sponsor of Dr. Edwin H. Colpitts, who will receive the Cresson Medal *in absentia*. Dr. Wright.

DR. WRIGHT: "Mr. President, I present, *in absentia*, Edwin H. Colpitts for an award.

"The phenomenal growth of the world's communication networks since the turn of the century is one of the miracles of the twentieth century. Our Medalist has played an outstanding role in this development between the years 1899, when he joined the American Bell Telephone Company in Boston, and 1937, when he retired as vice-president of the Bell Telephone Laboratories with 24 United States patents to his credit. He was directly responsible for the experimental work undertaken to verify the theoretical work of George A. Cambell on the loaded telephone line. The oscillator circuit bearing our Medalist's name is a classic and known to all communication engineers and students.

"I present, *in absentia*, Edwin H. Colpitts, of Orange, New Jersey, as a candidate for an Elliott Cresson Medal, awarded to him 'In consideration of his pioneering achievements leading to the development of practical systems of long distance communication, both by wire and radio.'"

MR. NALLE: "By virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to Dr. Edwin H. Colpitts, *in absentia*, this Elliott Cresson Medal and the Certificate and Report which accompany it. Dr. Allen, as Secretary of The Franklin Institute, will you see that this Medal and the Certificate and Report which accompany it are transmitted to Dr. Colpitts?"

DR. ALLEN: "Yes, indeed."

PRESENTATION OF THE LONGSTRETH MEDALS.

(To Raleigh J. Wise, Dunnellen, New Jersey, Telefax Research Engineer, The Western Union Telegraph Company, New York 13,

N. Y.; Nicholas F. Arone, Switchgear Division Laboratory, General Electric Company, Philadelphia, Pa., and Edwin H. Brink, Assistant Director of Research, Masonite Corporation, Laurel, Mississippi.)

MR. NALLE: "The next Sponsor, Mr. McClarren."

MR. MCCLARREN: "Mr. President, I present Raleigh J. Wise for an Award. He has been largely responsible for inventing and developing a special recording paper for use in facsimile reception of printed or written matter. Developed just prior to World War II, his recording paper, called 'Teledeltos,' was used extensively for recording all kinds of transmitted information such as maps, pictures, coded messages and for instrument records such as the fathometer and others.

"The Western Union Telegraph Company, with which Mr. Wise is associated, makes extensive use of 'Teledeltos' in its business. Special apparatus has been developed by it to expedite and facilitate the sending of messages from place to place. One such machine will automatically transmit and another will receive at a remote location an exact copy of a written message.

"'Teledeltos' has been made available commercially and is used by many concerns for various recording purposes.

"I present Raleigh J. Wise, of Dunnellen, New Jersey, as a candidate for an Edward Longstreth Medal 'In consideration of the development of a dry electrosensitive recording blank which can be stored and handled like ordinary paper with practically no deterioration or change and which can be permanently marked by simple means.'"

MR. NALLE: "Mr. Wise, by the same authority, I present to you an Edward Longstreth Medal, and the Certificate and Report which accompany it."

Mr. Nalle (*continued*): "I recognize Dr. Crampton as the next Sponsor."

DR. CRAMPTON: "Mr. President, I present Edwin H. Brink and Nicholas F. Arone for awards for an achievement of great usefulness that has contributed to the potential conversion of the entire industrial field from oil circuit breakers with their hazards to life and property, to an air device which has only a fraction of the industrial hazard.

"This has been accomplished through patient and successful search for an inorganic, moldable, thermoset, insulating compound embodying the necessary requirements for use in the construction of magne-blast circuit breakers.

"Here, powerful arcs resulting from such causes as lightning and short circuits, are destroyed by being forced into a series of interleaving, insulating barriers made of the new material Phospho-Asbestos developed by the Medalists. This material is a combination of chrysotile asbestos and phosphoric acid.

"I present Edwin H. Brink, Laurel, Mississippi, and Nicholas F.

Arone, of Philadelphia, to each of whom has been awarded an Edward Longstreth Medal 'In consideration of their persistent search and ingenuity, resulting in the discovery of Phospho-Asbestos and in the chemical and productional development of this uniquely useful product.'"

MR. NALLE: "Mr. Arone, by the same authority, I present to you an Edward Longstreth Medal, and the Certificate and Report which accompany it."

"Mr. Brink, likewise, by the same authority, I present to you an Edward Longstreth Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE POTTS MEDALS.

(To David Bigelow Parkinson, of Maplewood, New Jersey, Research Engineer, Bell Telephone Laboratories, New York 14, N. Y., and Clarence Anding Lovell, of Summit, New Jersey, Switching Research Engineer, Bell Telephone Laboratories, New York 14, N. Y.; and Eugene J. Houdry, of Ardmore, Pennsylvania, Founder, Houdry Process Corporation, Philadelphia 2, Pa.)

MR. NALLE: "I call upon Colonel Ely."

COLONEL ELY: "Mr. President, I present David Bigelow Parkinson and Clarence Anding Lovell for awards.

"These two scientists have always been interested in unusual electrical and mechanical contrivances and since joining the Bell Telephone Laboratories they have devoted their time to developing better devices for use in the Telephone System. As an outgrowth of this, the basic conception of the Electrical Gun Director came to Dr. Parkinson. Dr. Lovell became coordinator of the gun director development which followed. An amazing fact about this development is that men who had no previous experience with a complex Ordnance problem were able to apply the special electrical techniques used in the communications art to the solution of the military problem with a high degree of accuracy.

"I present David Bigelow Parkinson, of Maplewood, New Jersey, and Clarence Anding Lovell, of Summit, New Jersey, to each of whom has been awarded a Howard N. Potts Medal 'In consideration of their combined contributions, both to the theoretical and practical design of the Electrical Gun Director, one of the outstanding pieces of equipment developed during the war period, which development has further contributed largely to the theory and practical application of servo mechanisms and smoothing filters in general.'"

MR. NALLE: "Dr. Parkinson, by the same authority, I present to you a Howard N. Potts Medal, and the Certificate and Report which accompany it."

"Dr. Lovell, in the same manner, by the same authority, I present to you a Howard N. Potts Medal, and the Certificate and Report which accompany it."

MR. NALLE: "Mr. Masland."

MR. MASLAND: "Mr. President, I present Eugene J. Houdry for an award.

"Every once in awhile, progress is made by recognizing that an exceedingly difficult obstacle is in the way, and then climbing over the obstacle. Such was the case with the catalytic cracking of petroleum. In many catalytic processes, the active life of the catalyst can be measured in months. In catalytic cracking of petroleum, the active life of the catalyst must be measured in minutes.

"Mr. Houdry recognized this obstinate fact which had previoulsy stopped progress in the catalytic cracking of petroleum and set out to climb over the obstacle. He recognized the very short active life of catalysts suitable for cracking petroleum and he designed and built equipment which gave us high octane gasoline in large quantities during the war when we badly needed it; this in spite of the short life of the catalyst.

"I present Eugene J. Houdry, of Ardmore, Pennsylvania, as a candidate for a Howard N. Potts Medal 'In consideration of his leadership in the development of the important process of catalytic cracking of petroleum that bears his name, utilizing known basic chemical and physical facts.'"

MR. NALLE: "Mr. Houdry, by the same authority, I present to you a Howard N. Potts Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE LEVY MEDALS.

(To Jan A. Rajchman, Research Engineer, RCA Laboratories, Princeton, New Jersey, and William H. Cherry, Physicist, RCA Laoratories, Princeton, New Jersey.)

MR. NALLE: "The Sponsor for the Levy Medal, Dr. Boehne."

DR. BOEHNE: "Mr. President, I present Jan A. Rajchman and William H. Cherry for awards.

"The betatron or induction accelerator contains closed electron orbits which are the longest, longest lived, and most stable orbits known outside of the atoms themselves. In order for large electron energies to be gained, acceleration must take place over long electron paths. The current widespread development of the betatron as a generator of high energy electrons and gamma rays has created need for a comparatively detailed knowledge of the electron dynamics of induction acceleration.

"The paper by Rajchman and Cherry entitled, 'The Electron Mechanics of Induction Acceleration,' describes the radial and axial motions of electrons in the betatron by means of a potential function of forces. Previous studies assumed a potential of parabolic variation. The authors' extension of the analysis to non-parabolic regions describes more accurately their observed behavior of conventional instruments, especially when higher voltages of injection are employed. The new points of view presented are applicable to other electronic devices such as the magnetron and various electron optical lens systems of rotational symmetry.

"I present Jan A. Rajchman and William H. Cherry, of Princeton, New Jersey, as candidates, each for a Louis Edward Levy Medal, 'In recognition of their outstanding paper, "The Electron Mechanism of Induction Acceleration," appearing in the April and May 1947 issues of the JOURNAL OF THE FRANKLIN INSTITUTE.'"

MR. NALLE: "Dr. Rajchman, by the same authority, I present to you a Louis E. Levy Medal, and the Certificate and Report which accompany it."

"Mr. Cherry, likewise, by the same authority, I present to you a Louis E. Levy Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE HENDERSON MEDAL.

(To Charles Duncanson Young, of Whitford, Pennsylvania, Vice President (Retired), The Pennsylvania Railroad Company.)

MR. NALLE: "The next Sponsor, Mr. J. V. B. Duer."

MR. DUER: "Mr. President, I present Charles Duncanson Young for an award.

"Railroad motive power has been developed through the efforts of many able men, working towards the common aim of producing the most efficient, economical, and reliable locomotive for the lowest possible cost.

"Pioneer among motive power units is the steam locomotive, which has been developed to a degree of reliability, economy and effectiveness, as a transportation unit, little dreamed of by the builders of the first locomotives of this type.

"Outstanding among those who have contributed to this result is our Medalist, who, through his ingenuity in developing a method of testing completed locomotives, while stationary on a testing plant, as well as by his inventions and also by making available to railroad engineers, accurate information regarding the performance of locomotives tested, has made possible substantial increases in efficiency and reliability and reductions in cost of steam locomotives, thereby producing a more effective transportation unit.

"I present Charles Duncanson Young, of Whitford, Pennsylvania, as a candidate for the George R. Henderson Medal 'In consideration of his contribution to the scientific advancement of the steam locomotive which has resulted in improving the reliability and efficiency and reducing the cost of steam locomotives thereby producing a more effective transportation unit.'"

MR. NALLE: "General Young, by the same authority, I present to you the George R. Henderson Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE WETHERILL MEDAL.

(To Wendell Frederick Hess, Head, Department of Metallurgical Engineering, Rensselaer Polytechnic Institute, Troy, New York.)

MR. NALLE: "Dr. Adams, will you take the floor?"

DR. ADAMS: "Mr. President, I present Wendell Frederick Hess for an award.

"One feature of Dr. Hess's career is noteworthy. After studying and teaching Electrical Engineering and Physics for several years, he was transferred to the Department of Metallurgical Engineering of which he is now the head, as well as of the Welding Laboratory at the Rensselaer Polytechnic Institute.

"He thus brought into the metallurgical field a rare and much needed knowledge of analytical physics, so essential to the solution of many metallurigcal and welding problems, notably those involving transient heat flow.

"However, he is best known for his contributions to the field of Resistance Welding, where he started a veritable revolution which is still under way. Merely to list the specific contributions would take more time than is here available. His published papers already number more than fifty, and their sum total constitutes by far the largest contribution by any individual to the art and science of Resistance Welding. This work has already been recognized in the award of numerous prizes and several medals.

"I present Wendell Frederick Hess, of Troy, New York, as a candidate for a John Price Wetherill Medal 'In consideration of his outstanding contributions to the Art and Science of Welding, notably in the field of Electric Resistance Welding.'"

MR. NALLE: "Professor Hess, by the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE BALLANTINE MEDAL.

(To Ray Davis Kell, Director of Television Research, RCA Laboratories, Princeton, New Jersey.)

MR. NALLE: "Dr. Murray, Sponsor for the Ballantine Medal, is ill and cannot be with us. Mr. William Ellis will act as Sponsor in his place. Mr. Ellis."

MR. ELLIS: "Mr. President, I present Ray D. Kell for an award.

"The word that most people associate with our rapidly growing, present-day television service is: 'Marvelous.' Many refer to television as the newest invention in the communication field. Actually, it is the result of long and painstaking research and invention over a period of more than 20 years.

"In our Medalist we have a noted engineer, who, during *all* of these twenty years, has been researching and inventing, first with the General Electric Company, later with the Radio Corportaion of America, to make possible the clear, sharp pictures that merit the word: 'Marvelous.'

"In World War II it was he who, applying television to guided missiles, enabled the bombardier effectually to keep his target in view as his controlled bomb glided earthward.

"For the last word in home entertainment, we look forward to television in *color*. Under the direction of Mr. Kell, there has already been demonstrated a color system, which, when it emerges from the development stage, will permit the broadcasting of color images, yet will provide usable signals for our present black-and-white receivers. We wish him continued success in this important work.

"I present Ray Davis Kell, of Princeton, New Jersey, as a candidate for the Stuart Ballantine Medal 'In consideration of his outstanding pioneer work in Television; the adaptation of this means of communication to military needs and for his inventive contributions and leadership in the development of Color Television.'"

MR. NALLE: "Mr. Kell, by the same authority, I present to you the Stuart Ballantine Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDALS.

(To Wendell Meredith Stanley, Director of the Virus Laboratory, University of California, Berkeley, California; and Theodor von Kármán, Director of Guggenheim Aeronautics Laboratories, California Institute of Technology, Pasadena, California.)

MR. NALLE: "The next Sponsor, Dr. Palmer."

DR. PALMER: "Mr. President, I have the honor to present as a candidate for the Franklin Medal, Wendell Meredith Stanley.

"Such diseases as influenza, infantile paralysis, encephalitis, measles, smallpox, yellow fever, and the common cold, as well as many other diseases of both animals and plants, are caused by a virus. This is an infectious agent so small as to be invisible in an optical microscope but capable of being photographed in an electron microscope. Until a dozen years ago, each of the many kinds of virus then known was generally regarded as a sub-microscopic living organism, yet some authorities inclined toward the view that a virus is an inanimate chemical substance with the peculiar properties of mutation and reproduction in the host.

"When Dr. Stanley joined the Rockefeller Institute for Medical Research, he attacked the virus problem and in less than three years was successful in isolating a crystalline material that possessed all of the properties of the tobacco mosaic virus. Many other viruses have now been isolated in pure form and they all appear to be composed of giant chemical molecules of very complex structure known as nucleoproteins. From his laboratory also came the first information concerning the changes in chemical structure that accompany the mutation of a virus and give rise either to a more virulent strain or to a less virulent one, such as can provide immunity to millions of individuals against the more virulent strains of the same virus.

"During World War II you, Sir, as the result of your intensive study of the influenza virus, developed an effective vaccine by the application of a new method of purification by means of the centrifuge. Through the general use of this vaccine, influenza epidemics of the future should be greatly reduced in intensity or perhaps entirely prevented.

"Hence, Mr. President, 'In recognition of his discovery that a virus can be a nucleoprotein rather than a living organism, thereby establishing a significant turning point in virus research; and also of his development of an effective centrifuge-type influenza vaccine by means of which the intensity of future influenza epidemics may be greatly lessened,' I take pleasure in presenting Wendell Meredith Stanley."

MR. NALLE: "Dr. Stanley, by the same authority, I present to you a Franklin Medal, the Certificate and Report which accompany it, together with the Certificate of Honorary Membership."

MR. NALLE: "I recognize Mr. W. Laurence LePage, the Sponsor of Dr. Theodor von Kármán, who will receive the Franklin Medal *in absentia*. Mr. LePage."

MR. LEPAGE: "Mr. President, I have the honor to present to you, *in absentia*, as a candidate for the highest award made by The Franklin Institute, a man of great distinction in the field of Aeronautics.

"Dr. Theodor von Kármán is probably the outstanding mathematical and physical fundamentalist engaged in aeronautics today. But more than this, Dr. von Kármán is, at the same time, a very practical engineer.

Thus, we find in him a truly remarkable union of qualities as a result of which every new von Kármán discovery, every novel theoretical conception and each basic theorem is reduced to such practical terms as may be understood and employed by any engineer of normal capabilities.

"This remarkable aerodynamicist has been responsible for the establishment of two great aeronautic institutions, the Aeronautical Institute of the University of Aachen and the Guggenheim Aeronautics Laboratory of the California Institute of Technology. His work has covered such a broad mathematical and mechanical field in aerodynamics, aeronautics and theory of air flow, and has been so outstanding in each contribution, that it has been practically impossible to select any one study as foremost. Furthermore, his great success in stimulating, coordinating and supervising the work of others has added appreciably to the depth and breadth of his own contributions.

"Dr. von Kármán, more than any other single individual, is responsible for the development of super-sonic air flow theory and its practical application. He propounded the method of Fourier integrals in the development of super-sonic wing theory, established the mathematical existence of the shock wave in super-sonic air flow and the practical desirability of the swept back plan form for wings of super-sonic aircraft.

"Sir, upon citation which reads: 'In recognition of his outstanding engineering and mathematical achievements, particularly those relating to the development of advanced aerodynamic conceptions which have directly influenced the progress of aeronautical design, and for his unusual leadership whereby some measure of his own genius is constantly instilled in those who work with him.'

"I present to you, *in absentia*, Theodor von Kármán, Director of the Guggenheim Aeronautics Laboratory of the California Institute of Technology as a candidate for the Franklin Medal."

MR. NALLE: "By the same authority, I present to Dr. Theodor von Kármán, *in absentia*, this Franklin Medal, and the Certificate and Report which accompany it, together with the Certificate of Honorary Membership. Dr. Allen, as Secretary of The Franklin Institute, will you see that these are transmitted to Dr. von Kármán?"

DR. ALLEN: "Yes, indeed."

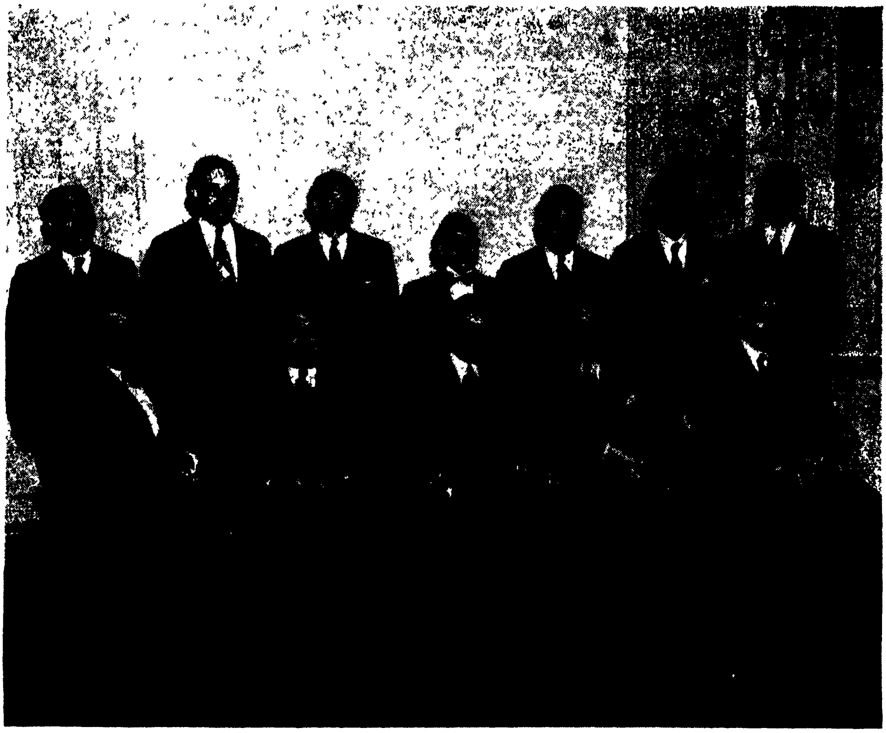
Upon the conclusion of each award, the recipient briefly expressed his appreciation.

MR. NALLE: "Will the Sponsors kindly resume their seats at their respective tables?"

MR. NALLE (*continued*): "It now gives me great pleasure, and we deem it a privilege, to have the opportunity to present Dr. Wendell

Meredith Stanley who will speak to us about 'Recent Advances in Virus Research.'[1] Dr. Stanley."

Mr. Nalle, after Dr. Stanley had read his paper, conveyed to him the thanks of the Institute.



RECIPIENTS OF AWARDS FOR 1948.

Seated (*left to right*): Eugene J. Houdry (Potts); Clarence Anding Lovell (Potts); Wendell Meredith Stanley (Franklin); Charles Duncanson Young (Henderson); William H. Cherry (Levy).

Standing (*left to right*): Raleigh J. Wise (Longstreth); David B. Parkinson (Potts); Ray Davis Kell (Ballantine); Nicholas F. Arone (Longstreth); Edwin H. Brink (Longstreth); Jan A. Rajchman (Levy); Wendell F. Hess (Wetherill).

MR. NALLE: "Dr. Theodor von Kármán, our other Franklin Medalist and speaker, was unable to return from Europe in time to be with us. He has, however, sent to us his paper on 'Progress in Aviation'[2] which he had intended to give. We ask Mr. LePage, his Sponsor, if he will deliver this short address for Dr. von Kármán. Mr. LePage."

The President then thanked Mr. LePage for reading Dr. von Kármán's paper, and dismissed the assembly with a cordial word and declared the meeting adjourned.

---

[1] See page 445 for "Recent Advances in Virus Research."

[2] See page 451 for "Progress in Aviation."

STATED MONTHLY MEETING, WEDNESDAY, NOVEMBER 17, 1948.

The Stated Monthly Meeting of The Franklin Institute was held November 17th in the Lecture Hall of The Franklin Institute. The President, Mr. Richard T. Nalle, presided and called the meeting to order at 8:15 P.M. There were approximately 100 people present. The President stated that the monthly meeting for October was the annual Medal Day Exercises, and because of the length of the minutes they had not yet appeared in the Journal of The Franklin Institute. Therefore, their approval would not be asked for until a later date.

The President then called upon the Secretary, Dr. Henry B. Allen, to give the report of the Membership and to make any announcements. The report of the Membership for the month of October was presented as:

Honorary	2
Active	41
Associate	21
Student	22

The total Institute Membership as of October 31, 1948 was 5260.

The Secretary reminded the members and friends of the Institute that the Atomic Energy Exhibit was now on display, and would continue on exhibit through January 30, 1949.

The announcement was then made by the Secretary that the "Star of Bethlehem" show would be demonstrated in the Planetarium beginning November 26th through December.

The Secretary then told the audience about the Laboratories for Research and Development, which now boast of approximately 270 personnel.

The President then introduced Dr. V. K. Zworykin of the RCA Laboratories in Princeton, New Jersey. Dr. Zworykin spoke briefly on the origin of electronic aids for the blind and reviewed the early history of the instruments. He said that the work fell into two classes; one using the characteristics of the letter to generate a signal and the other recognizing the letters electrically and producing sound in response. Dr. Zworykin introduced Mr. L. E. Flory who discussed the first category and demonstrated the direct signal method. Mr. Pike was then introduced by Dr. Zworykin, who demonstrated the more complex instrument for aiding the Blind.

At the conclusion of the meeting, Dr. Corner, chairman of the Committee on Sensory Devices at the Carnegie Institution of Washington, D. C., presented to The Franklin Institute for safekeeping this instrument used in the latter demonstration. The President accepted the instrument on behalf of The Franklin Institute.

The meeting was adjourned at 9:30 P.M., with a vote of thanks to the three speakers of the evening.

Henry B. Allen,
Secretary.

MUSEUM NOTES.

FELS PLANETARIUM.

In one of the Gospels we read of a miraculous star which guided the Wise Men to the manger of young Jesus and, very naturally, the mention of a star inspires people to ask astronomers what this object may have been. Quite frankly, no precise answer to this question can be given. The details are much too obscure and there is no other mention of this event in either sacred or secular literature.

Over a period of about a dozen years the staff of the Fels Planetarium have delved into matters astronomical and historical in the attempt to arrive at a concise and satisfactory suggestion concerning this cherished tradition of the Christian faith. This research has involved investigation of the nationality and even the personality of one who could be called a Wise Man, and many translations of the story have been consulted in the attempt to arrive at unadulterated meanings of the words that are used.

The evidence of the astronomers and of the Gospel itself is really quite clear in indicating that the commonly accepted interpretation of a brilliant, blazing star must be in error. But there was something in the sky that, to those who are called the Wise Men, would have been received as an omen of an important event.

During December for the past several years, it has been our custom in the Fels Planetarium to turn time back through almost 2000 years to reproduce the skies for the period preceding and following the birth of Jesus. By combining the Scriptural story with the established historical and cultural background of the period, we are able to arrive at an answer not only to the question of what was the star of Bethlehem, but also to the far more important question of the date of the birth of Jesus. He was born in the spring of the year 6 B.C. according to the interpretation which is used by the astronomers of the Fels Planetarium.

Following the scientific presentation, the traditional story of the Holy Night and of the arrival of the Wise Men in Bethlehem will be told in Scriptural readings, familiar music, tableaux and interpretative lighting effects.

Other special demonstrations are also under consideration, but each month a newly designed presentation of some phase of astronomy is given. For those who want to know how the calendar we now use has come into being, the January, 1949, demonstration is recommended. Those who attend the "Story of the Calendar" showings will also hear a recommendation for an improved calendar which will preserve the best of what we now have and will eliminate the factors which trouble us from year to year. The February demonstration will tell our visitors how to identify and how to increase their enjoyment of the brilliant stars which stud our winter sky. Further plans for the 1949 program in the Fels Planetarium will be announced as they crystallize.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, November 10, 1948.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, NOVEMBER 10, 1948.

DR. THOMAS D. COPE, *Acting Chairman*, in the Chair..

The following report was presented for final action:

No. 3191: Work of Arthur M. Young on the Stability of Helicopters.

This report recommended the award of an Edward Longstreth Medal to Arthur M. Young, of Paoli, Pennsylvania, "In recognition of his successful application of flying scale models to test unproven ideas in helicopter flight."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays, and Saturdays from 9 A.M. until 5 P.M., and Wednesdays and Thursdays from 2 P. M. until 10 P.M.

RECENT ADDITIONS.

BIOGRAPHY.

PEPPER, WILLIAM. The Medical Side of Benjamin Franklin. 1911.

BOTANY.

BALY, EDWARD CHARLES CYRIL. Photosynthesis. 1940.
BOYCE, JOHN SHAW. Forest Pathology. Second Edition. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BERGMANN, ERNST DAVID. Isomerism and Isomerization. 1948.
EGLOFF, GUSTAV AND HULLA, GEORGE. Alkylation of Alkans. Vol. 1. 1948.
HOUGEN, O. A. AND DODGE, F. W. The Drying of Gases. 1947.
HUNTRESS, ERNEST HAMLIN. The Preparation, Properties, Chemical Behavior and Identification of Organic Chlorine Compounds. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

SMITH, ARTHUR WHITMORE. Electrical Measurements in Theory and Application. Fourth Edition. 1948.
SOLLER, THEODORE; STARR, MERLE A. AND VALLEY, G. E. Cathode Ray Tube Displays. 1948.

ENGINEERING.

ALLEN, J. Scale Models in Hydraulic Engineering. 1947.

GEOLOGY.

PEARL, RICHARD M. Popular Gemology. 1948.
SHERLOCK, ROBERT LIONEL. The Permo-Triassic Formations. 1947.
U. S. BUREAU OF MINES. Mineral Resources of the United States. 1948.

HISTORY.

ATKINS, EDWIN FARNSWORTH. Sixty Years in Cuba. 1926.
BURCHARD, JOHN ELY. Q.E.D. M.I.T. in World War II. 1948.

MANUFACTURE.

HAINES, RAY E. The Home Crafts Handbook. 1948.

MATHEMATICS.

ORE, OYSTEIN. Number Theory and its History. 1948.
RIDER, JOHN FRANCIS AND USLAN, SEYMOUR D. Understanding Vector and Phase. 1947.
SCARBOROUGH, JAMES BLAINE. Fundamentals of Statistics. 1948.

METALLURGY.

WILLIAMS, ROBERT S. AND HOMERBERG, VICTOR O. Principles of Metallography. Fifth Edition. 1948.

NATURAL HISTORY.

OSBORN, FAIRFIELD. Our Plundered Planet. 1948.

PHYSICS.

AMERICAN PHYSICAL SOCIETY. DIVISION OF ELECTRON OPTICS. Preparation and Characteristics of Solid Luminescent Materials. 1948.

FAIRES, VIRGIL MORING; BREWER, ALEXANDER V. AND SIMMANG, CLIFFORD M. Problems on Applied Thermodynamics. Revised Edition. 1948.

ROSENFELD, L. Nuclear Forces. 1948.

ROTHERY, WILLIAM HUME. Atomic Theory for Students of Metallurgy. 1948.

PLASTICS.

BROWN, D. WARBURTON. Handbook of Engineering Plastics. 1946.

RADIO.

STRUTT, MAXIMILIAN JULIUS OTTO. Ultra and Extreme Short Wave Reception. 1947.

PUBLICATIONS RECEIVED.

Fundamentals of Physical Science, by Konrad Bates Krauskopf. Second Ed., 676 pages, 15 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., 1948. Price, $4.50.

Vacuum Tube Amplifiers, edited by George E. Valley, Jr. and Henry Wallman. 743 pages, 16 × 23 cm., tables, drawings and illustrations. New York, McGraw-Hill Book Co., 1948. Price, $10.00.

The Specifications and Law on Engineering Works, by Walter C. Sadler. 493 pages, 14 × 22 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Simplified Design of Structural Timber, by Harry Parker. 218 pages, 13 × 20 cm., tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.25.

Physical Geology, by Chester R. Longwell and others. Third Ed., 602 pages, 15 × 24 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Nuclear Radiation Physics, by R. E. Lapp and H. L. Andrews. 487 pages, 15 × 22 cm., tables, drawings and illustrations. New York, Prentice-Hall, Inc., 1948. Price, $6.00.

Radio at Ultra-high Frequencies, Volume II. (1940–1947), edited by Alfred N. Goldsmith and others. 485 pages, 15 × 23 cm., tables, drawings and illustrations. RCA Review, Radio Corp. of America, RCA Laboratories Division, Princeton., N. J., 1948. Price, $2.50.

Supersonics—The Science of Inaudible Sounds, by Robert Williams Wood. 162 pages, 13 × 20 cm., drawings and illustrations. Brown University, Providence, R. I., 1939. Price, $2.00.

Conversion of Petroleum, by A. N. Sachanen. Second Ed., 602 pages, 15 × 23 cm., tables and drawings. New York, Reinhold Publishing Corp., 1948. Price, $11.00.

Natural Philosophy Through the Eighteenth Century and Allied Topics (Commemoration Number to mark the 150th Anniversary of the foundation of The Philosophical Magazine), edited by Allan Ferguson. 164 pages, 17 × 25 cm., Port. London, Taylor & Francis, Ltd., 1948. Price, 15s (paper).

The Stuff We're Made of, by W. O. Kermack and P. Eggleton. 356 pages, 13 × 19 cm., tables and illustrations. New York, Longmans Green & Co.; London, Edward Arnold & Co.; 1948. Price, $3.00.

Sources of Engineering Information, by Blanche H. Dalton. 109 pages, 16 × 24 cm. Universtiy of California Press, 1948. Price, $4.00.

BOOK REVIEWS.

TWO-DIMENSIONAL FIELDS IN ELECTRICAL ENGINEERING, by L. V. Bewley. 204 pages, 16 × 24 cm., drawings and illustrations. New York, The Macmillan Co., 1948. Price, $5.50.

This most worth while book by Professor Bewley fulfills a need for a text which covers some of the introductory problems of field theory in several branches of engineering. It serves admirably in attempting to bridge a gap between various subjects which require field treatment and as a new and perhaps more basic and sound approach to problems in electromagnetic fields and hydrodynamics than has been used previously.

This new book is of unusual interest in that the subject matter presents the mathematical and physical concepts through illustrative problems which are quite practical. The fact that these problems are drawn from several fields emphasizes the power of the field treatment as a working tool, not only for the student but for the practicing engineer. It is seen to be an aid in precise and accurate reasoning, coupled with an easy means of more readily understanding the mathematical results.

The level on which the book is written is that of a senior student in electrical engineering, and it presupposes a knowledge of calculus and differential equations. Through practical illustrations the reader is led through many phases of mathematical physics, including elements of the theory of functions, the Cauchy-Riemann equations, Laplace's equation, sources and sinks, contour integrals, conformal mapping and the important Schwarz-Cristoffel transformation, and the theory of images. These subjects and others are amply illustrated with practical problems such as: eddy current and skin effects, various shaped electromagnetic fields, vortex flow, Bordas mouthpiece, and many others.

From the description listed above, it may rightly be inferred that the bulk of the work consists of solutions of Laplace's equations for particular boundary conditions. Most subsections of the book have collections of problems useful for the classroom and for self study, serving to stimulate thinking along the generalized ideas presented in the various chapters. In addition to the problems listed, it may be good mathematical exercise to re-plot many of the figures in the text starting with the basic equations for the physical concepts.

S. CHARP.

HIGH FREQUENCY MEASURING TECHNIQUES USING TRANSMISSION LINES, by E. N. Phillips, W. G. Sterns, and N. J. Gamara. 58 pages, diagrams, 21 × 28 cm. New York, John F. Rider Publisher, Inc., 1947.

This publication attempts to formalize some of the techniques involved in carrying out high frequency measurements using a slotted concentric transmission line. The procedures outlined have been evolved and are used to answer the needs imposed by measurements in the ultra-high frequency spectrum, where the slotted concentric transmission line is almost the only instrument available to measure physical quantities such as wave length, impedance, velocity of propagation, electrical length, and attenuation in four terminal networks.

The following particular measurement techniques, which can be made with a standardized slotted concentric transmission line, are described: surge impedance and electrical length of the slotted transmission line; measurement of the load impedance attached to the load end of the slotted line, using either a knowledge of all the line constants or by a short circuit method which does not require a knowledge of the series components which connect the load to the line; method of correction for finite attenuation in the measuring section of the line and for attenuation in cable sections; far-end cable load measurements by a method dependent upon short circuit, or a double load, or surge impedance; determination of the electrical characteristics of radio frequency transmission cable section, such as surge impedance and propagation constant; and the determination of the length of ultra-high frequency cable by the procedure commonly utilized in low-frequency communication circuits.

The authors present the techniques in a series of data sheets which list: the order in which a particular measurement is made; the procedure of making the measurement; a sketch showing the section of the line where the particular measurement is made; and the formula or other pertinent data necessary to make a computation for the physical quantity under investigation from the quantities measured as noted under procedures. These data sheets comprise the major portion of the booklet.

Although there is a wealth of information included, there is much lacking to make this publication of use to the reader on a first reading. Certain terms are introduced in early sections and are either not defined or are defined in later sections. As a result, some sample calculations are made with no statement on the data sheets as to the algebraic form of the equations which are being solved. Many of the diagrams are inadequately labeled or lack identification. Typical of this latter fault is the frequent reference to a Transmission Line Calculator or "Smith Chart," which chart is not shown until some time after its introduction, and then it is illustrated with neither explicit instructions for the use of such diagram, nor even with an explanation as to what the various lines on the chart represent. As a result, the interested reader must study the data sheets in detail and follow through all the numerical examples in order to understand clearly what is being done.

In spite of the shortcomings noted above, it is felt that this booklet may find applications as a reference work in laboratories concerned with ultra-high frequency measurements, and for engineers concerned with such work.

S. CHARP.

TELEVISION—HOW IT WORKS, by John F. Rider. 203 pages, 21 × 28 cm., drawings and illustrations. New York, John F. Rider, 1948. Price, $2.70 (paper).

Typical of many other John F. Rider publications, "Television—How It Works" is written expressly for the radio serviceman. The book gives a thorough, practical, non-mathematical approach to the underlying principles of television with the major emphasis placed, of course, on receiving rather than transmitting equipment.

The flow of text material follows a logical pattern, opening with general background material and then proceeding smoothly from the antenna to the sound and picture reproducers. Throughout the text, frequent reference is made to the circuitry of a number of the receivers on the market today.

Although the book was written by four different authors, the over-all job of editing and coordination has been very well done, so that there is no overlapping nor conflict from chapter to chapter.

There are several statements made in the text which may be misleading to the serious reader. For instance, in Chapter 3 on Television Receiving Antennas, the impression is created that all reflectors have a tendency to raise the input resistance of a dipole, whereas the input resistance can be either raised or lowered by the parasitic reflector depending upon spacing and length. Also, in Chapter 6 on The Video I-F and Detector Section, it is stated that the curves of individual i-f. stages are additive to produce the over-all i-f. curve. Actually, the individual i-f. stage curves multiply together to give the over-all response. The book contains very few references to other sources of material, and it is hoped that this situation will be rectified in any future editions by the inclusion of a bibliography or footnote references for the reader who wishes to broaden his knowledge on the subject of television.

The final chapter on Alignment and Servicing is quite general, but none the less it is as complete as possible, barring the actual inclusion of individual receiver manufacturers' maintenance data.

This book is recommended not only to the serviceman, but to anyone interested in the fundamentals of television systems. It is believed that it can be particularly valuable to the purchasers of "build-your-own" television receiver kits.

ROBERT S. GRUBMEYER.

READINGS IN THE PHYSICAL SCIENCES, edited by Harlow Shapley and others. 501 pages, 16 × 24 cm., drawings. New York, Appleton-Century-Crofts, Inc., 1948. Price, $3.00.

The aim of the editors in preparing this book is to bring to the general student an understanding of the fundamental nature of the physical world and the skills by which this nature is discerned. The book is a collection of fifty-eight articles abstracted from published papers, books, and lectures in several scientific fields of endeavor, with emphasis on contemporary work. But the writings of early workers have not been neglected, since reference to their labors seems to give perspective to a volume which not only attempts to present some of the principles of the scientific method, but also, through the selections, a brief history of science.

The choice of articles is not at all haphazard. A definite plan was in mind, namely, to approach the problems in science by five paths: history; an examination of the scientific method and its implications; research; tools and apparatus; and bibliographies in the fields considered. Original articles such as those by Galileo, Newton, and Copernicus, to mention a few, attune one more readily to appreciate the problems encountered in the present.

The mental attitude of the scientist, as it develops an eagerness for knowledge through the use of observation and experiment, serves to illustrate the scientific method.

But science is based upon research, and the articles by some of the greatest research men of our times and of the past help to make the stories of some of their work quite vivid. And research without tools is impossible. The need for the latter, and how they help us to learn more and more about the unknown in interrelated fields becomes apparent when it is realized that discoveries and developments of tools in one field are often used in other fields which may not be too closely related.

The plan of approach to the appreciation of the status of scientific achievement today is carried out by a series of specialized articles in each of the following fields: science and scientific method; astronomy; geology; mathematics; physics; and chemistry. Each of these six parts of the book includes a good-sized bibliography for further and more detailed readings in the subject.

While the editors are credited with compiling the works of the various authors, the original works themselves, mostly written in English, have a style and clarity individual to the scientist-writer. As a result, the reader is carried along by the sheer intense interest which the scientist imparts to his story. When Franklin describes his kite experiment, it is done so explicitly that almost anyone could repeat and observe the phenomena. The article by Sir James Jeans on "Exploring the Atom," written in his philosophical style, tells the story of the elementary particles of physics in a factual, yet a most readable manner. As a result, the reader, while not necessarily a scientist nor one in training for a career in science, can gain some insight into the minds of two scientists of widely varying interests.

Nor are modern day problems neglected. The controversial Tennessee Valley Authority is introduced through an article by David E. Lillienthal and the abstract of the now famous Smyth report "Atomic Energy for Military Purposes" brings the reader to face face with aspects of science of interest to our national and international welfare.

While not a text book as we ordinarily consider some books, this volume would serve admirably for a course in the survey of the sciences, or the history of science, or simply one concerned with readings in the physical sciences.

S. CHARP.

ENGINEERING CONTRACTS AND SPECIFICATIONS, by Robert W. Abbett. Second ed., 327 pages, 15 × 24 cm., tables. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.75.

In this second edition of a book on how to write engineering contracts and specifications from an engineering point of view, the author, a consulting engineer, presents some of the legal and business aspects of the engineering profession in compact form, making this book most suitable as a ready reference as well as a text book. Originally written as a text book for college students, it should be of interest to practicing engineers, architects, contractors, and all persons who are interested in the fundamental legal aspects of this subject.

Basic principles are stressed throughout the volume. The twelve chapters include the following subjects: legal considerations and relationships among engineers, contractors, and owners; elements of contracts, liens, and labor laws; types of construction contracts; bidding procedures; insurance matters of interest to engineering and construction work; surety bonds and arbitration; engineering services contracts; and specifications determination and writing.

The sections on contracts are clearly written and are quite expository, covering such forms of contracts as competitive bid, lump sum, unit price, and cost plus. The bidding procedures commonly encountered in engineering construction work are outlined and there are reproduced complete samples of Advertisements to Bid forms in current use. Likewise, in the several sections on contracts, examples are given of current "standard" forms. These contracts are of a time-tested form and represent sound current practice in the United States. They are included primarily to illustrate the text and may, at the same time, serve as a guide in the preparation of similar documents for specific projects.

While throughout the book the various specification, bid and contract methods are discussed with emphasis along the lines of private practice, the comparative government procedures for public works projects are examined. This comparison between private and public working arrangements is one of the most important features of this book. This is the reason that it is so different from other volumes treating contracts from the engineer's point of view.

Many engineering schools do not require their students to study the legal aspects of the profession for which they are in training. It would be most advantageous if at some time in his training the student were to read a book of this type, if for no other reason than to make him realize some of the legal responsibilities of a practicing engineer.

S. CHARP.

INTRODUCTION TO APPLIED MATHEMATICS, Francis D. Murnaghan. 389 pages, 16 × 23 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Professor Murnaghan's book, based on a one-year graduate course which he gave for two decades at Johns Hopkins University, should prove a welcome addition to the growing list of mathematical works published in recent years for the benefit of scientists and engineers.

Chapter 1 consists of a clear presentation of cartesian vector analysis in two, three, and *n* dimensions. Chapter 2, entitled "Linear Vector Functions," includes a presentation of the concept of characteristic values and characteristic vectors. Chapter 3 presents a concise discussion of the theory of orthonormal functions. The functions of the orthonormal set are treated as base-vectors, so that this chapter should serve as a simple introduction to the study of function-spaces. Chapter 4 deals with orthogonal curvilinear co-ordinates in two and three dimensions, and includes a discussion of vector analysis in such systems. Chapter 5 presents a discussion of the most common methods of solving Laplace's equation in two and three dimensions, with physical interpretations in terms of electrostatics. Chapter 6 gives a straightforward presentation of the theory of spherical harmonics and Bessel functions, and in Chapter 7 there is given a very simple and lucid presentation of the one-dimensional boundary value problem. The excellent discussion of the Green's function deserves particular note. Chapter 8 gives a very clear discussion of the Fredholm and Hilbert-Schmidt theories of linear integral equations. Chapter 9 is devoted to the calculus of variations, with emphasis on applications to analytical mechanics. The concluding chapter deals with operational methods for the solution of initial-value problems of ordinary linear differential equations.

In a book of moderate length it is quite impossible to treat adequately all the most common methods of applied mathematics. In particular, the reviewer regrets that it was necessary to confine the discussion of complex-variable methods to brief sections in Chapters 4 and 5, and that the opening chapters do not contain an introduction to the theory of tensors. Numerous other extensions of the contents could also be recommended. Nevertheless, Professor Murnaghan deserves great credit for his clearly written presentation of some of the most important mathematical methods now employed in scientific research. Finally, it may be remarked that the author has wisely chosen the level of rigor, avoiding extreme pedantry on the one hand and looseness on the other.

BERNARD EPSTEIN.

INDUSTRIAL ELECTRONICS REFERENCE BOOK, by Electronics Engineers of the Westinghouse Electric Corp. 680 pages, 23 × 30 cm., tables, drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $7.50.

Most electrical engineers concerned with the production and distribution of electric power at commercial frequencies are familiar with the book entitled "Electric Transmission and Distribution Reference Book" written by the electrical engineers of the Westinghouse Electric Corp. Now out of this same large organization, authored by thirty-seven expert physicists and electronics engineers, comes a reference work with the most modern information available today on the industrial applications of electronics. From the first page to the last the reader is made well aware that the correct facts are presented about the various subjects considered. In the opinion of this reviewer, this reference work has no equal in regard to scope and clarity of presentation.

The Industrial Electronics Reference Book was written by a group of practicing engineers and it is meant to be read primarily by other practicing engineers who may be concerned with the application and utilization of electronic apparatus as tools in industry. In addition, research workers and designers engaged in the development of new industrial devices may gain a more far-reaching understanding of the possibilities and limitations of industrial equipment as used today. Students may find this book of material aid, and it may have application for use in courses in industrial electronics on either the undergraduate or graduate level. The more than one thousand references, contained in bibliographies to be found at the end of most of the chapters, cover specific problems mentioned in the book.

The work first introduces the fundamental and theoretical concepts of electron production and control, with some consideration of the theories of emission and forces on electrons in the presence of electric and or magnetic fields. Design, construction, and operation of the various types of electronic and gaseous tubes are then considered. The field is rather completely covered, as witness sections devoted to: electrical conduction in gases; vacuum tubes, klystrons, and magnetrons; hot and cold cathode gas tubes and their static and dynamic characteristics; photoelectric devices; X-ray tubes; cathode-ray tubes; and ultraviolet radiators.

A theoretical and practical treatment is accorded electronic components and circuits, emphasizing the use of electronic devices as rectifiers, amplifiers, oscillators, and controllers. A good basic theory of antennas and transmission lines is presented, with discussions of some experimental measuring techniques, the use of a transmission line chart of the type used by communications engineers, and certain design curves for antennas.

The basic design features of many pieces of industrial equipment are reviewed in the remainder of the book, in many instances setting forth their limitations. Typical of the subject matter covered in these sections are: power rectifiers and inverters; radio frequency heating; power line carrier systems; electronic instrumentation; and electronic controllers. Because of the rapidly changing aspects of electronic applications, the sections devoted to specific pieces of equipment are written somewhat thinly so that many basically different pieces of apparatus can be included in such a survey. That material which is presented is done so authoritatively and represents the most up-to-date information available.

While the book is primarily a reference work, it can be safely said that if any engineer were to read and digest the subject matter to be found in this work, he would have quite a liberal education in the subject of industrial electronics as the status of the art is today.

S. CHARP.

SCIENCE, SERVANT OF MAN, A LAYMAN'S PRIMER FOR THE AGE OF SCIENCE, by I. Bernard Cohen. 362 pages, illustrations, 14 × 20 cm. Boston, Little, Brown and Co., 1948. Price, $4.00.

With the increasing emphasis on scientific research by government agencies, the layman who foots the bill should be interested in knowing why he should be expected to do so, and what direction such scientific activities should take. Dr. Cohen has presented an instructive and interesting account of the potentialities of pure scientific research as a source of practical benefits.

Prompted by the National Science Fund (National Academy of Sciences), Dr. Cohen has had the benefit of assistance from many specialists, including several whose work he describes. The author, who is an instructor in the History of Science at Harvard University, has written other books on the history of science and is well qualified for the task he has undertaken.

Moreover, this is no bare recital of facts and dates about important inventions and discoveries. To clarify the importance of pure scientific research for the layman, the book presents the practical consequences of such research by means of selected case histories. In these, emphasis has been placed on the experience of the scientist himself, tracing the history of ideas, how they were discovered, the effect they had on the state of science and how they were put to use in the service of mankind.

The introductory chapters review the conditions necessary for scientific discovery and the point is made that discoveries are often dependent upon the existence of a proper scientific atmosphere. This explains in part the occurrence of simultaneous discovery as well as the rediscovery of older facts, known but not exploited. In this latter case a discovery may be made at a time when the scientific atmosphere is not propitious and it consequently lies dormant until such time as conditions are more favorable for futher investigation.

The case histories on the application of science are presented in three categories. First is that in which the practical applications of fundamental research are merely the "by products of the research for truth in the scientific laboratory." Examples are to be found in some of the new weed-killers which derived from investigating the question "Why do plants bend toward the light?", and in nylon, which was the outgrowth of research into high polymers.

The second type involves the situation in which a practical application seems likely and the research is directed towards that end. Instances of this type are the work done on blood groups and blood transfusions, and the development of a satisfactory hybrid corn. Generally such research is distinguished by being aimed at supplying a definitely felt need.

The third group of case histories is more unusual. In these the scientific facts had been known and studied for a long time before a use could be found for the knowledge. A striking example is the use made of "foraminifera" which has been studied by paleontologists in the various fossil formations and which, in the last few years when oil was vital, formed the basis of new techniques in petroleum prospecting. The author emphasizes that if this basic knowledge of these fossils had not been available, it would not have been possible to make this practical application in an entirely different field. He also points out the improbability of any group of scientists having devised a particular application until the scientific background had been set. For example, the visualization of any such technique as the use of X-rays in medicine would have been undoubtedly out of the question until the discovery of X-rays.

In two concluding chapters Dr. Cohen discusses science as the servant of man and how we can get the most out of science. He points out that society as a whole is responsible for what is done with the results of scientific research, and that the scientists should not be blamed if some development such as atomic energy is used for bombs instead of peaceful purposes.

Dr. Cohen has produced a stimulating and thought-provoking book. It not only entertains and instructs, but it conveys a message as to the importance of a free and untrammeled science, which should be liberally supported in the knowledge that practical applications generally result, even if they are unseen at the time. For those desiring to read further, a helpful bibliography is included with indication by use of a symbol of those titles which may prove most interesting to the layman.

G. E. PETTENGILL.

COMPUTING MECHANISMS AND LINKAGES, by Antonin Svoboda, edited by Hubert M. James. 359 pages, 16 × 23 cm., tables, drawings, and plate. New York, McGraw-Hill Book Co., 1948. Price, $4.50.

This volume of the Radiation Laboratory Series was written by one of the best designers of linkage computers, one who had studied and designed these particular computing elements for many years prior to the past war and his association with the Radiation Laboratory. As a result the work is one which is authoritative, representing some of the ideas and methods developed by the author and his close foreign collaborators.

The subject matter of the book is concentrated wholly in the field of the design of continuously acting computing mechanisms. In particular, it deals principally with the mathematical design of elements for bar-linkage computers. Such computers have a wide field of application, especially where the computer must be small and light-weight, as when it is to be carried in an aeroplane or guided missile. While in this volume there will be found some comment on the mechanical features of bar linkages, the mechanical design of these components is not discussed in detail, just sufficiently to give the analyst or analytical designer a basis for reasoning in the development of the "cells" for a complete computer.

The author considers a complex mechanical computer to be composed of a number of elementary computers which are essentially basic bar linkages and mechanisms which perform the elementary arithmetic functions. These latter mechanisms are described and analyzed in detail with the aid of clear sketches. Among the elements so presented are: eight varieties of adders or differentials; slide and nomographic multipliers; mechanical resolvers and component solvers; six varieties of cams; and friction wheel and ball integrators.

It is shown that simple bar linkages can be designed to perform all the elementary functions noted above except to establish relations between differentials, that is, they cannot perform the functions of integrators. It is pointed out, however, that these linkages serve quite well as function generators, although in many cases the design is not unique.

Problems of bar-linkage computer design are considered with emphasis on those mechanisms having one and two degrees of freedom. Examples of design procedures and the theory of design and operation are amply illustrated for the cases of: harmonic transformers, singly and in series; the three-bar linkage; and linkage combinations with one degree of freedom. In each of these categories an analysis is made of the errors involved due to structural defects as well as those due to non-ideal fitting of functions. Nomographic and geometric methods of analysis are presented. Two degree-of-freedom linkages and function generators are considered, as well as an analysis of the adjustment of linkage constants.

Throughout most of the book the reader is shown, by worked-out examples, how to apply the design theory of these mechanisms.

This book does not seem to be written for a general type of reader, but rather for one definitely interested in linkage desgin. It should, therefore, appeal to mechanical engineers and instrument designers, especially those designers who continually have to meet specifications for small size and light weight devices.

S. CHARP.

ELECTRONS IN GASES, by Sir John Townsend. 166 pages, 15 × 24 cm., tables and drawings. London, Hutchinson & Co., Ltd., 1947. Price, 25s.

This monograph on the actions of electrons and ions in gases was written by a master of both experimental technique and theoretical concepts, and should be of interest to every serious advanced student or worker in the field of experimental electronic physics and engineering. Patterned somewhat after the author's earlier work "Electricity in Gases" (1915) the book contains an account of some complete investigations of the motions of electrons in gases. The point of view is not that of a description of electronic motion as is understood in applications to electron lenses, but rather a consideration of electron motion as it affects our knowledge of atomic physics.

The book deals primarily with the experimental side of this subject, starting out with an explanation of how the effects of the collisions of electrons with molecules are deduced from experiments with small currents, in uniform fields of force, and in gases at various pressures.

From a knowledge of the distribution of the velocities of agitation and the mean free paths, the author derives expressions for the coefficient of diffusion of these electrons as they move in a stream, showing that this coefficient is equal to $l\bar{U}/3$, where l = mean free path and $\bar{U}$ = mean velocity of agitation. Electronic motion in electric and magnetic fields is considered in the presence of diffusion and expressions are derived in forms which allow their comparison with the results of experiment.

While the author presents much material which is in the nature of original contribution to this restricted field, he deals comprehensively with the work of other physicists. Experiments measuring the diffusion of streams of electrons and ions are described, indicating the effects due to the presence of small amounts of water vapor. The mobilities and energies of electrons during motion and as a result of collisions are considered in gases such as: hydrogen, argon, neon, and helium. Numerous graphs and tables are presented showing the relationships between gas pressure, electric field, mean velocity and diffusion ratio.

The author's theories on ionization by collision are deduced from the early experiments which he conducted as far back as 1900. The many objections which certain leading scientists made to his theories are discussed, and it is shown that on the basis of further experiments by the author and others that ionization is definitely caused by collisions of electrons. Again with regard to the determination of ionization potentials in several gases, experiments as carried out by many physicists are described and their results tabulated. It is shown that the energy losses as a result of collisions vary for different gases and for different electron energies. A theory of the action of positive ions is presented and it is shown to be in accordance with the experiments of the discharges in uniform fields and of discharges between coaxial cylindrical electrodes.

The one fault which this reviewer finds with the book is the lack of reference to any experimental work after the year 1938, the greatest number of references being dated in the first two decades of this century. While it may be true that most of the basic experimental work was done many years ago, still it would seem that our scientific journals must surely contain worth while material on the subject of electrons in gases from the point of view considered in this monograph. Their absence seems to make the book somewhat incomplete.

S. CHARP.

BOOK NOTES.

TABLES OF BESSEL FUNCTIONS OF FRACTIONAL ORDER, prepared by the Computation Laboratory of the National Applied Mathematics Laboratories, National Bureau of Standards. Volume I, 413 pages, tables, 20 × 27 cm. New York, Columbia University Press, 1948. Price, $7.50.

The Computation Laboratory of the National Applied Mathematics Laboratories, formerly known as the Mathematical Tables Project, has been preparing useful tables for ten years. This latest volume comprises tables for Bessel functions of fractional order $J_\nu(x)$ for $\nu = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}$. These functions are important in their own right and also can be used as approximations to solutions of more complicated differential equations encountered in wave theory. Practical applications of the functions may be found in the fields of elasticity, heat conduction and hydrodynamics. The bibliography supplies a guide to some of these applications while the introduction gives helpful hints on the use of the tables.

G. E. P.

THEORY OF PROPELLERS, by Theodore Theodorsen. 164 pages, illustrations, charts, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $6.00.

This work is based on a series of reports by the National Advisory Committee for Aeronautics, and incorporates the developments which resulted from the research program on propellers conducted by the N.A.C.A. during the war. The material is presented in nine chapters with several appendices and an index. Several illustrations are shown on model wake spirals and the book also contains many diagrams and charts. The volume is published as one of the McGraw-Hill Publications in Aeronautical Science.

W. P.

ENGINEERING APPLICATIONS OF FLUID MECHANICS, by J. C. Hunsaker and B. G. Rightmire. 494 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $5.00.

Two members of the faculty of the Massachusetts Institute of Technology have written this introductory textbook on fluid mechanics, based on some twelve years of experience in teaching an undergraduate course in the subject. Emphasis has been placed on an adequate understanding of flow phenomena rather than familiarity with many practical devices.

The first part of the volume considers the more general principles of fluid mechanics, while the second part treats of the theory in particular applications. Among the topics discussed in this latter section are wing theory, lubrication, both hydrodynamic and boundary hydraulic turbines, pumps, propellers, fluid coupling and torque converters. Because of the importance of models in cases of confused type of flow, the authors have treated the theory of dimensions and physical similitude at some length. A selected bibliography is appended to most chapters and the problems have been gathered together in one section at the end of the volume.

G. E. P.

STRENGTH OF MATERIALS, by Joseph Marin. 464 pages, illustrations, 16 × 24 cm. New York, Macmillan Co., 1948. Price, $4.75.

The material in this book has been selected for use as a text in a first course in strength of materials. The subject of the book is divided in four parts—simple stresses, combined stresses, statically indeterminate stresses, and special topics compiled into twelve chapters and two appendices. Part 1 covers simple tension and compression, torsion, bending and axial loads; part 2, combined stresses; part 3, statically indeterminate stresses; and part 4, riveted and welded joints. At the end of each chapter there is a collection of problems. The book is well illustrated and complete with well arranged contents and index.

W. P.

FUNDAMENTALS OF STATISTICS, by J. B. Scarborough and R. W. Wagner. 144 pages, tables, 16 × 23 cm. Boston, Ginn and Company, 1948. Price, $2.40.

A textbook presenting in rather concentrated form the fundamental principles and techniques of statistical analysis. Mathematics through elementary calculus is a prerequisite, as quite detailed proofs are offered of some of the theorems. Problems and answers are included.

G. E. P.

PROBLEMS ON APPLIED THERMODYNAMICS, by V. M. Faires, A. V. Brewer and C. M. Simmang. Revised edition, 151 pages, tables and diagrams, 15 × 23 cm. New York, Macmillan Company, 1948. Price, $1.70 (paper).

Some eleven hundred problems are presented in this paper-bound volume. Designed primarily for use with Faires' *Applied Thermodynamics*, they are adaptable to any text emphasizing the general energy equation and which uses terminology in accordance with the more recent codes and standards. Since some instructors may prefer not to have answers given, they have been omitted for some problems. Steam tables and charts are given in an appendix.

G. E. P.

THE THEORY OF MATHEMATICAL MACHINES, by Francis J. Murray. Revised edition, 139 pages, diagrams, 21 × 28 cm. New York, King's Crown Press, 1948. Price, $3.00.

This interesting presentation on various mathematical machines now appears in a revised edition. Opportunity has been taken to correct such errors as had been noted and to make some additions. These latter are principally in Part III dealing with the solution of problems and include two new chapters on "Electronic Digital Computers" and "Accuracy, Noise and Stability." The addition of an index will also make the material more useful.

G. E. P.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Abstract of **The Activation Mechanism and Physico-Chemical Properties of *Clostridium histolyticum* Proteinase.**—WALTER KOCHOLATY AND LAURA E. KREJCI (*Archives of Biochemistry*, **18**:1, 1948). *Cl. histolyticum* proteinase has been obtained as an apparently homogeneous protein by a combination of precipitation procedures and electrophoresis. It gives no positive reaction for sulfhydryl or disulfide groups, and contains very little tyrosine or tryptophane. Like pepsin, it exhibits increased electrophoretic mobility in solutions of increased ionic strength.

The enzyme shows increased activity in the presence of ferrous ions and cysteine. The extent of hydrolysis when enzyme and activator are added simultaneously to the subtrate was found to be the same as when addition of the activator is deferred until digestion by the unactivated enzyme is complete. These results were interpreted to indicate that the greater activity in the presence of the activator is the result of a change in, or extension of, the specificity of the enzyme.

CURRENT TOPICS.

A Simple, Accurate Microhardness Testing Device (*Metal Progress*, Vol. 54, No. 2).—This apparatus, a wartime development in Europe, uses an inverted metallographic microscope, whereby the exact location of the spot to be tested can be fixed. The objective is then replaced by another holder in which a pyramidal diamond is accurately centered, and specimen is lowered onto the diamond's point. A counterbalanced lever system regulates the pressure between diamond and specimen (the indenting load) between limits of 0.5 and 500 g., and the resulting impression is measured by micrometer eyepiece in the usual manner. The tester differs from other microhardness testers principally because of the facts that the necessary load is applied direct to the specimen (thus simplifying the construction) and that the specimen instead of the indenter is movable. The wide range of utility for microhardness testing can be illustrated by the following examples, which have been chosen primarily as applications to purely practical questions: testing of a wire 0.004 in. in diameter; determination of hardness of the working surface of small dental drills, thin twist drills, metal fragments and file teeth; testing of metal foils for hardness; and the determination of the hardness of thin surfaces, such as electrodeposited layers. Also, good results can be obtained by using a section of metal cut at an acute angle to the surface. The apparatus is also well adapted to measuring the hardness of brittle materials, by using a sufficiently small indentation load. This simple device can be used by any first class laboratory technician, once he has been trained in the routine.

Pioneer Aluminum Crane of Canadian Design (*Canadian Machinery and Manufacturing News*, Vol. 59. No. 9).—The latest, and perhaps most interesting adaptation of aluminum is its use for the structural members of a crane about to be installed at Arvida. It was built for the rod mill of the Aluminum Co. of Canada. Designed and fabricated by Dominion Bridge Co., the new crane is of the lattice girder type and has a lifting capacity of 15 tons. The 97-ft. girders are fabricated entirely of high strength aluminum alloys, extruded in the shapes of channels and angles and rolled into plates. The rivets and bolts used in fabrication are also of special alloyed aluminum.

The strength of the aluminum alloys used is approximately the same as that of structural steel, but in order to prevent excessive deflection it was necessary to increase the depth of the girders. To conserve headroom the trolley rails were attached to the inside faces of the girders so that the trolley would not extend above their top surfaces.

The weight of a standard crane fabricated in steel would have been 94,000 lb.; the actual weight of the aluminum crane (complete) is 51,735 lb. As a result of this considerable saving in weight, it was found possible to install a motor of 30 per cent less horsepower than that required for a standard crane. Maintenance will be simpler and less expensive as the crane bridge will never require painting. A reduction in weight of supporting steelwork was also found possible.

Automatic Projector.—LaBelle Industries, Inc., Oconomowoc, Wis., has acquired all sales rights to the PRC automatic slide projector, formerly distributed by the Picture Recording Company, Chicago.

This unit, which has been known to the trade as Model 300 PRC, is now identified as LaBelle Model 301 Automatic Magazine Loading Slide Projector.

In operation, the new LaBelle is different from any other projector in existence. It does all feeding, changing and restacking of slides automatically, efficiently and in perfect order. A pushbutton switch on a remote control cord gives real advantages to the operator, who can stand in front of his audience if giving a lecture, or relax comfortably in an easy chair, away from the projector.

A metal magazine, loaded with either 75 two-by-two ready mounts or 35 glass slides, is inserted into the top of the LaBelle projector.. Pressing the control switch releases the slides, one at a time, into a special chamber where they are prewarmed and conditioned for the heat in the projection aperture. The pre-warming operation prevents warping or buckling and insures correctly focused projections.

Each slide moves automatically into the right position in the projection aperture. A superior optical system increases light intensity and projects the image through coated lenses that reduce reflections. The image appears on the screen in sharp detail, always in perfect focus, and remains there until the pushbutton is operated again.

A double-action shutter completely blocks out the picture on the screen when slides are changed. A new view appears instantly, without flickering or the usual slide "wipe."

After leaving the projecting aperture, all slides, kept in correct sequence and proper position, are automatically restacked in an interchangeable metal container at the bottom of the machine. This container, when removed from the projector after an exhibition, can either be inserted into the top for another showing or filed for future use.

The LaBelle Projector makes it unnecessary to handle slides or ready mounts after they are once arranged, and they remain free of fingerprints or scratches. The objective lens is controlled by a "micro-focusing" adjustment. The "micro-tilt" quickly adjusts the alignment of the projected image on the screen.

The LaBelle retails at $125.00, a price that includes a handsomely styled carrying case with a screen in its detachable, standup cover. This new projector appeals to sportsmen, color-photo enthusiasts, school executives, sales managers, advertising men, sales training supervisors, traveling salesmen and lecturers.

Nuclear Charts Prepared by Westinghouse School Service.—Six lithographed wall charts in two colors illustrating the important areas of nuclear physics have been prepared by the Westinghouse Electric Corporation's School Service for use in classes in schools and colleges.

Measuring 25 by 36 in. and made of heavy stock, the charts are accompanied by a 32-page book of valuable supplementary information. The price for the complete set is $1.00.

The first chart portrays ten basic particles important in nuclear physics, illustrating and describing the proton, neutron, alpha particle, beta particle, gamma ray, deuteron, positron, mesons, and the neutrinos. The second chart explains how nuclei are put together, depicting some 635 isotopes and 75 isomers. It also illustrates and explains mass defect, binding energy, and Einstein's mass-energy formula.

Natural and man-made nuclear reactions are depicted in the third chart which includes solar reactions, natural radioactivity, nuclear fission, as well as typical reactions induced by various kinds of atomic bullets. Chart No. 4 portrays eleven types of apparatus for detecting and inducing nuclear reactions including the ionization chamber, Geiger counter, Wilson cloud chamber, and modern atom smashers.

The fifth chart shows five areas for the useful application of atomic energy. The construction and operation of the atomic pile are described. Illustrations of tracers, research, medical therapy, and the atomic bomb are included.

The last chart covers the major theoretical, experimental, and engineering achievements in nuclear physics, pointing out 38 contributions, from the discovery of natural radioactivity and X-rays to the current developments.

Camera for Diagnosis of Heart Ailments.—The Fairchild Camera & Instrument Corp. has announced that its fully-automatic 70-mm. Fluoro-Record Camera, which has been proven to be ideal for routine and mass radiography, is now available with a film speed-up modification that permits its use in the angiographic study of heart conditions.

A modified version of the original F-212 Fluoro-Record Camera, this new instrument gives increased film transport speed permitting exposures at the rate of approximately 17 every 24 seconds, to provide an adequate record for accurate diagnosis. Each 100-ft. roll of 70-mm. film supplies 375 negatives, 2½ by 3 in., permitting easy reading without high magnification.

Radiologists are now offered added economy and versatility with Fairchild's new gear shift speed changing mechanism which makes the Fluoro-Record Camera available for either angiocardiography or regular chest X-ray purposes.

No-Dip Paint Brush (*Science Illustrated*, Vol. 3, No. 11).—A continuous-flow paint brush, which eliminates the tiresome dipping of the brush in the paint, has been invented by Russell C. Engren, who employed the principle of gravity feed to solve the problem. Engren inserts valve outlets at the bottom of paint buckets and connects them with light, flexible hose to brushes with hollow heels. Paint flows continuously into the brush bristles, subject to control by the bicket valve and a simple clip. Housewives as well as painters tested the Air Flow Brush, found it saved an average of 38 per cent in time and paint. Contractors reported as much as 50 per cent higher painting production per man hour. Saving in brushes is also substantial, since cleaning bristles from heel to tip keeps them useable until they are completely worn out.

Du Pont Makes Titanium (*Nucleonics*, Vol. 3, No. 4).—The Du Pont Company has announced that it has begun the small scale manufacture of titanium metal, a new basic raw material for industrial development. A pilot plant of 100-lb. daily capacity has been successfully placed in operation at the

Newport, Delaware, plant of the Pigments Department. This, so far as the company knows, is the first time ductile titanium metal has been produced for commercial exploration. The U. S. Bureau of Mines has been producing the metal for research purposes.

Reports of the Bureau of Mines and other organizations indicate that the properties of titanium are such that it may be used for reciprocating mechanical parts and in jet engines where heat and pressure are great.

Most titanium metal which has been available heretofore has been in powder form. Du Pont is producing it now in sponge form and will shortly be producing ingots weighing up to 100 lb. Running 99.5 per cent plus of purity, the metal is ductile and can be rolled and drawn readily. It has high resistance to breaking or pulling and has high strength against distortion. The company will supply small samples without charge to industrial and university laboratories for testing.

Stronger than Steel (*Plastics*, Vol. 8, No. 10).—The American Chemical Society reported the development of a new plastic-cloth material which is stronger than steel and withstands shell fragments as successfully as armor plate. Easy to make from common textiles and the polyester type of plastic, the material can be molded into automobile bodies, water pipes, boat hulls and many other useful products. The textiles contributing the greatest strength are woven glass fiber, a synthetic cellulose fiber called Fortisan, tide-cord rayon, nylon, and Steralon, a non-woven cotton cloth originally designed for babies disposable diapers. The report stated that, pound for pound, a multi-layered sandwich of Fortisan and plastic is twice as strong as structural steel.

Plating Tungsten Alloys.—The metal tungsten may have new industrial uses as a result of research at the University of Wisconsin.

A rare, though not precious metal, tungsten's unusual properties have long been known to scientists. They include extreme hardness and the highest melting point of any known metal.

In combination with other metals, tungsten has been used for electrical contact points, light bulb filaments, high speed machine tools, and other electrical equipment.

A method for plating tungsten alloys now has been developed by M. L. Holt, University of Wisconsin professor of chemistry, and his graduate assistants. He recently reported on his work to the Electrochemical Society at its annual meeting in New York.

Professor Holt, assisted by Luther E. Vaaler, plated tungsten alloys on such metals as iron, copper, and brass, giving these metals an extremely hard, heat-resistant outer surface, useful where much friction and wear are encountered.

The alloys which Professor Holt plated were composed of from 5 to 55 per cent of tungsten with nickel, cobalt, or iron.

"The plated tungsten when properly heat-treated is quite hard for an alloy deposit," Professor Holt said, "and resistant to wear and corrosion. Our approach now, of course, is to find ways to plate pure tungsten."

Refining tungsten ore, much of which is mined in Southern China, presents great problems to industry. There is literally nothing in which it can be melted—the 6000°F. required to melt tungsten is higher than any substance which might be used for a container.

By chemical means the oxide of tungsten is recovered from the ore and heated in an electrical furnace with hydrogen gas. The hydrogen and oxygen combine, leaving tungsten powder which is pressed into bars. The bars are subjected to a heavy electrical current, raising the temperature of the tungsten to a point where it fuses into a rod.

Radiant Heating by Electricity (*Engineers' Digest*, Vol. 5, No. 7).—For many years electrical energy has been employed for room heating and in Great Britain the most popular method is by the use of portable electric fires. In European countries and the U. S. A. a convector type of electric heater is commonly employed, and this is becoming popular in Great Britain for although it is less efficient than the exposed element heater, its fire risk is also less.

Intermittent heating by either of these types of electric heaters is not only uneconomical, but introduces a bad load factor as the use of these units for house heating only partially coincides with the off peak load. These operating inefficiencies can be eliminated and a more convenient and healthy application of electrical energy to space heating secured by the application of low temperature radiant heat by the "Dulrae" system of electric panel warming.

"Dulrae" is a thin fabric, about 1 mm. thick, in which resistance wires are woven, and manufactured in 100-ft. long rolls, 48 in. or 24 in. wide, with multiwired conductors in the outer edges. The resistances are arranged in such a manner that at multiples of the length, the material may be cut across without injuring the element. The best operating results are obtained by fixing "Dulrae" material to ceilings, and for this purpose the elements are loaded at about 16 watts per square foot, which in normal temperatures will warm the surface of the ceiling to about 90 F. A thermostat, with a differential of plus or minus 1 F., is provided in each individual room for temperature regulation, which, unless intentionally changed, is maintained steadily without requiring manual operation of any kind. To reduce heat losses, the ceiling above the "Dulrae" material is insulated over the whole area.

Where land is of great value and rents are exceptionally high, the installation of a "Dulrae" system is a very economical proposition, for, although electricity for heating costs about four times as much as oil fuel, the saving of labor, and the extra rents receivable for the increased area of floor space (rendered available by the abolition of the boiler-room, fuel chamber, chimney, ventilating shaft, and the stoker's dressing room) show a balance in favor of the electric system.

Blade Milling Machine (*Engineers' Digest*, Vol. 5, No. 7).—A new type of blade milling machine developed by Sulzer Brothers, Winterthur, Switzerland, is a vertical copy milling machine with hydraulic control, rigid spindle and moving table. It is equipped with a normal cylindrical milling cutter which machines the whole length of the blades at one pass in a longitudinal direction. The feed motion of the work table is a combined movement consisting of rotation at a freely adjustable speed and a longitudinal motion controlled by a cam. This cam, which determines the profile to be milled, is mounted on a second revolving table whose rotary and longitudinal movements correspond exactly with those of the work table. On the cam runs a roller which controls the longitudinal displacement of the two tables by way of a pilot valve and a servomotor in such a way that the roller remains exactly in its central position at all times. If the diameter of the roller is equal to that of the milling cutter, the profile milled is thus an exact reproduction of the cam form. In general two profile cams are used per blade, one being for the front of the blade and one for the back. Blades up to approximately 200 mm. in length can be manufactured on these machines.

Chemical Mist Can Kill Fungi Too.—Blowing a mist of concentrated spray material over trees and fields has proved a notably effective new method of dealing with a good many insect pests. Noting the new equipment developed for this work, farmers and orchardists have been asking the U. S. Department of Agriculture whether it is not practical to use the same type of machinery for fighting plant diseases that are caused by spores of fungus.

At first thought the prospect did not seem encouraging. Nearly all insects move about on the plant they are attacking, and one of the great merits of DDT and other new poisons is that they remain deadly for some time, so that if insects of certain types merely come in contact with a tiny particle of the insecticide remaining on a leaf it causes "jitters" and kills the insect.

Many plant diseases are caused by the germination of spores of a fungus. Millions of these spores move in the air and shower plants constantly. The great majority of the spores die without causing any ill effect. Most drop to the ground. Others land on dry foliage and drop off or dry out. Still others ride on insects which may never drop a spore at a favorable place. But with millions of spores to waste, some land on moist spots on leaves, perhaps in dew drops. These germinate, spread, and multiply. With this situation the prospect did not seem promising for mist application of fungicides. It did not seem probable that the mist particles would cover the leaves well enough to prevent spore germination and the build-up of infection that follows.

With equipment available, however, S. F. Potts of the Bureau of Entomology and Plant Quarantine and A. E. Dimond of the Connecticut Agricultural Experiment Station, have tested the mist blower method of applying a fungicide for control of the anthracnose that attacks sycamores and London planes. They report a fair degree of control—much better than they anticipated.

It will be necessary, the scientists say, to make tests with other concentrated fungicidal sprays, and to observe results with other diseases of plants. But the results open a promising field for further experiment and investigation. The workers do not regard the method as promising for of the fungus causing Dutch elm disease which is carried by an insect with spores deposited in feeding injuries or under the bark where mist blown fungicides do not penetrate.

INDEX TO VOLUME 246

July to December, 1948.

References with (N) are Notes from the National Bureau of Standards and references with (C) are Current Topics.

Aberrations of electron lenses. Electron lenses of hyperbolic field structure (Rüdenberg), 311, 377
Abrasion-resistance. Line shot blast room with rubber, 444(C)
Adhesion. A study of chemically deposited silver mirrors (Morgan and Myers), 363
Aeronautics. Progress in aviation (von Kármán), 451
Aerosols. Business teams with research, 52(C)
Airborne fire control apparatus. Research work of The Franklin Institute during World War II on airborne fire control apparatus (McClarren), 273
Aircraft. America's most powerful aircraft jet engine in production, 340(C)
Aluminum. Pioneer aluminum crane of Canadian design, 531(C)
Aluminum alloys. Stress-corrosion tests of high-strength aluminum alloys, 344(N)
Amplifiers. The transitor—a new amplifier, 438(C)
Atomic fuel. Operation bootstrap, 441(C)
Atomic power. Westinghouse announces formation of atomic power division, 450(C)
Atomic radiation. Zeeman effect of neutral nitrogen and oxygen, 169(N)
Aviation. Progress in aviation (von Kármán), 451

Bacteria and vibrations, 292 (C)
Ballistics. A theory of the internal ballistics of the "Hoch-und-Niederdruck Kanone" (Corner), 233
Basic research. Research work of The Franklin Institute during World War II on airborne fire control apparatus (McClarren), 273
  Science in national security (Ritchie), 223
Bearings. Frictional and thermal behavior in journal-bearing lubrication, 343(N)
Benjamin Franklin half dollar. The story of the new Benjamin Franklin half-dollar (Ross), 53
Bentonite. Exposure to air improves some Wyoming-type bentonite, 443(C)
Biology. Biological effects of high energy radiations, 360(C)
Blade milling machine, 535(C)
Blind guidance by ultrasonics, 122(C)
Bomb sights. Research work of The Franklin Institute during World War II on airborne fire control apparatus (McClarren), 273
Book Reviews:
  Abbett, Robert W.: Engineering contracts and specifications, 523
  Arguimbau, Lawrence Baker: Vacuum-tube circuits, 85
  Bachman, C. H.: Techniques in experimental electronics, 86
  Bell, A. E.: Christian Huygens and the development of science in the seventeenth century, 264
  Bewley, L. V.: Two-dimensional fields in electrical engineering, 521
  Boas, Walter: Physics of metals and alloys, 264
  Burchard, John: Q.E.D. M.I.T. in World War II, 431
  Chymia, annual studies in the history of chemistry, 263
  Cohen, I. Bernard: Science, servant of man, 525
  Cook, Arthur L.: Elements of electrical engineering, 432
  Crocker, William: Growth of plants. Twenty years' research at the Boyce Thompson Institute, 180
  Dobson, J. F. and Brodetsky, S., translators: Nicolaus Copernicus, 178
  Drake, Rollen H.: Aircraft engines, 175
  Faires, Virgil Moring: Applied thermodynamics, 431
  Faires, Virgil Moring: Elementary thermodynamics, 86

Faires, V. M., Brewer, A. V., and Simmang, C. M.: Problems of applied thermodynamics, 529
Ham, C. W. and Crane, E. J.: Mechanics of machinery, 434
Hammer, Bernard W.: Dairy bacteriology, 179
Harrington, R. H.: The modern metallurgy of alloys, 430
Heumann, Gerhart W.: Magnetic control of industrial motors, 432
Hunsaker, J. C. and Rightmire, B. G.: Engineering applications of fluid mechanics, 529
Industrial electronics reference book, 525
Jacobson, C. A.: Encyclopedia of chemical reactions, 178
Journal of the Society of Motion Picture Engineers. Index, January, 1936 to December, 1945, 176
Killefer, D. H.: The genius of industrial research, 179
Kimbark, Edward Wilson: Power system stability, 85
King, Reno C., Jr.: Practical marine engineering, 177
Lichty, Lester C.: Thermodynamics, 261
Marin, Joseph: Strength of materials, 529
Maurain, Charles: La Foudre, 178
Mikulaschek, W.: Die dissertationen der eidgenossischen technischen hochschule, 1909–1946, eine systematische bibliographie, 84
Mineral resources of the United States, 263
Moore, Charlotte E.: Atomic energy levels, as derived from the analyses of optical spectra, 259
Morse, Philip M.: Vibration and sound, 434
Murnaghan, Francis D.: Introduction to applied mathematics, 524
Murray, Francis J.: The theory of mathematical machines, 529
Nauth, Raymond: The chemistry and technology of plastics, 433
Pearl, Richard M.: Popular gemology, 434
Philips, E. N., Sterns, W. G., and Gamara, N. J.: High frequency measuring techniques using transmission lines, 521
Pierce, J. A. and others, ed.: Loran, long range navigation, 176
Pierce, Willis Conway and Haenisch, Edward Lauth: Quantitative analysis, 434
Preparation and characteristics of solid luminescent materials, 262
R.C.A. Engineers: Frequency modulation, 86
Ralston, A. W.: Fatty acids and their derivatives, 258
Rider, John F.: Television—how it works, 522
Scarborough, J. B. and Wagner, W. R.: Fundamentals of statistics, 529
Severns, William H. and Degler, Howard E.: Steam, air, and gas power, 433
Shand, S. James: Eruptive rocks, 262
Shapley, Harlow (editor): Readings in the physical sciences, 522
Spangenberg, Karl R.: Vacuum tubes, 85
Spriegel, William R.: Industrial management, 429
Stepanoff, A. J.: Centrifugal and axial flow pumps, 177
Svoboda, Antonin: Computing mechanisms and linkages, 526
Tables of Bessel functions of fractional order, 528
Taylor, Donald W.: Fundamentals of soil mechanics, 261
Theodorsen, Theodore: Theory of propellers, 528
Townsend, Sir John: Electrons in gases, 527
Venable, William Mayo: The interpretation of spectra, 260
Vincent, E. T.: Supercharging the internal combustion engine, 175
von Bergen, Werner and Mauersberger, Herbert R.: American wool handbook, 84
Wostrel, John F. and Praetz, John G.: Household electric refrigeration, 177
Zeluff, Vin and Markus, John: What electronics does, 84
Brazing. New technique facilitates brazing stainless steel parts, 92(C)
Break-in procedure. Fast break-in procedure for Dodge cars, 450(C)
Brookhaven Laboratory. Weather towers near completion at Brookhaven Laboratory, 269(C)

Camera for diagnosis of heart ailments, 533 (C)
Carbonyl iron powders, 185(C)
Ceilometers for Army and Navy, 361(C)
Charts. Nuclear charts prepared by Westinghouse school service, 532(C)

Chemical deposition. A study of chemically deposited silver mirrors (Morgan and Myers), 363
Chemical mist can kill fungi, too, 536(C)
Chromated protein films inhibit corrosion, 63(N)
Classical method. Method of solving differential equations with constant coefficients (Lyon), 159
Committee on technical information. Special committee on technical information formed, 442(C)
Conductivity cell for viscous materials, 65(N)
  Measurement of the dielectric properties of soils and water at 3.2 cm. wave length (Straiton and Tolbert), 13
Conestoga wagon (Coulson), 215
Conservation. Results of conservation, 310(C)
  Two and a half acres per stomach, 158(C)
  United Nations Scientific Conference on the Conservation and Utilization of Resources (UNSCCUR), 272(C)
Control devices, automatic. Stabilization of carrier-frequency servomechanisms (Sobczyk). I. Gain phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187
Core baking. New Thermonic core baking tunnel, 271(C)
Corner, J.: A theory of the internal ballistics of the "Hoch-und-Niederdruck Kanone," 233
Corrosion. Chromated protein films inhibit corrosion, 63(N)
Cosmic rays. The effective path length through a vertical cosmic-ray telescope (Witmer and Pomerantz), 293
Coulson, Thomas: The Conestoga wagon, 215
Cranes. Pioneer aluminum crane of Canadian design, 531(C)
Cylinder wear. Laboratory for study of cylinder wear, 495(N)

Dehydrofreezing, 94(C)
De-icing. Heat conducting glass keeps ice from windshield, 185(C)
Dental amalgam. New motion picture on dental amalgam released, 498(N)
Dentistry. New motion picture on dental amalgam released, 498(N)
Diagnosis, medical. Camera for diagnosis of heart ailments, 533(C)
Dielectric properties. Measurement of the dielectric properties of soils and water at 3.2 cm. wave length (Straiton and Tolbert), 13
Differential equations. A modified method of iteration of the Picard type in the solution of differential equations (Gwoh-Fan Djang), 453
Diffusers. The effect of screens in wind-tunnel wide-angle diffusers, 341(N)

Elasticity. Elastic truths measure stretch in fabrics, 93(C)
Electrical sparks and lightning discharge. The mechanism of lightning discharge (Loeb), 123
Electromagnetism. Generalized magneto-ionic theory (Gerson and Seaton), 483
Electron lenses of hyperbolic field structure (Rüdenberg), 311, 377
Electron microscope. Electron lenses of hyperbolic field structure (Rüdenberg), 311, 377
  Electron optical Schlieren effect, 165(N)
Electron microscopy of wool fibers by a shadowed replica method, 66(N)
Electron optics. Electron lenses of hyperbolic field structure (Rüdenberg), 311, 377
Electron tubes. Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes (Swann), 149
Electronics. Ceilometers for Army and Navy, 361(C)
  New Thermonic core baking tunnel, 271(C)
Electrostatic lenses. Electron lenses of hyperbolic field structure (Rüdenberg), 311, 377
Engine manufacture. Fast break-in procedure for Dodge cars, 450(C)
Engines. Laboratory for study of cylinder wear, 495(N)
Equipment. What's new in equipment, 443(C)
Eye-dropper technique proves effective method of testing steel, 310(C)
Eye-protective glasses. Properties and use of eye-protective glasses, 250(N)

Fading. Control of fading lamps with light-sensitive paper, 167(N)

Feed-back, a-c. Stabilization of carrier-frequency servomechanisms (Sobczyk). I. Gain-phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187.
Fibers. Du Pont announces new synthetic,—Orlon, 482(C)
Fire-retardant paint. Paint defies flame, 52(C)
Fluoroscopy. Camera for diagnosis of heart aliments, 533(C)
  Tube expected to make fluoroscope 500 times brighter, 214(C)
Forest fires. Protect forests from fire, 362(C)
Franklin, Benjamin. A check list of Franklin imprints in the Library of The Franklin Institute (Pettengill), 351
Franklin imprints. A check list of Franklin imprints in the Library of The Franklin Institute (Pettengill), 351
The Franklin Institute:
  Annual report of the President 1947, 71
  Biochemical Research Foundation Notes:
    Ely, J. O. and Ross, M. H.: A method of ultraviolet photomicrography, 87
    Enns, T.: Radiation survey meter, 181
    Kocholaty, Walter and Krejci, Laura E.: *Abstract of* The activation mechanism and physicochemical properties of *Clostridium histolyticum* proteinase, 530
    Zittle, Charles: A.: *Abstract of* Blood group substance from intestinal mucosa and its precipitation from borate, 359
    Zittle, Charles A.: Inhibitors for ribonucleinase, 266
    Zittle, Charles A.: Manometric determination of carboxypeptidase, 435
  Committee on Science and the Arts: Abstracts of Proceedings, June 9, 1948, 78; October 13, 1948, 426; November 10, 1948, 518
  Day, Charles, Lecture: The full responsibility of management (Meinel), 44
  Library. Publications received: 83, 174, 265, 350, 429, 520
  Library Notes: 80, 171, 256, 349, 427, 519
  Medal Day Proceedings, 503
  Museum Notes: 256, 348, 427, 517
  New members elected December 1, 1947 to June 1, 1948, 78
  Stated Meetings: Minutes, October 20, 1948, 425; November 17, 1948, 517
Frost control with air circulators, 502(C)
Frozen food. Dehydrofreezing, 94(C)
Fuel. Operation bootstrap, 441(C)
Fungi. Soil organisms in vital contests, 93(C)
  Chemical mist can kill fungi, too, 536 (C)

Gas storage. Natural gas stored in underground pipes, 438(C)
Gas turbine for electric utility, 440(C)
  Czech builds road-type gas-turbine engine, 444(C)
Geiger-Mueller counters. The effective path length through a vertical cosmic-ray telescope (Witmer and Pomerantz), 293
Generators. Quick starting house turbine-generator unit, 444(C)
Germ-less air, 92(C)
Gerson, N. C. and Seaton, S. L.: Generalized magneto-ionic theory, 483
Glass. Heat conducting glass keeps ice from windshield, 185(C)
Glycol vaporizers. Germ-less air, 92(C)
Gun design. A theory of the internal ballistics of the "Hoch-und-Niederdruck Kanone" (Corner), 233
Gwoh-Fan Djang: A modified method of iteration of the Picard type in the solution of differential equations, 453

Hafnium oxide, 450(C)
Heating. Radiant heating by electricity, 534(C)
Honey bees. Measure honey in "bee-years," 272(C)
Hormones. Symposium on steroid hormones, 442(C)
Humidity test apparatus, 421(N)
Hydrated limes. Expansive characteristics of hydrated limes, 420(N)
Hydraulics. The stroboflow, an oscilloscope for hydraulics, 183(C)
Hygrometers. Humidity test apparatus, 421(N)

Image amplifier. Tube expected to make fluoroscope 500 times brighter, 214(C)
Induction heating equipment available without capital investment, 340(C)
Industrial management. The full responsibility of management (Meinel), 45
Insecticides. Business teams with research, 52(C)

Integrating sphere. Large photometric sphere, 61(N)
Isotopes. Measurement of radioactive isotopes, 419(N)
Iteration. A modified method of iteration of the Picard type in the solution of differential equations (Gwoh-Fan Djang), 453

Jet engine. America's most powerful aircraft jet engine in production, 340(C)

Kimball, W. S.: Optimum ratio between crank arm and connecting rod for smooth operation of reciprocating engines, 409
Kinematics of machines. Optimum ratio between crank arm and connecting rod for smooth operation of reciprocating engines (Kimball), 409

Laminated wood. Wood Research Laboratory facilities enlarged, 270(C)
Languages for science (Moon and Spencer), 1
Lightning discharge. The mechanism of lightning discharge (Loeb), 123
Light-sensitive paper. Control of fading lamps with light-sensitive paper, 167(N)
Lime. Expansive characteristics of hydrated limes, 420(N)
Linear differential equations. Method of solving differential equations with constant coefficients (Lyon), 159
Liquid rheostats, 185(C)
Loeb, Leonard B.: The mechanism of lightning discharge, 123
Loveland, R. P., Menihan, H. M., and Trivelli, A. P. H.: Analysis of the particle-size distribution of the grains producing a photographic image, 459
Lubrication. Frictional and thermal behavior in journal-bearing lubrication, 343(N)
Lyon, W. V.: Method of solving differential equations with constant coefficients, 159

Machine design. Optimum ratio between crank arm and connecting rod for smooth operation of reciprocating engines (Kimball), 409
Magnetic pulley. Puri-Pulley magnetic pulley, 502(C)
Magneto-ionic theory. Generalized magneto-ionic theory (Gerson and Seaton), 483
Manometer. New type manometer, 184(C)
Manufacturing advances in wartime Germany, 94(C)
Marine engineering. Optimum ratio between crank arm and connecting rod for smooth operation of reciprocating engines (Kimball), 409
McClarren, Ralph H.: Research work of The Franklin Institute during World War II on airborne fire control apparatus, 273
Measurement of the dielectric properties of soils and water at 3.2 cm. wave length (Straiton and Tolbert), 13
  Large photometric sphere, 61(N)
  New type manometer, 184(C)
Mechanical engineering. Optimum ratio between crank arm and connecting rod for smooth operation of reciprocating engines (Kimball), 409
Mechanism of lightning discharge (Loeb), 123
Medicine. Recent advances in virus research (Stanley), 445
Meinel, William J.: The full responsibility of management, 44
Menihan, H. M., Loveland, R. P., and Trivelli, A. P. H.: Analysis of the particle-size distribution of the grains producing a photographic image, 459
Meteorology. Weather towers near completion at Brookhaven Laboratory, 269 (C)
Microfilm camera for unusually large copy, 482(C)
Microhardness. A simple, accurate microhardness testing device, 531(C)
Microwaves. Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes (Swann), 149
Milling machine. Blade milling machine, 535(C)
Military preparedness. Science in national security (Ritchie), 223
Minting process. The story of the new Benjamin Franklin half-dollar (Ross), 53
Mirrors. A study of chemically deposited silver mirrors (Morgan and Myers), 363
Moon, Parry and Spencer, Domina Eberle: Languages for science, 1
Morgan, Raymond and Myers, Ralph D.: A study of chemically deposited silver mirrors, 363
Myers, Ralph D. and Morgan, Raymond: A study of chemically deposited silver mirrors, 363

National security. Science in national security (Ritchie), 223
Natural gas stored in underground pipes, 438(C)
Networks, a-c. differentiating and stabilizing. Stabilization of carrier-frequency servomechanisms (Sobczyk). II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187.
Networks, phase-shifting. Stabilization of carrier-frequency servomechanisms (Sobczyk). II. Design formulae for proportional-derivative networks, 95.
No-dip paint brush, 533(C)
Nuclear charts prepared by Westinghouse school service, 532(C)
Nuclear physics. A radio-frequency mass spectrometer, 498(N)
  Du Pont makes titanium, 533(C)
  The effective path length through a vertical cosmic-ray telescope (Witmer and Pomerantz), 293
  Westinghouse announces formation of atomic power division, 450(C)
Nutrition. Two and a half acres per stomach, 158(C)
Nyquist diagrams, a-c. Stabilization of carrier-frequency servomechanisms (Sobczyk). I. Gain-phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187.

Ordinary linear differential equations. Method of solving differential equations with constant coefficients (Lyon), 159
Ordnance. Science in national security (Ritchie), 223
Orlon. Du Pont announces new synthetic, —Orlon, 482(C)

Paint defies flame, 52(C)
Paper. Resin bonding—a new development in papermaking, 252(N)
Particle accelerators. 70-million volt synchrotron, 441(C)
Particle-size distribution. Analysis of the particle-size distribution of the grains producing a photographic image (Loveland, Menihan and Trivelli), 459
Pennsylvania history. The Conestoga wagon (Coulson), 215
Pennycress. Valuable oil from weed seed, 158(C)
Pettengill, George E: A check list of Franklin Imprints in the Library of The Franklin Institute, 351
Photography. Analysis of the particle-size distribution of the grains producing a photographic image (Loveland, Menihan, and Trivelli), 459
Photometry. Large photometric sphere, 61 (N)
Physical instruments. The effective path length through a vertical cosmic-ray telescope (Witmer and Pomerantz), 293
Plastics. Stronger than steel, 534(C)
  What's new in plastics, 310(C)
Plating tungsten alloys, 534(C)
Pomerantz, Martin A. and Witmer, Enos E.: The effective path length through a vertical cosmic-ray telescope, 293
Powder metallurgy. Carbonyl iron powders, 185(C)
Power. Developed water power of the world, 232(C)
Projector. Automatic projector, 532(C)
Protective coatings. Chromated protein films inhibit corrosion, 63(N)
Public lands in Colorado and Utah reserved for uranium drilling, 270(C)
Pulleys. Puri-Pulley magnetic pulley, 502 (C)

Radar. Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes (Swann), 149
Radiant heating by electricity, 534(C)
Radiations. Biological effects of high energy radiations, 360(C)
Radio-frequency mass spectrometer, 498(N)
Radio signals sent from rocket at 72-mile height, 362(C)
Radioactivity. Measurement of radioactive isotopes, 419(N)
  Mechanical hands, 458(C)
  Radioactive chemical precautions, 44(C)
Rare metals. Hafnium oxide, 450(C)
Rental-purchase plan. Induction heating equipment available without capital investment, 340(C)
Reproduction. Microfilm camera for unusually large copy, 482(C)
Resin bonding—a new development in papermaking, 252(N)

Rheostat. Liquid rheostat, 185(C)
Ritchie, Scott B.: Science in national security, 223
Rockets. Rocket motor test station, 183(C)
Rolling mills. X-ray gage controls sheet thickness, 44(C)
Ross, Mrs. Nellie Tayloe: The story of the new Benjamin Franklin half-dollar, 53
Rubber. Line shot blast room with rubber, 444(C)
Rüdenberg, Reinhold: Electron lenses of hyperbolic field structure, 311, 377
Rural electrification. From 19 per cent to 90 per cent, 361(C)

Safety engineering. Mechanical hands, 458 (C)
  Radioactive chemical precautions, 44(C)
Sampling. Eye-dropper technique proves effective method of testing steel, 310(C)
Schlieren effect. Electron optical Schlieren effect, 165(N)
Screens, wind-tunnel. The effect of screens in wind-tunnel wide-angle diffusers, 341 (N)
Seaton, S. L. and Gerson, N. C.: Generalized magneto-ionic theory, 483
Servomechanisms, carrier-frequency. Stabilization of carrier-frequency servomechanisms (Sobczyk). I. Gain-phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187.
Signalling. Radio signals sent from rocket at 72-mile height, 362(C)
Silver mirrors. A study of chemically deposited silver mirrors (Morgan and Myers), 363
Silver reclaiming kit, 439(C)
Slide projector. Automatic projector, 532(C)
Sobczyk, Andrew: Stabilization of carrier-frequency servomechanisms. I. Gain-phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier-phase shift, 187.
Soils. Measurement of the dielectric properties of soils and water at 3.2 cm. wave length (Straiton and Tolbert), 13
Sparks, electrical, and lightning discharge. The mechanism of lightning discharge (Loeb), 123
Spectrometry. A radio-frequency mass spectrometer, 498(N)
  New crystals for infrared spectrometry, 249(N)
Spencer, Domina Eberle and Moon, Parry: Languages for science, 1
Springs. Variable-resistance spring transducer, 423(N)
Stability analysis of a-c. systems. Stabilization of carrier-frequency servomechanisms (Sobczyk). I. Gain-phase margin diagrams of controller characteristics, 21. II. Design formulae for proportional-derivative networks, 95. III. Methods of obtaining required carrier phase-shift, 187.
Stainless steel. New technique facilitates brazing stainless steel parts, 92(C)
Stanley, Wendell Meredith: Recent advances in virus research, 445
Steam pipes for hottest turbine to run dull red, 440(C)
Steel. Eye-dropper technique proves effective method of testing steel, 310(C)
  Stronger than steel, 534(C)
Steroid hormones. Symposium on steroid hormones, 442(C)
Straiton, A. W. and Tolbert, C. W.: Measurement of the dielectric properties of soils and water at 3.2 cm. wave length, 13
Stress-corrosion tests of high-strength aluminum alloys, 344(N)
Stroboflow. The stroboflow, an oscilloscope for hydraulics, 183(C)
Swann, W. F. G.: Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes, 149
Synchroton. 70-million volt synchroton, 441 (C)
Synthetic language. Languages for science (Moon and Spencer), 1

Technical information. Special committee on technical information formed, 442(C)
Telemeter. Radio signals sent from rocket at 72-mile height, 362(C)
Telescope, cosmic-ray. The effective path length through a vertical cosmic-ray telescope (Witmer and Pomerantz), 293
Test stations. Rocket motor test station, 183(C)

Testing machines. A simple, accurate micro-hardness testing device, 531(C)
Textiles. Control of fading lamps with light-sensitive paper, 167(N)
  Elastic truths measure stretch in fabrics, 93(C)
  Electron microscopy of wool fibers by a shadowed replica method, 66(N)
Thallium halide crystals. New crystals for infrared spectrometry, 249(N)
Thrust augmentation. Increased uses of turbojet by augmentation, 122(C)
Titanium. Du Pont makes titanium, 533(C)
Tolbert, C. W. and Straiton, A. W.: Measurement of the dielectric properties of soils and water at 3.2 cm. wave length, 13
Transducer. Variable-resistance spring transducer, 423(N)
Transitor—a new amplifier, 438(C)
Transmission lines. Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes (Swann), 149
Transmitting tubes. Certain matters concerning scaling in the magnetron with special reference to the relative efficiency of magnetrons of different sizes (Swann), 149
Trivelli, A. P. H., Loveland, R. P., and Menihan, H. M.: Analysis of the particle-size distribution of the grains producing a photographic image, 459
Tungsten. Plating tungsten alloys, 534(C)
Turbines. Gas turbine for electric utility, 440(C)
  Quick starting house turbine-generator unit, 444(C)
  Steam pipes for hottest turbine to run dull red, 440(C)
Turbojet. Increased uses of turbojet by augmentation, 122(C)

Ultrasonics. Blind guidance by ultrasonics, 122(C)
United Nations Scientific Conference on the Conservation and Utilization of Resources (UNSCCUR), 272(C)
Universal language. Languages for science (Moon and Spencer), 1
Uranium. Public lands in Colorado and Utah reserved for uranium drilling, 270 (C)

Vibration. Bacteria and vibrations, 292(C)
Virus research. Recent advances in virus research (Stanley), 445
Viscous materials. Conductivity cell for viscous materials, 65(N)
Vision. Properties and use of eye-protective glasses, 250(N)
von Kármán, Theodore: Progress in aviation, 451

Water power. Developed water power of the world, 232(C)
Wave propagation. Measurement of the dielectric properties of soils and water at 3.2 cm. wave length (Straiton and Tolbert), 13
Weapon analysis. A theory of the internal ballistics of the "Hoch-und-Niederdruck Kanone" (Corner), 233
Wear. Laboratory for study of cylinder wear, 495(N)
Weather towers near completion at Brookhaven Laboratory, 269(C)
Weed oil. Valuable oil from weed seed, 158 (C)
Wheat clean-up, 214(C)
Wind machines. Frost control with air circulators, 502(C)
Wind tunnel. The effect of screens in wind-tunnel wide-angle diffusers, 341(N)
Witmer, Enos E. and Pomerantz, Martin A.: The effective path length through a vertical cosmic-ray telescope, 293
Wood Research Laboratory facilities enlarged, 270(C)
Wool fibers. Electron microscopy of wool fibers by a shadowed replica method, 66(N)
Wyoming-type bentonite. Exposure to air improves some Wyoming-type bentonite, 443(C)

X-ray gage controls sheet thickness, 44(C)
  Biological effects of high energy radiations, 360(C)

Zeeman effect of neutral nitrogen and oxygen, 169(N)

I.A.R.I. 75

INDIAN AGRICULTURAL RESEARCH
INSTITUTE LIBRARY, NEW DELHI.

Date of Issue	Date of Issue	Date of Issue
1 SEP 1993		

GIPNLK—H-40 I.A.R.I.—29-4-55—15,000